The Sun

Part 2

Luc-Fabrice Tremel | 16.02.2024

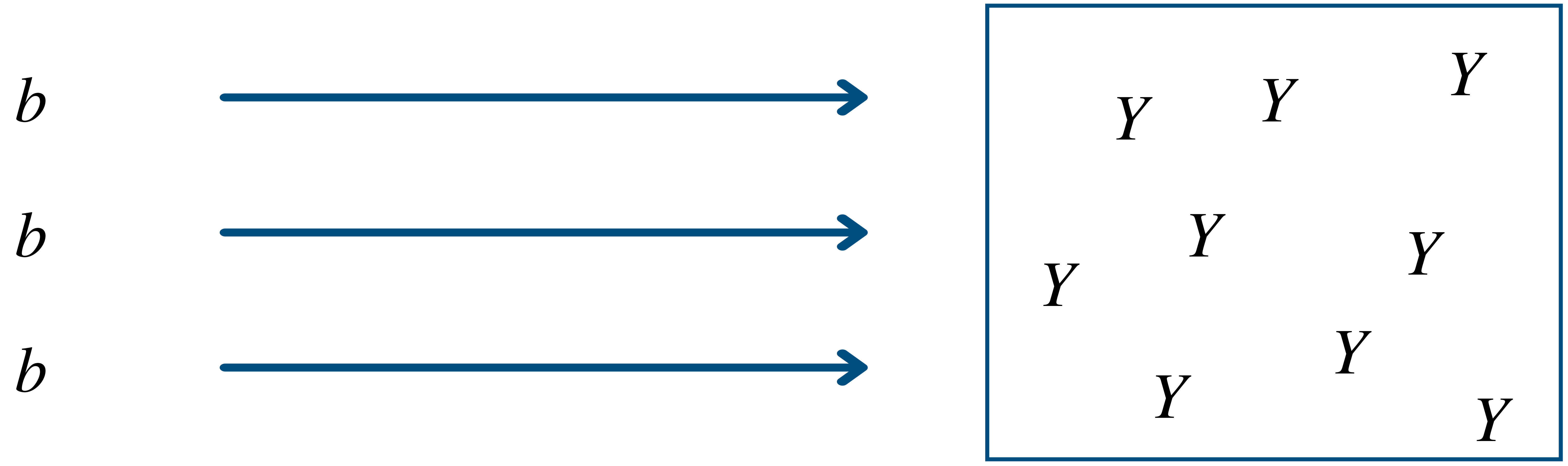
Table of Contents

Fusion in the Sun

Fusion in the Sun

Cross Sections and Nuclear Reaction Rates

Reaction: $b + Y \rightarrow P + d$



Def. Cross section: $\sigma(v) = \dfrac{\text{number of reactions per particle } Y \text{ per unit time}}{\text{number of incident particles } b \text{ per unit area per unit time}}$

Fusion in the Sun

Cross Sections and Nuclear Reaction Rates

Number of reactions per unit time per unit volume: $r_{bY} \propto N_b N_Y \sigma(v) v$

Def. Reaction rate: $r_{bY} \propto N_b N_Y \int_0^\infty \sigma(v) v \phi(v)\, dv = N_b N_Y \langle \sigma v \rangle_{bY}$

N_b, N_Y = particle number density; $\phi(v)$ = velocity distribution function

Fusion in the Sun

Cross Sections and Nuclear Reaction Rates

Solar conditions ⇒ Maxwell-distribution: $\phi(v) = 4\pi v^2 \left(\frac{\mu}{2\pi k_B T}\right)^{3/2} \exp\left(\frac{-\mu v^2}{2k_B T}\right) dv$

Reaction rate per particle pairs: $\langle \sigma v \rangle_{bY} = \sqrt{\frac{8}{\pi\mu(k_B T)^3}} \int_0^\infty \sigma(E) \exp\left(\frac{-E}{k_B T}\right) E \, dE$

E = relative kinetic energy; $\mu = \frac{m_b m_Y}{m_b + m_Y}$ = reduced mass

Fusion in the Sun

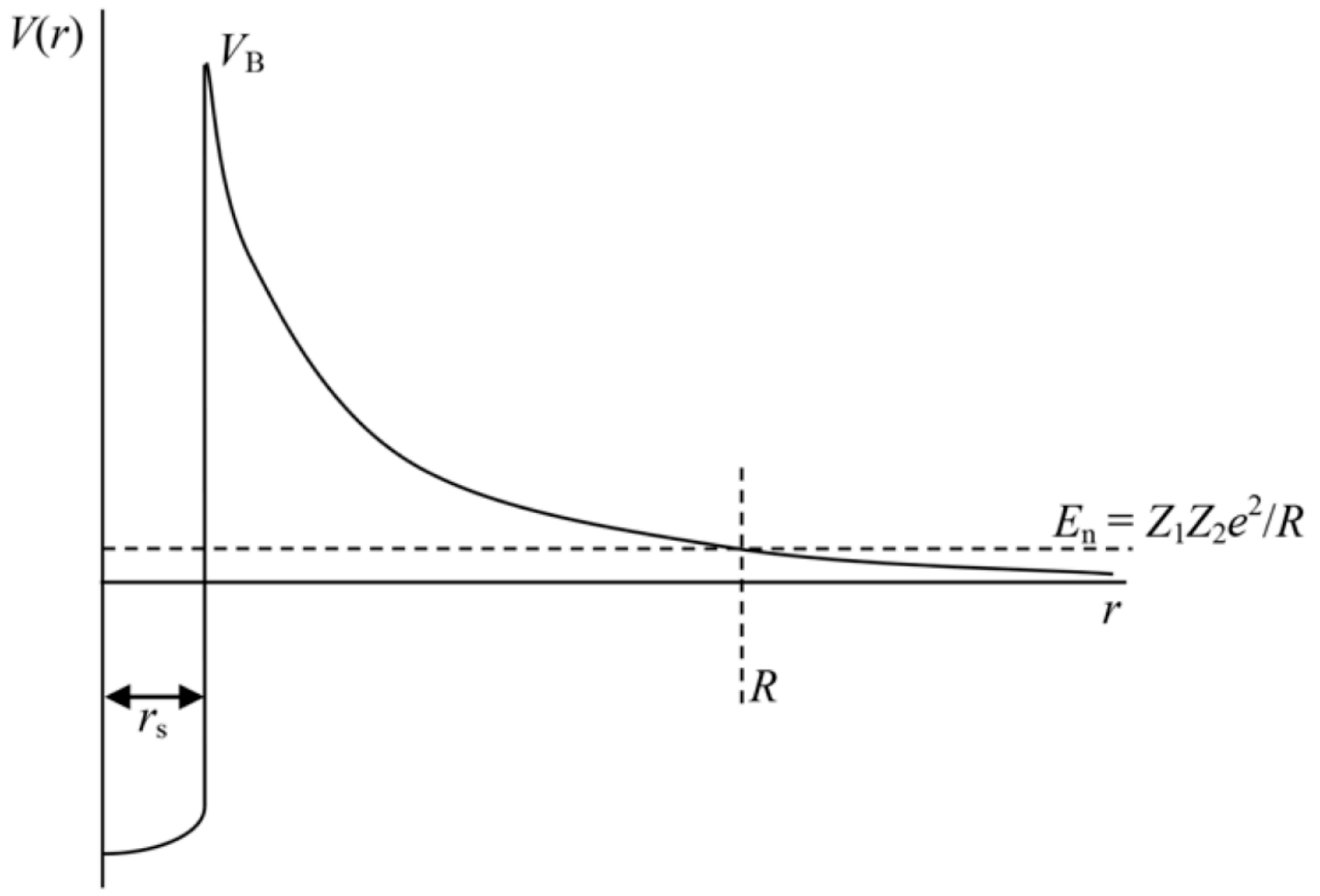
Cross Sections and Nuclear Reaction Rates

Gamow factor: $\sigma \propto \exp(-2\pi\eta(E))$

Geometrical factor: $\sigma \propto (\Delta r)^2 \propto \frac{1}{E}$

$$\eta(E) = \frac{Z_1 Z_2 \alpha}{v(E)}$$ = Sommerfeld parameter

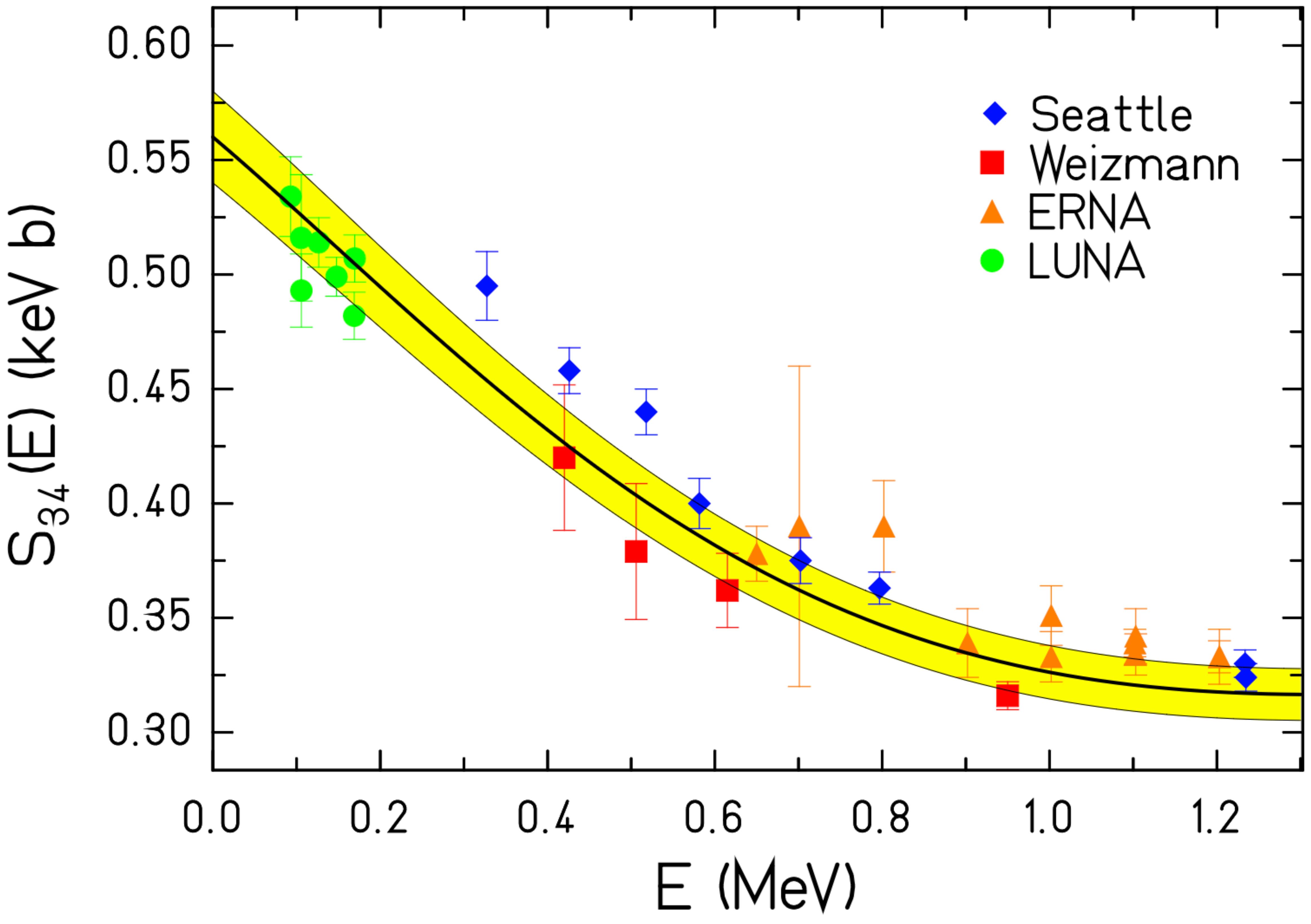
$$\Rightarrow \sigma(E) = \frac{S(E)}{E} \exp(-2\pi\eta(E))$$



James MacDonald. Structure and Evolution of Single Stars: An introduction. Morgan Claypool Publishers, November 2015.

Fusion in the Sun

Cross Sections and Nuclear Reaction Rates



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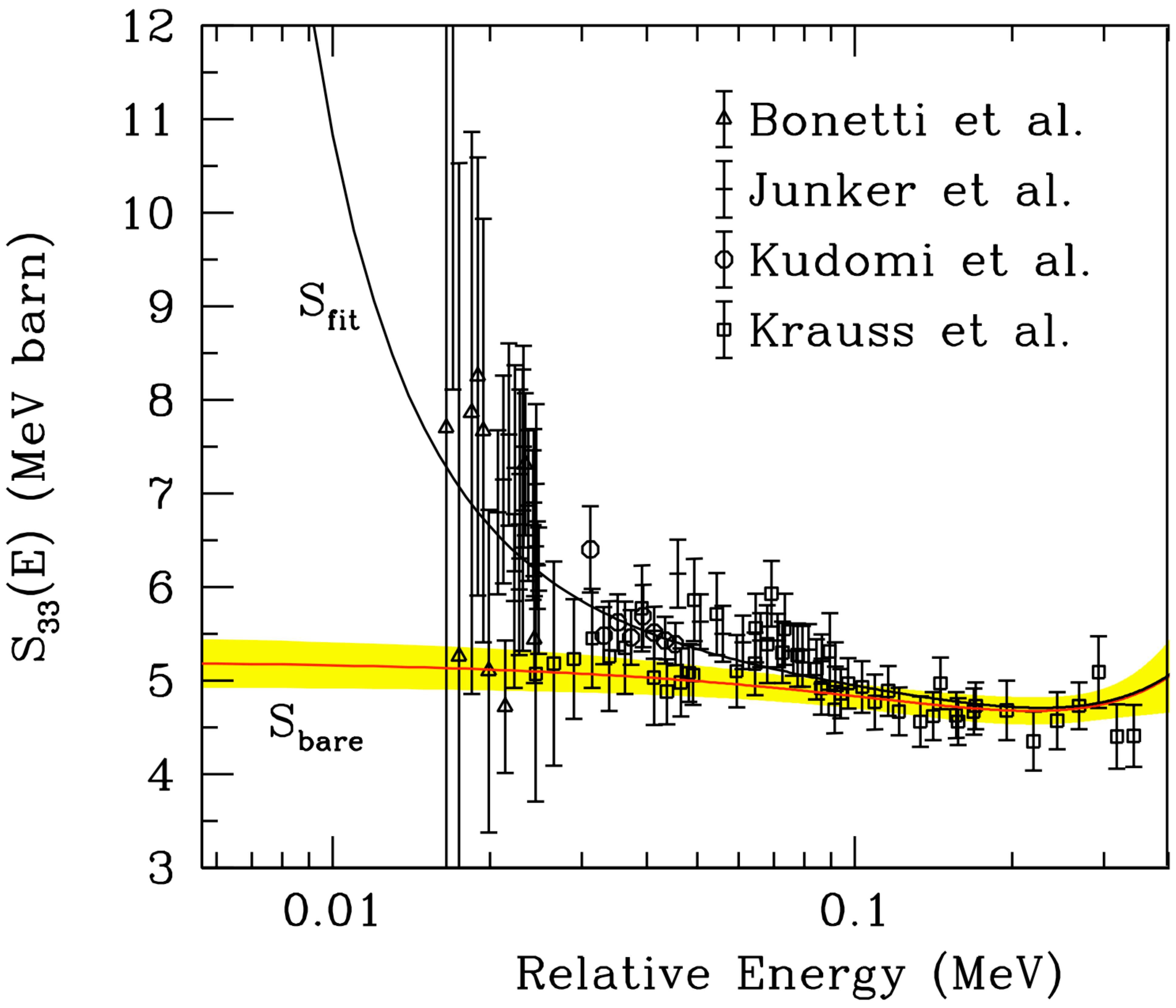

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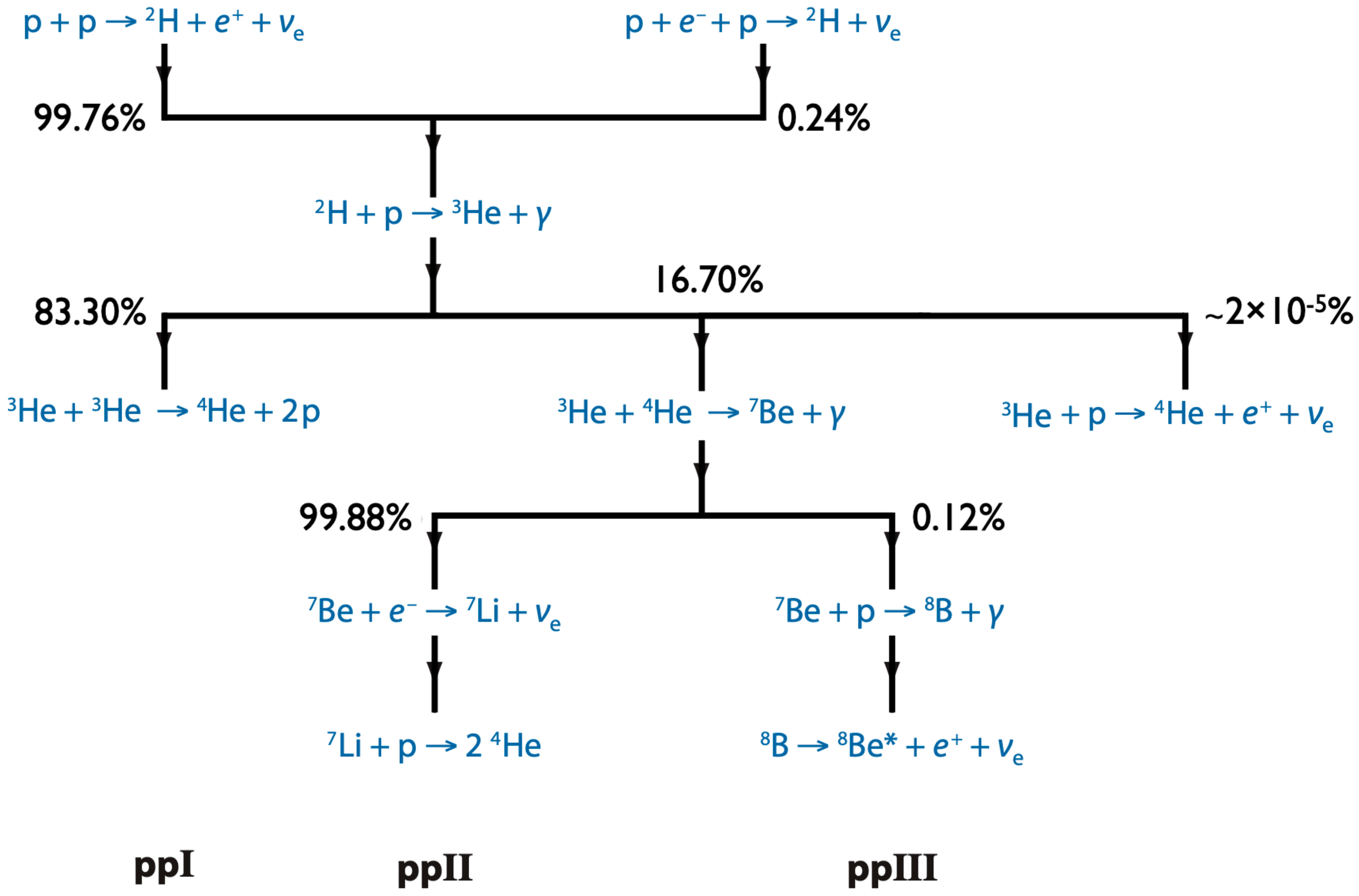
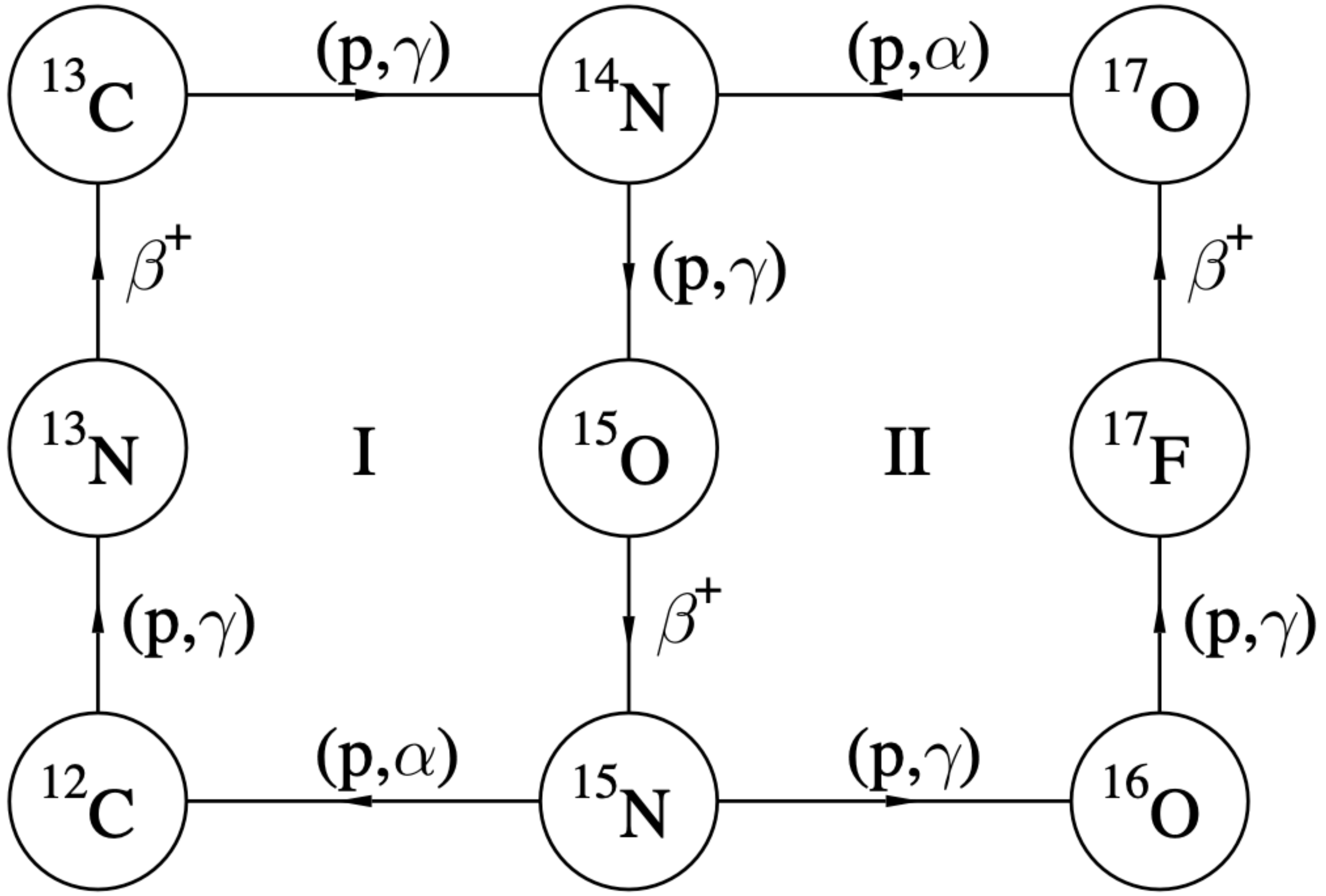
Fusion in the Sun

The PP Chain and CNO bi-Clycle





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Fusion in the Sun

The PP Chain and CNO bi-Clycle

- Energy released per ^{4}He-nucleus: $\approx 26 \text{ MeV}$
- Total radiation of the sun: $\approx 4 \times 10^{26} \text{ W}$

$\Rightarrow \approx 4 \times 10^{38} \text{ protons/s} \;\Rightarrow \approx 669 \times 10^6 \text{ t}/s$

- Total mass of the sun: $\approx 2 \times 10^{30} \text{ kg}$

$\Rightarrow$ lifespan of about 50 bn. years

ppI

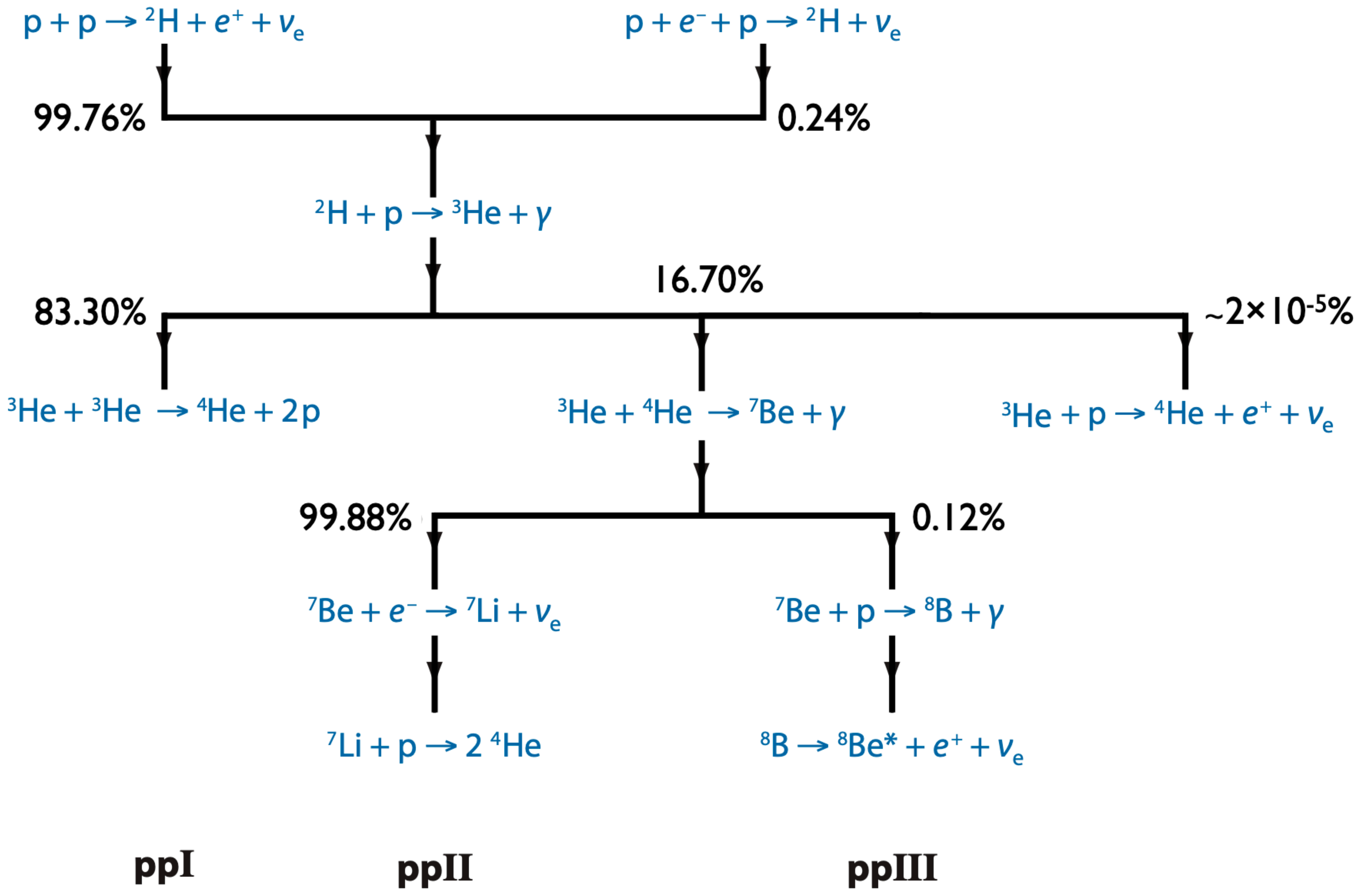
$$p + p \;\rightarrow\; {}^2\text{H} + e^- + \nu_e$$

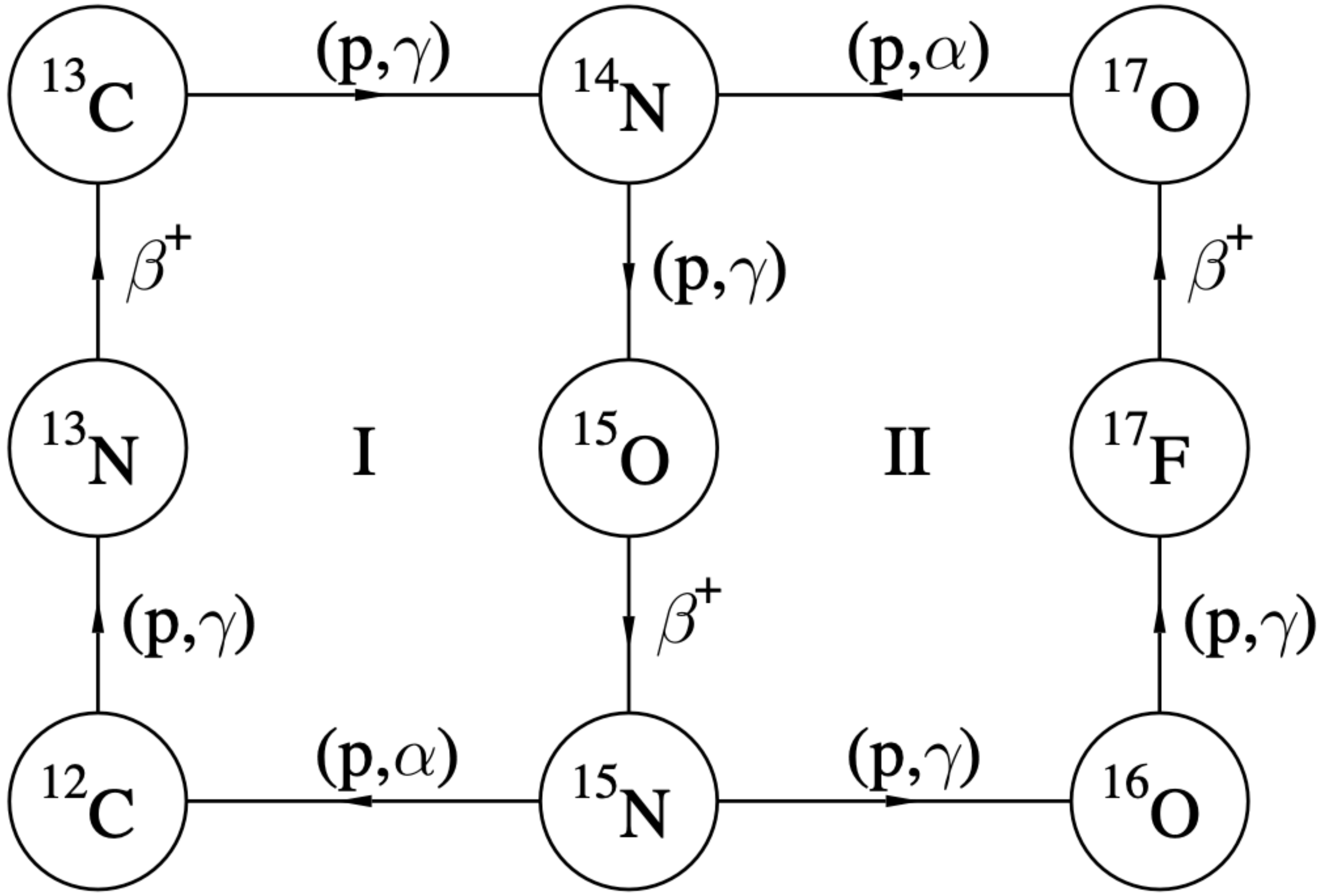
$${}^2\text{H} + p \;\rightarrow\; {}^3\text{He} + \gamma$$

$${}^3\text{He} + {}^3\text{He} \;\rightarrow\; {}^4\text{He} + 2p$$

Fusion in the Sun

The PP Chain and CNO bi-Clycle





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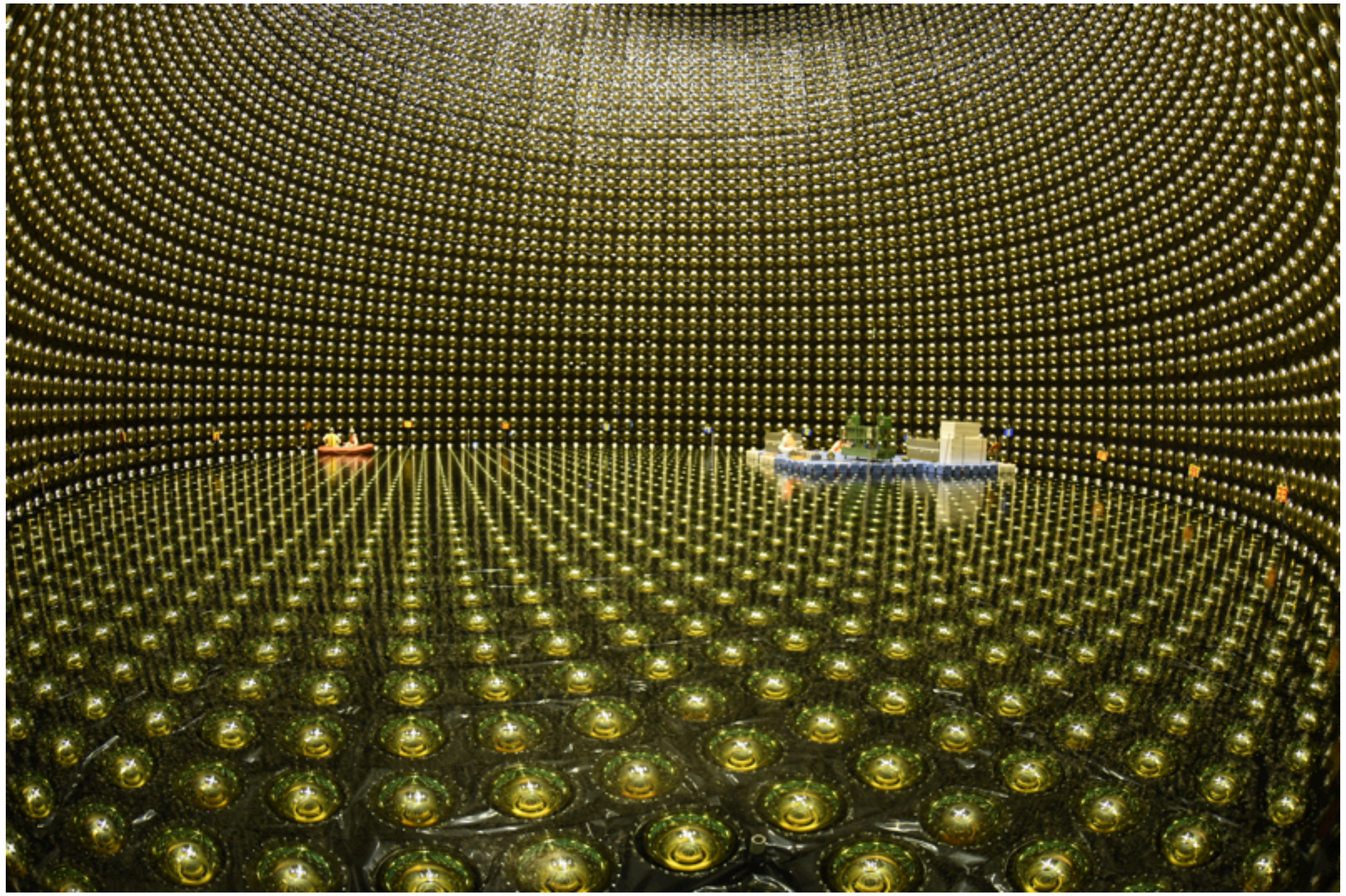
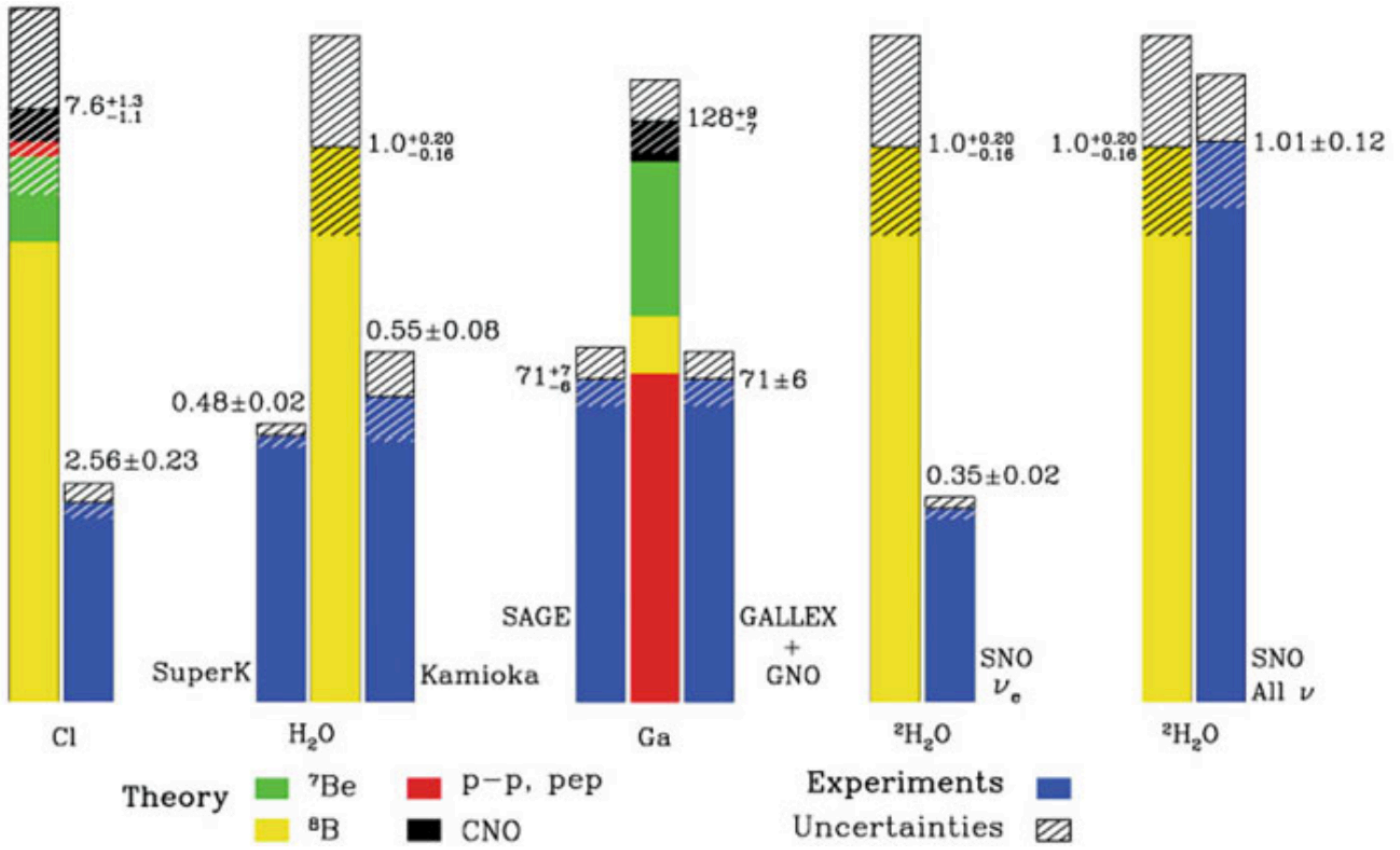
Fusion in the Sun

Solar Neutrinos



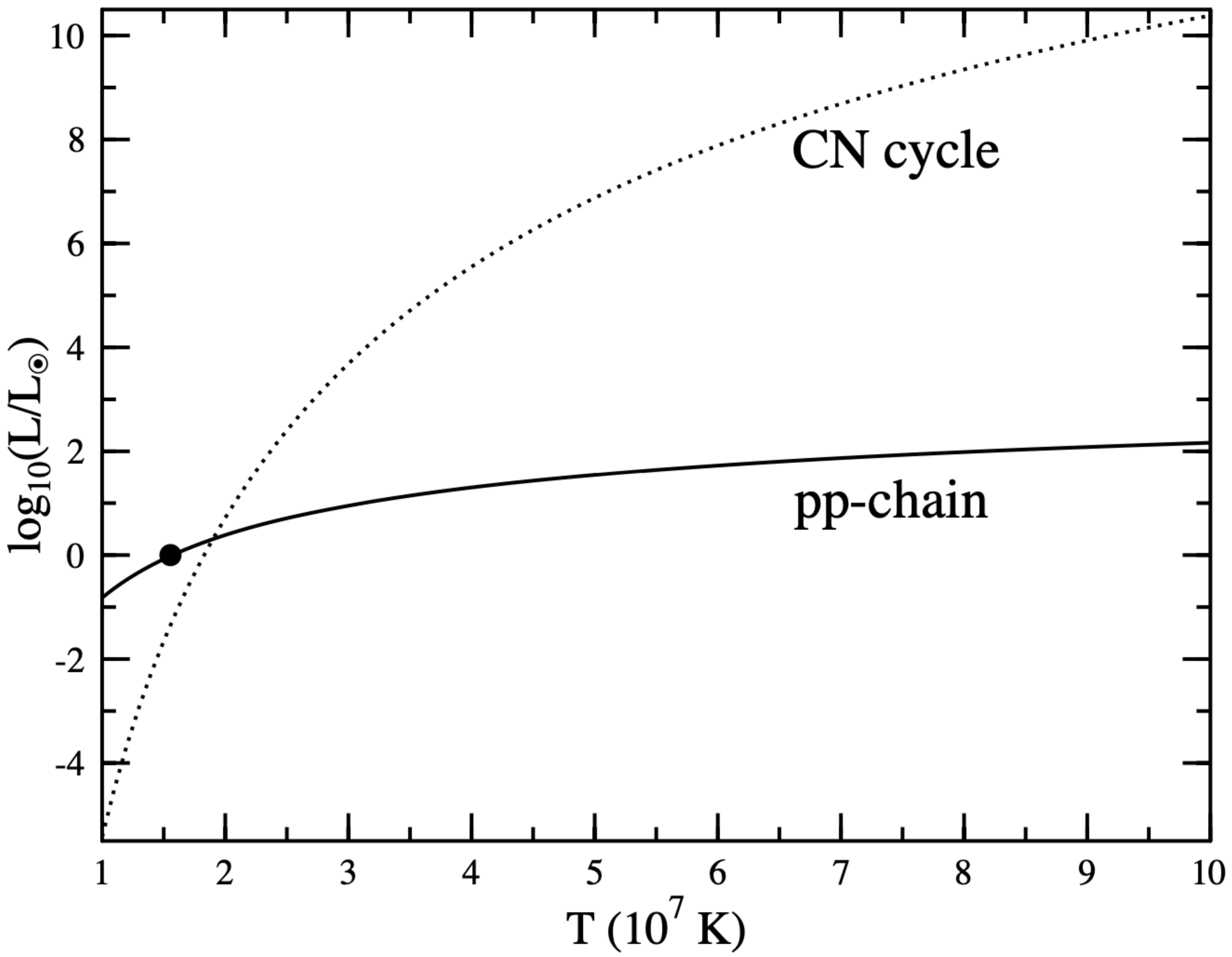
Kamioka Observatory, ICRR (Institute for Cosmic Ray Research), The University of Tokyo



Karl-Heinz Spatschek. Astrophysik. Springer Berlin Heidelberg, 2021.

Fusion in the Sun

Energy Production under Different Conditions



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The Evolution of Stars

The Evolution of Stars

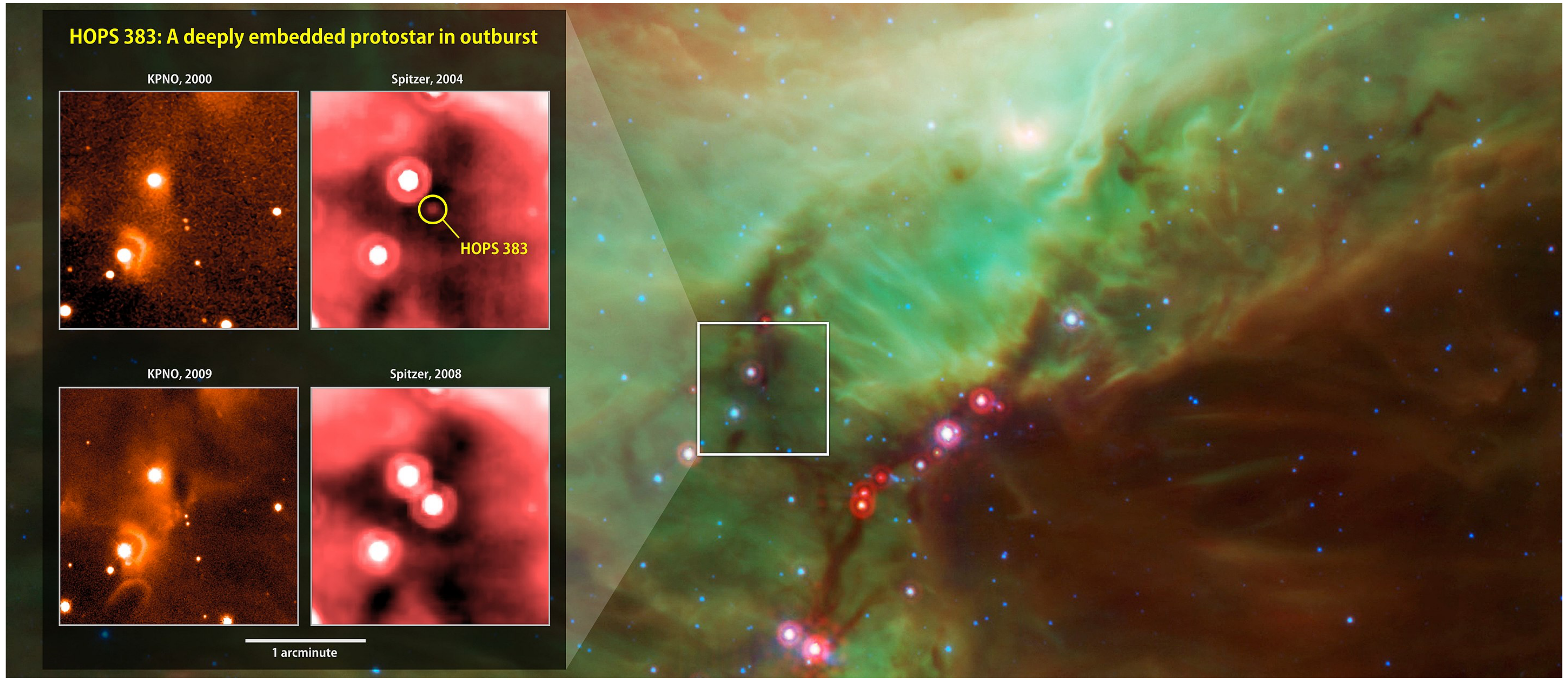
The Formation of Protostars

Jeans mass:

$$M_J \approx 4 \times 10^5 \left(\frac{T}{100K}\right)^{3/2} \frac{M_\odot}{\sqrt{n}}$$

Characteristic time scale:

$$\tau_J \approx 10^8 \frac{1}{\sqrt{n}} \text{ years}$$



NASA/JPL-Caltech/Univ. of Toledo; background: E. Safron et al. http://photojournal.jpl.nasa.gov/figures/PIA18928_fig1.jpg

ISM phase	n (atoms cm^{-3})	T (K)	Filling factor
GMCs	10^2–10^6	10–100	0.01
Warm neutral medium	0.2–0.5	6×10^3–10^4	0.1–0.2
Warm ionized medium	0.2–0.5	8×10^3	0.2–0.5
Hot ionized medium	10^{-4}–10^{-2}	10^6–10^7	0.3–0.7

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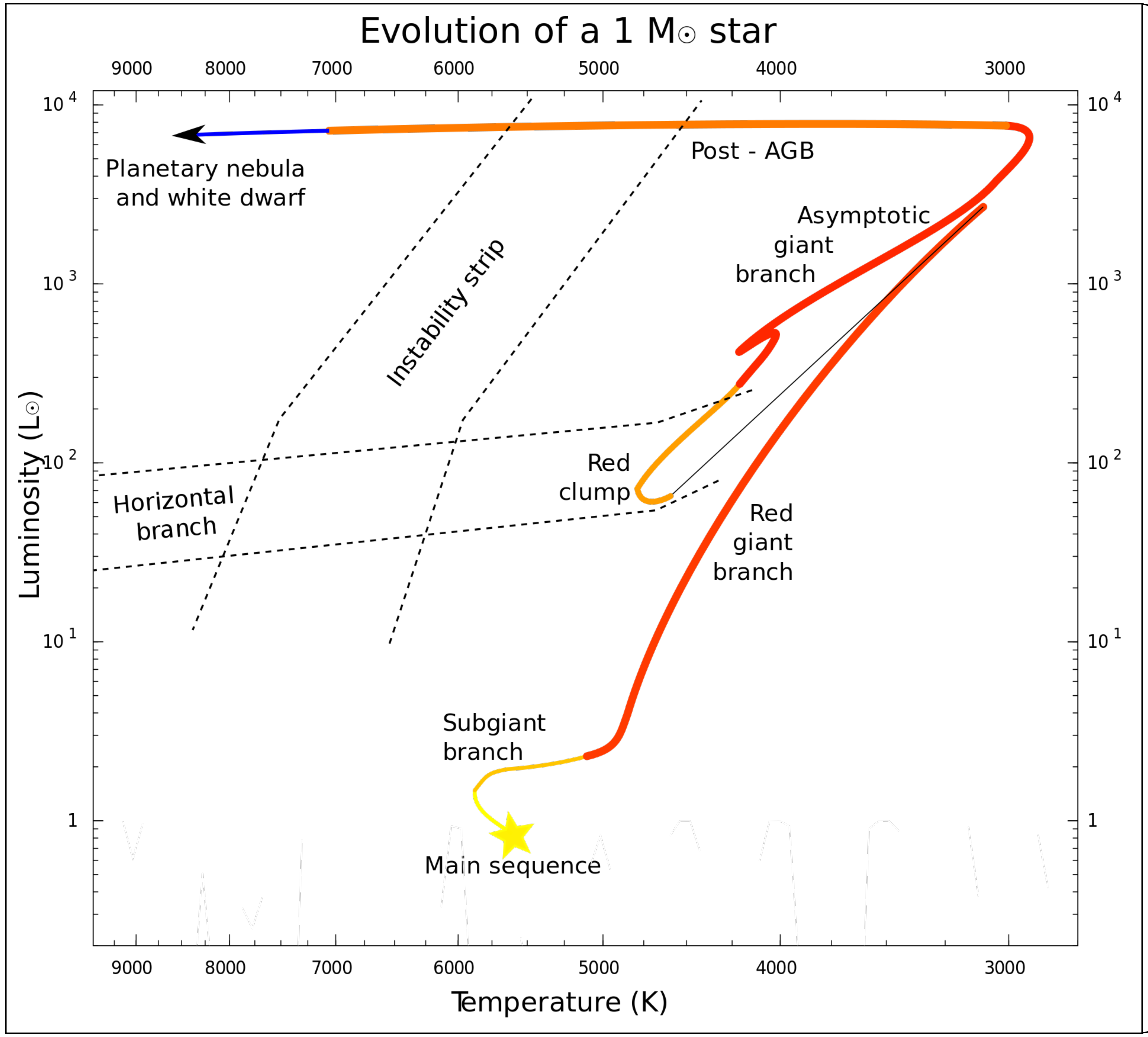
The Evolution of Stars

Hertzsprung-Russel Diagram



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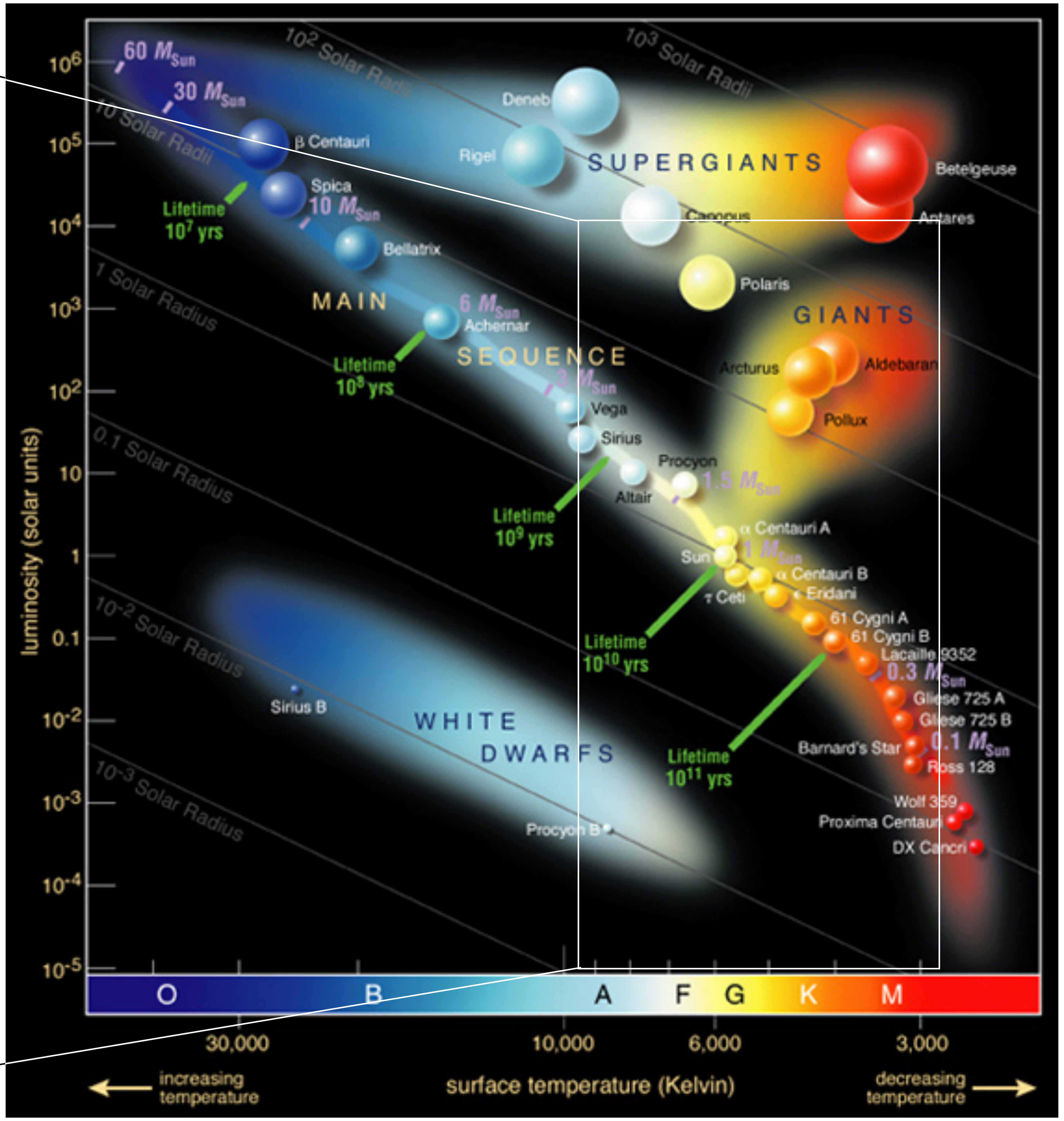
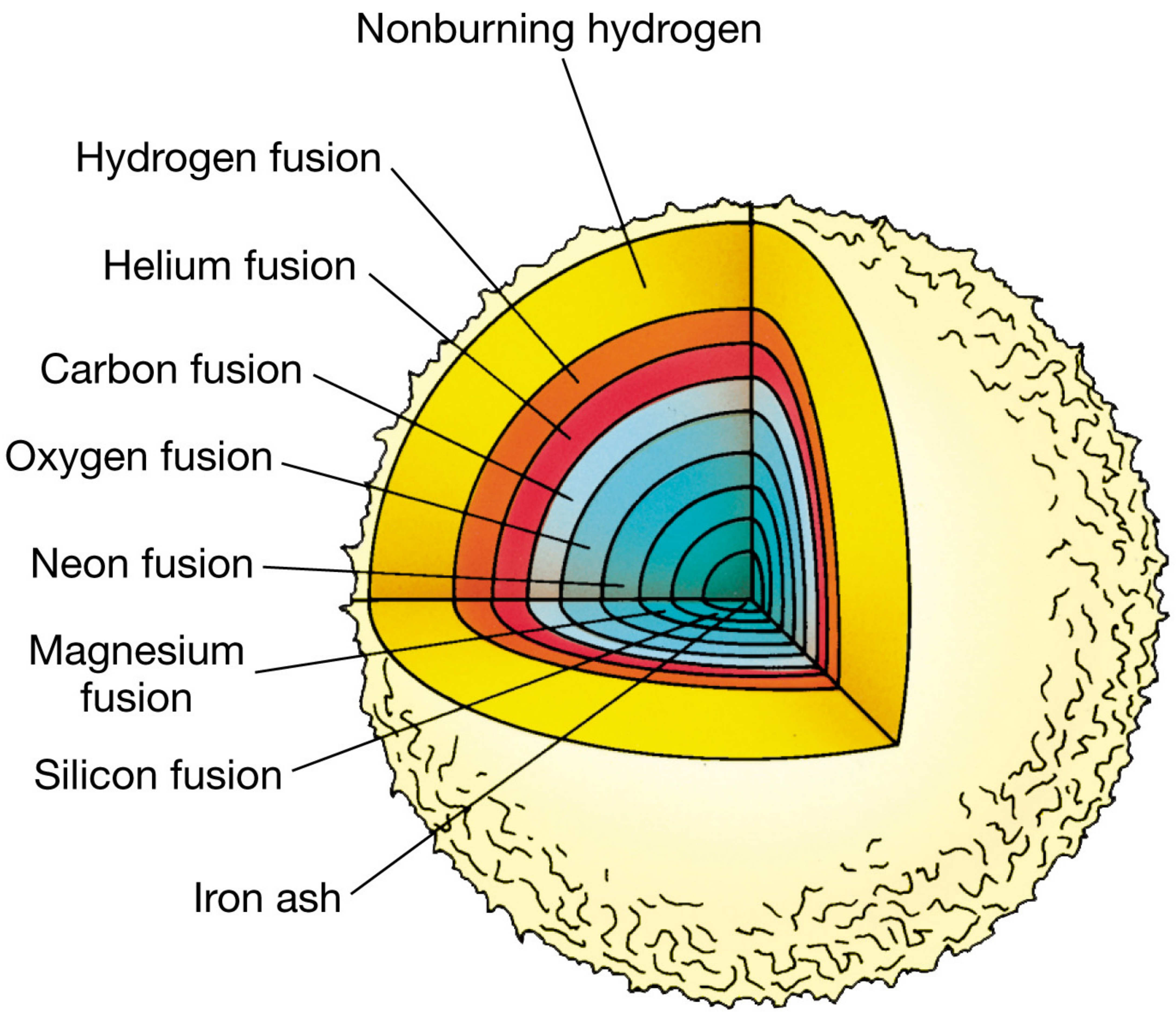


ESO - CESAR. The Hertzsprung diagram where the evolution of sun-like stars is traced. https://www.cosmos.esa.int/web/cesar/the-hertzsprung-russell-diagram

The Evolution of Stars

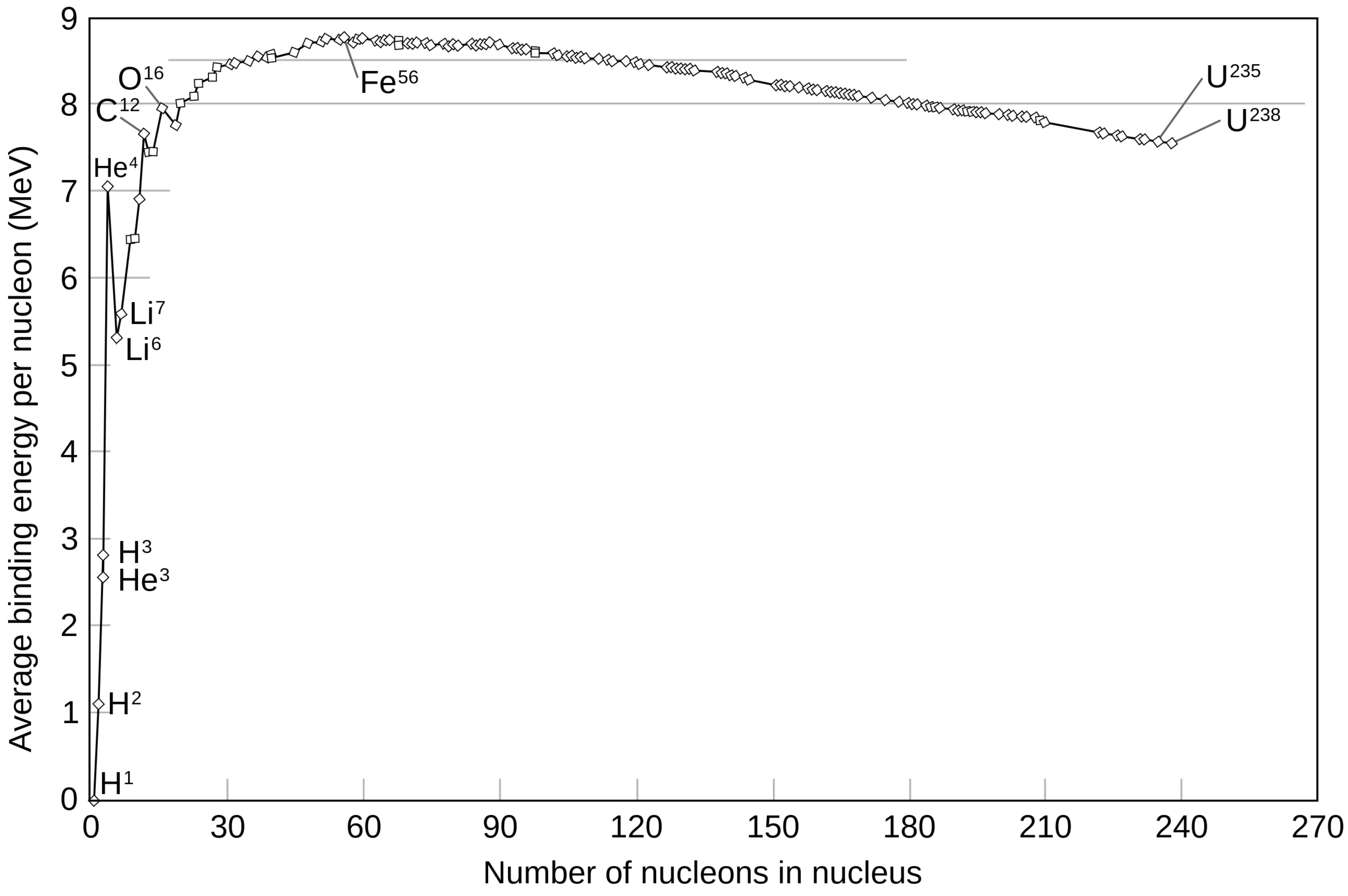
Hertzsprung-Russel Diagram



ESO - CESAR. The most metallic layers are in the most internal part of the stars. https://www.cosmos.esa.int/web/cesar/the-hertzsprung-russell-diagram

The Evolution of Stars

Hertzsprung-Russel Diagram



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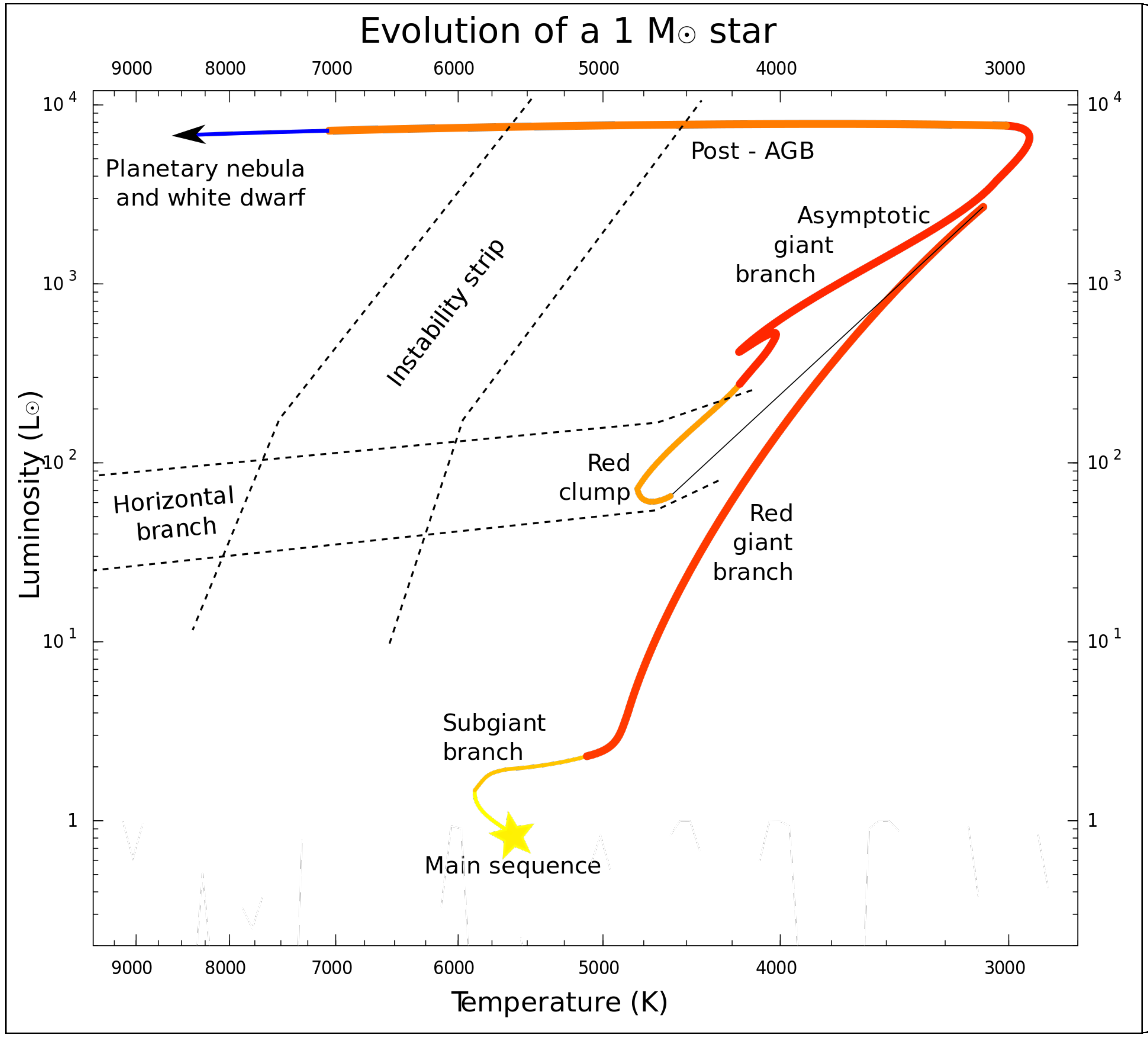
The Evolution of Stars

Hertzsprung-Russel Diagram



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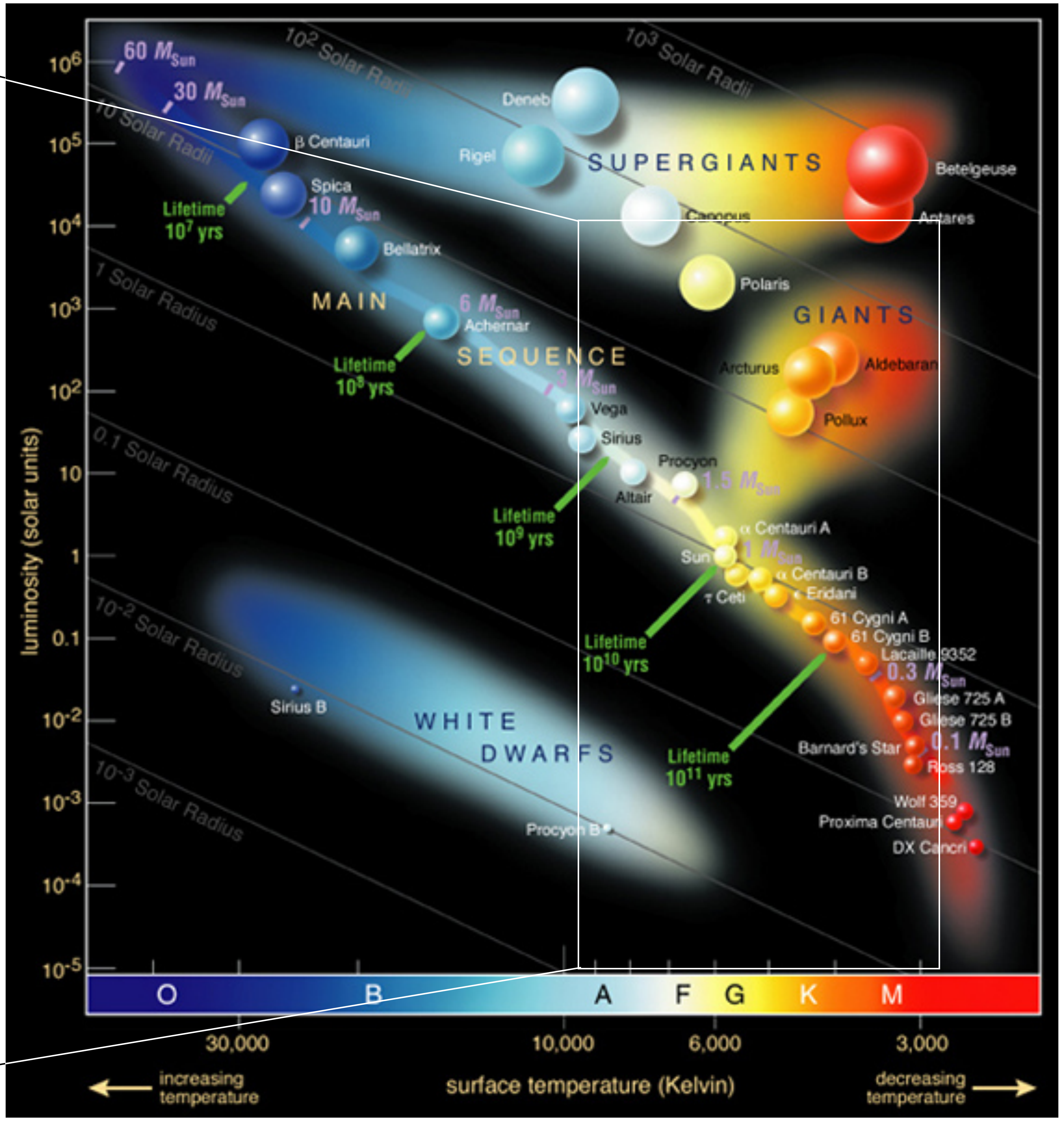

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NASA, ESA, CSA, STScI. Cassiopeia A (NIRCam). https://www.nasa.gov/missions/webb/nasas-webb-stuns-with-new-high-definition-look-at-exploded-star/

The Evolution of Stars

Neutron Stars

Chandrasekhar limit $\approx 1.4 M_\odot < M <$ Oppenheimer-Volkoff limit $\approx 2 - 2.17 M_\odot$

inverse β-decay: $p + e^- \rightarrow n + \nu_e$

ESO. First Image of a Black Hole. https://www.eso.org/public/images/eso1907a/

Degenerate Plasmas

Degenerate Plasmas

The Degeneracy Limit

A plasma is degenerate, if $T \approx E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$

Debye length $\lambda_D = \left(\frac{\varepsilon_0 T}{n_0 e^2}\right)^{1/2} \quad \rightarrow \quad \lambda_F = \left(\frac{2\varepsilon_0 E_F}{3n_0 e^2}\right)^{1/2}$ Thomas-Fermi length

Summary

Summary

1. Fusion in the Sun
    - astrophysical S-factor allows measurements of the cross sections
    - pp chain main energy source of the sun
    - solar neutrinos allow view directly into the core of the sun
2. The Evolution of Stars
    - the sun will become a red giant and end as a white dwarf
3. Degenerate Plasmas
    - quantum mechanics can’t be neglected at high densities

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