Bayesian design of plasma diagnostics

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A framework for diagnostics design based on Bayesian probability theory is introduced. The design is assessed in figures of an expected utility which measures the information gain from measurements. The approach is applied on design studies for a multichord interferometer for Wendelstein 7-X. © *2006 American Institute of Physics.* [DOI: 10.1063/1.2336457]

I. DESIGN OF PLASMA DIAGNOSTICS

The design of plasma diagnostics is a typical task to be resolved along the preparation of fusion experiments. Requirements for a design process are (a) highest accuracy of measurements, (b) high resolution $(t, \mathbf{x})$, (c) robustness, and (d) extensibility. In addition, the design process has to meet with constraints such as accessibility or economic restrictions.

A typical design process consists of a number of considerations based on typical use cases or—synonymously—set of assumptions on the expected physical scenarios and possibilities for the technical realization of the measurement. In a real experiment, however, the data produced by the diagnostics are ultimately employed for the resolution of physical issues, or in other words the quantitative assessment of hypotheses.

One can conclude that—as the analysis of data—the design of plasma diagnostics is a complicated task which includes all aspects of plasma physics and the analysis of noisy and often incomplete data. But the design task exhibits two more challenges, that is, (1) one has to deal with *expected data* of a future experiment and (2) one needs to quantify the benefit of a design consideration. The latter point allows one for design decisions (e.g., is it justified to add an additional laser beam in a multichord interferometer).

Consequently, the design process is generally considered to be an intricate problem and many (correct) design decisions are based on experience from former setups. Since the overall process requires the inclusion of many different aspects, approaches to quantify the *expected utility* of a diagnostics are beneficial for design considerations.

This article summarizes a framework which addresses the issues raised and provides a mathematical formulation to support the design process. Results for an application on design considerations of the interferometer setup for Wendelstein 7-X are shown in (Ref. 1).

II. DESIGN STRATEGY

Figure 1 shows a simplified workflow of the overall design procedure. The essential part is the optimization of the expected utility of the setup with respect to design parameters (e.g., the chord position). In order to formulate the design procedure as an optimization problem, the quantities to be measured are to be specified. Beyond crude estimates of ranges, it is intended to employ predictive modeling for Wendelstein 7-X (W7-X) specific physical issues, e.g., confinement studies. This philosophy is reflected by the choice of "start-up" diagnostics for Wendelstein 7-X which is oriented to resolve physical issues of stellarator optimization.[2]

In assistance, the robustness of the results is assessed by any available additional information, e.g., global scaling laws in the case of confinement studies. The resulting expected quantities are fed to a virtual instrument which implements the forward model of the measurement and—an essential part—describes the error statistics (likelihood). For the range of expected data, the optimization procedure may also include a weighting with respect to the quantity ranges of interest. Technical and physical considerations enter the process at different stages. For a proof of results the analysis of data from the virtual instrument is part of the design strategy.

III. BAYESIAN EXPERIMENTAL DESIGN

Lindley[3] proposed an approach based on decision theory to design. It begins with the choice of an appropriate utility function reflecting purpose and costs of an experiment. For the quantification of the *utility* of a setup, the Kullback-Leibler distance U_{KL} measures the information gain from the ignorance on a quantity φ before a measurement to the knowledge after data D are taken,

$$U_{\mathrm{KL}}(D,\eta) = \int d\varphi P(\varphi|D,\eta) \log \frac{P(\varphi|D,\eta)}{P(\varphi)}. \qquad (1)$$

It is measured in bits, if the base-2 logarithm is used.

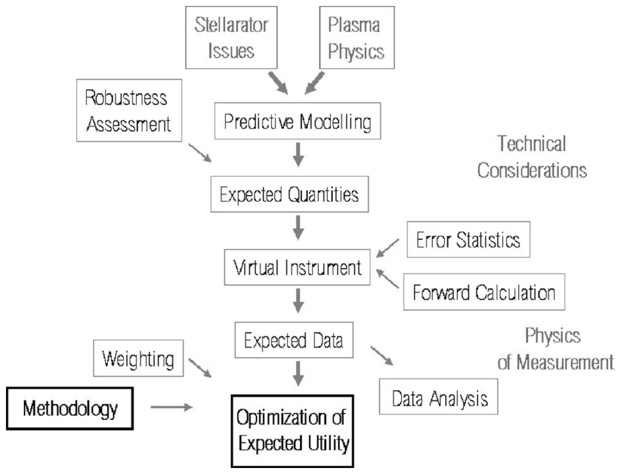

FIG. 1. Elements of the design procedure.

Uncertainties are encoded as probability density functions P. The conditional probability $P(\varphi|D)$ means the probability that φ is true given the data D. Please note, that U_{KL} depends both on the data D and the design parameters η which are the optimization parameters.

An integration over the range of expected data, where the evidence of the data is represented by the probability density function $P(D|\eta)$, yields the expected utility function EU,

$$\mathrm{EU}(\eta) = \int dDP(D|\eta)U_{\mathrm{KL}}(D,\eta), \tag{2}$$

which is a function of the design parameters η only. It is a measure of the mean information gain from the data, averaged over the expected data space. The principle of Bayesian diagnostic design is the maximization of the EU with respect to η.

Using Bayes theorem,

$$P(\varphi|D,\eta) = \frac{P(D|\varphi,\eta)\cdot P(\varphi)}{P(D|\eta)}, \tag{3}$$

the EU is given by

$$\mathrm{EU}(\eta) = \int\int dDd\varphi P(D|\varphi,\eta)P(\varphi)\log\frac{P(D|\varphi,\eta)}{P(D|\eta)}. \tag{4}$$

This formulation uses the likelihood $P(D|\varphi,\eta)$ and a probability density function $P(\varphi)$ which reflects the range of interest and weighting for φ. The likelihood can be regarded as a representation of a diagnostics model. It contains the forward calculation and the error statistics of the forward model parameters. The expected utility is now a function of the design parameters η only and is subject of optimization studies.[4]

IV. OPTIMIZATION STUDIES FOR THE W7-X INTERFEROMETER

The error statistics of the measurement has crucial impact on the expected utility.[5] For the examples presented here, a constant error is chosen which is about a few percent, depending on the actual data value. For the creation of virtual data, a parametrized density function is used,

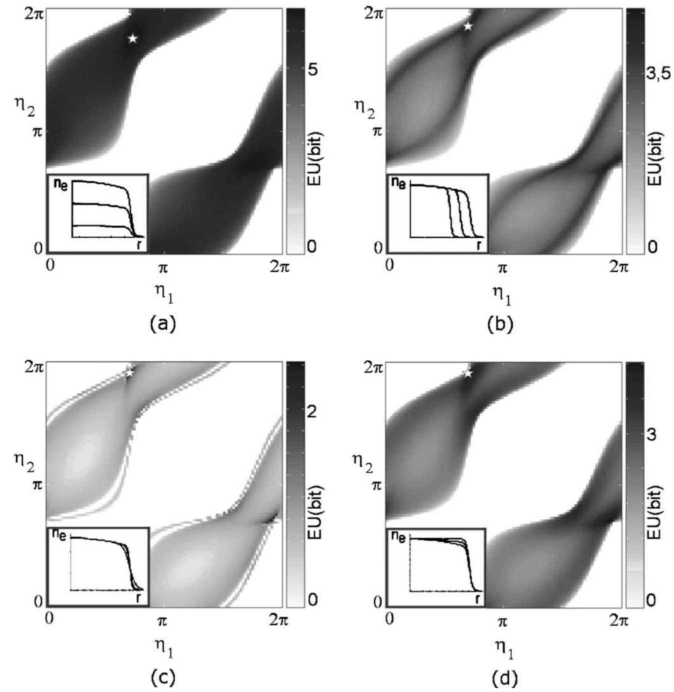

FIG. 2. Expected Utility for estimation of maximum density (a), gradient position (b), steepness (c), and bulge (d). The data space is generated by a variation of (a) $\varphi_1=0,\ldots,5\times10^{20}\ \mathrm{m}^{-3}$, (b) $\varphi_2=0.6,\ldots,0.95$, (c) $\varphi_3=1,\ldots,30$, and (d) $\varphi_4=-1,\ldots,0$. The star symbol marks the maximum of the EU. The insets show the corresponding density profile variation where the maximum ordinate is $n_e=1\times10^{20}\ \mathrm{m}^{-3}$ and the abscissas are effective radii ($r_{\mathrm{eff}}/r_{\mathrm{max}}$).

$$n_e(\varphi_1,\varphi_2) = \varphi_1\frac{1+\varphi_4(r/r_{\mathrm{max}})^2}{1+[(r/r_{\mathrm{max}})^2/\varphi_2^2]^{\varphi_3}}. \tag{5}$$

The parameters $\varphi_1,\ldots,\varphi_4$ represent the maximum density, position of the edge gradient, steepness, and bulge of the density distribution.

Figure 2 shows the expected utility for a single beam, where the optimization target is the reconstruction of each parameter φ_i. The result is displayed in figures of two angles θ_1 and θ_2 which represent the starting and end point of a chord on a circumventing circle, as indicated in Fig. 3.

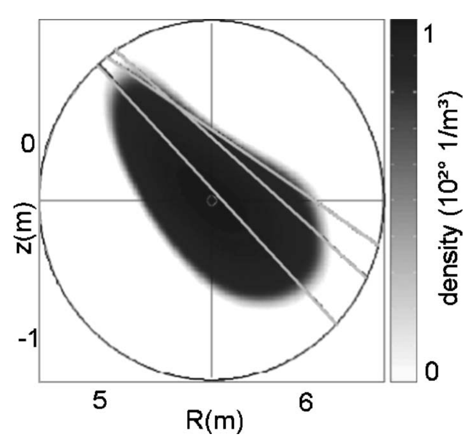

FIG. 3. Density distribution in the toroidal plane for the W7-X interferometer. The chords shown represent the optimum reconstruction of data (cf. Fig. 2). The center beam corresponds to Fig. 2(a), the outmost beam to Figs. 2(c) and 2(d).

The results indicate the different impacts of shaping. Coarsely, the chord represents the maximum signal-to-noise ratio (SNR) chord for the respective parameter. For the maximum [Fig. 2(a)] a beam traversing the plasma center yields best SNR. Since the shaping effects of the other parameters are most distinct at the plasma edge, the resultant reconstruction has maximum information gain for sightlines traversing the edge region. The parameter steepness and bulge have nearly coinciding optimum chords as to be seen in Fig. 3 where the outmost beam is degenerated (bulge and steepness).

The benefit of this approach is a quantification of the design quality which also allows the diagnostician to estimate the robustness of a design chosen.

Technical constraints arise due to restricted access to the plasma vessel through the ports. Figure 4 indicates accessible chords in figures of the parametrization chosen. The effect of the technical restrictions can be quantified and compared to ideal access. A translation of the expected utility to measurement uncertainties is straightforward but depends on the forward function of the virtual instrument.

V. DISCUSSION

Bayesian diagnostics design is applied on plasma diagnostics. The design allows for a quantification of design considerations and estimates for their robustness. The reconstruction of density profiles by means of a multichannel infrared interferometer at Wendelstein 7-X shows how measuring capabilities can be detected and complicated entanglements of measurement and geometry revealed. The impact of technical boundary conditions can be quantified as well as the information gain by the inclusion of additional chords.

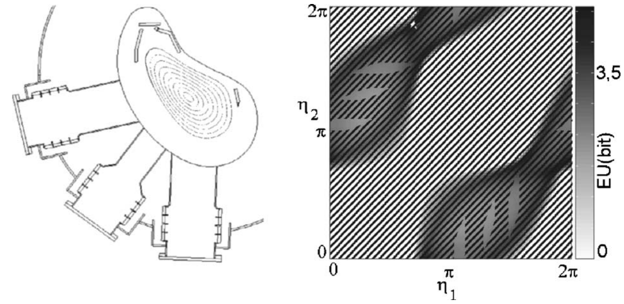

FIG. 4. Restricted access to the plasma due to port system (left), accessible chord positions in EU plot for the gradient position case (lightened areas, right).

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