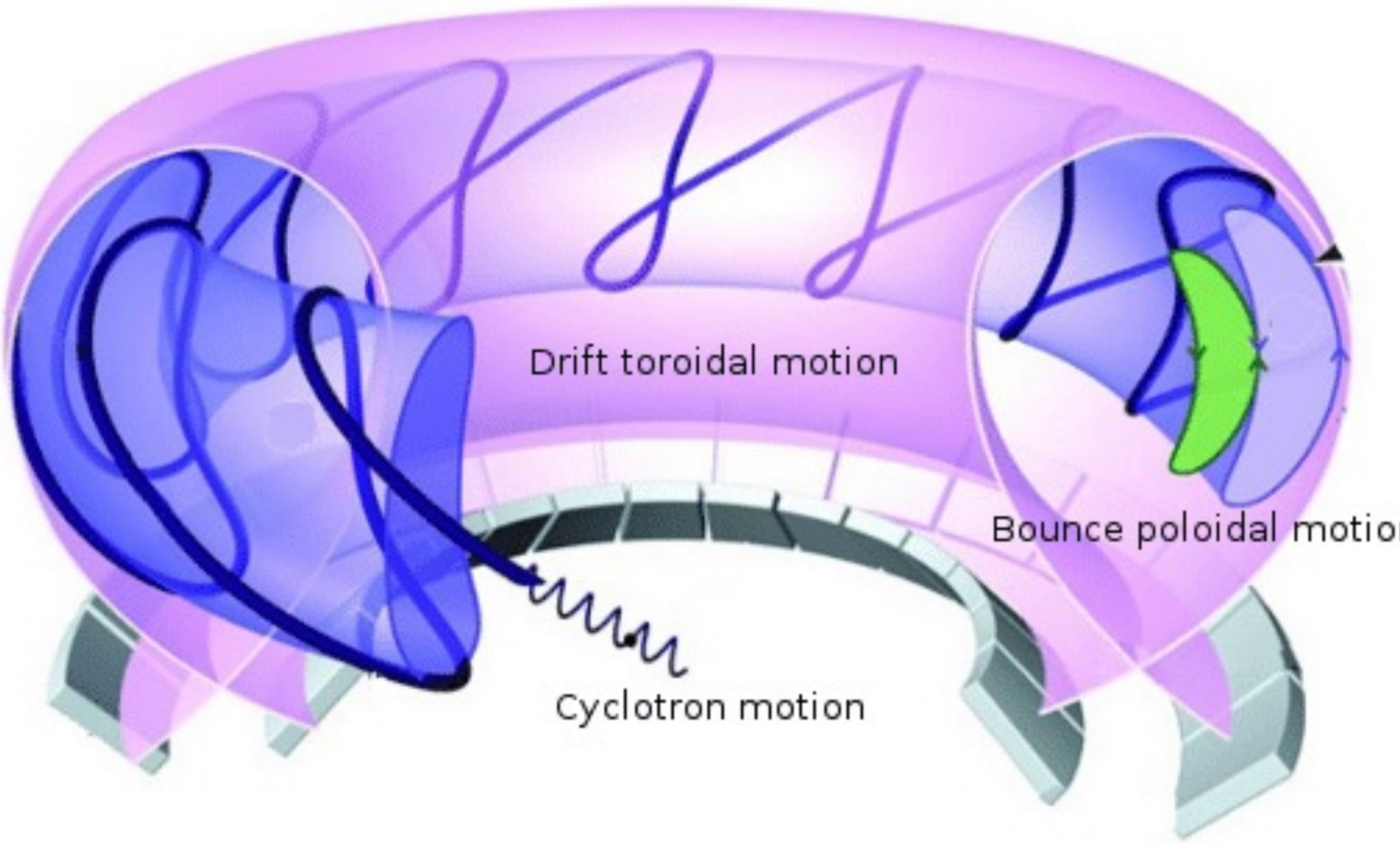

Charged particle motion in inhomogeneous magnetic fields

by Davide Cremese

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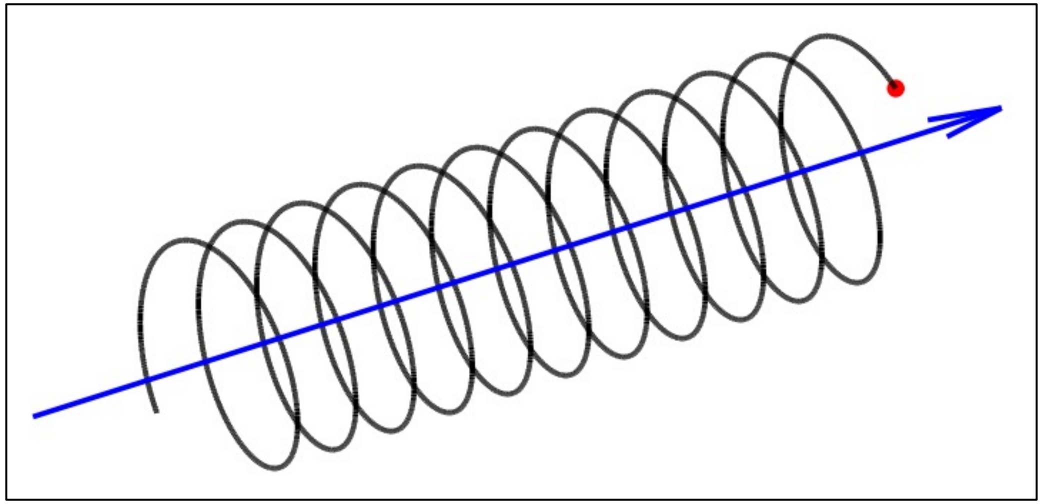
Gyration

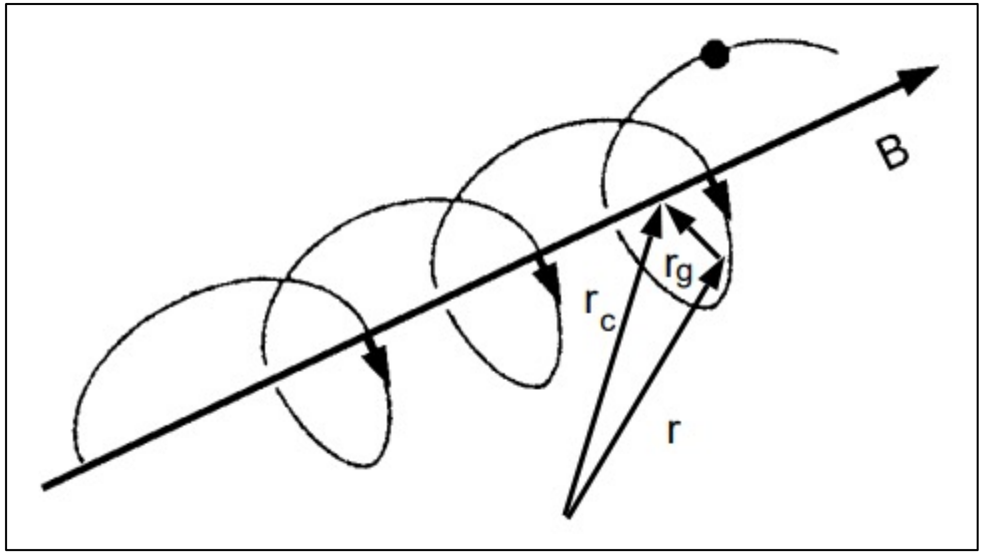
- Lorentz Force $\boldsymbol{F_L} = q \cdot (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$
- Magnetic Field results in a circular motion (gyration)
  - Gyro-/ Larmorradius $r_L = \frac{m v_\perp}{|q| \cdot B}$
  - Gyro-/ Cyclotron-Frequency $\omega_c = \frac{|q| \cdot B}{m}$
- Gyromotion can be used for diagnostics and plasmaheating



Source: Simulation of Charged Particle Orbits in Fusion Plasmas

Guiding Center Approach

- Introducing an external force $\boldsymbol{F}$
- General case: $m\dot{\boldsymbol{v}} = \mathbf{F} + \mathrm{q}\dot{\boldsymbol{v}} \times \boldsymbol{B}$
  - $\boldsymbol{B}$ and $\boldsymbol{F}$ constant
- Motion of $\boldsymbol{B}$ and $\boldsymbol{F}$ in a superposition
  - Guiding Center: $\boldsymbol{r}_c = \boldsymbol{r} + \boldsymbol{r}_g$
  - Gyromotion: $\boldsymbol{r}_g = \frac{m}{q \cdot B^2} \boldsymbol{v} \times \boldsymbol{B}$
- $\boldsymbol{v}_c \ll \boldsymbol{v}_g \rightarrow \boldsymbol{r}_c$ approximates the trajectory



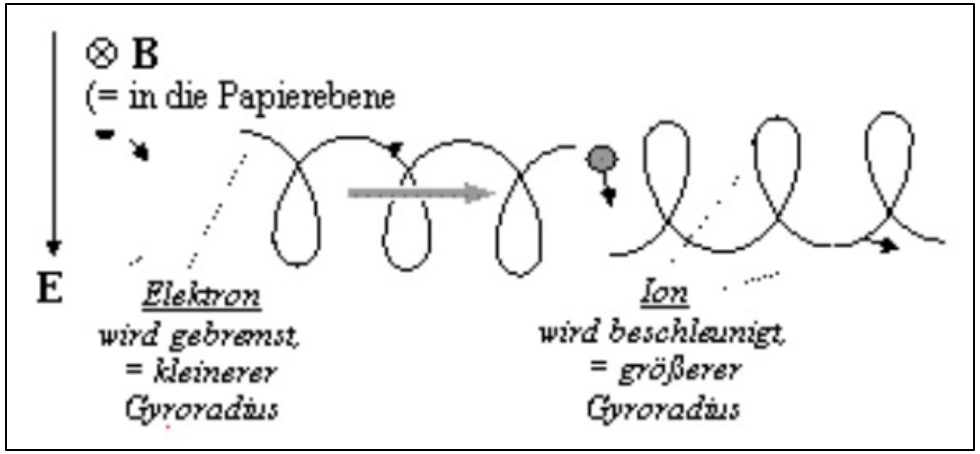
Source: Einführung in Plasmaphysik I, S.Günter

Guiding Center velocity $\boldsymbol{v_c} = \boldsymbol{v}_\parallel + \frac{\boldsymbol{F} \times \boldsymbol{B}}{q \cdot B^2}$

Drift velocity $\boldsymbol{v}_D = \frac{\boldsymbol{F} \times \boldsymbol{B}}{q \cdot B^2}$

Particle Drifts

$\boldsymbol{E}\times\boldsymbol{B}$-Drift



Source: Einführung in Plasmaphysik I, S.Günter

- External Force: $\boldsymbol{F} = q \cdot \boldsymbol{E} \rightarrow \boldsymbol{v}_D = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^{\mathbf{2}}}$
- Drift independent of charge
- Leads to global movement perpendicular to $\boldsymbol{E}$ and $\boldsymbol{B}$
- E-field causes the Gyroradius to change periodically

Particle Drifts
Non-uniform B-field

What happens if the strength of the B-field changes? How can we derive the drift motion?

- Taylor expanding the B-field: $B_z(x, y, z, t) = B_0 + \frac{\partial B_z}{\partial y}\left(y - y_{gc}\right) + O(\varepsilon_r^2) + O(\varepsilon_t^2)$

  $\varepsilon_r = \left|\frac{\nabla B}{B}\right| r_L \ll 1, \qquad \varepsilon_t = \left|\frac{\frac{\partial B}{\partial t}}{\omega_c \cdot B}\right| \ll 1$

- Introducing the first adiabatic invariant μ:
  - Particle performs periodic motion in one generalized coordinate q with the associated momentum p_q
    → $\oint p_q dq$
  - Action remains conserved even with a small perturbation ε in all powers of ε

- Our case: periodic motion is the gyration

$$\oint p_q dq = \int_0^{2\pi} m v_\theta r d\theta = \int_0^{2\pi} m v_\perp r_L d\theta = 2\pi m v_\perp r_L = 2\pi m v_\perp \frac{m v_\perp}{qB} =$$

$\frac{4\pi m}{q}$ μ = constant

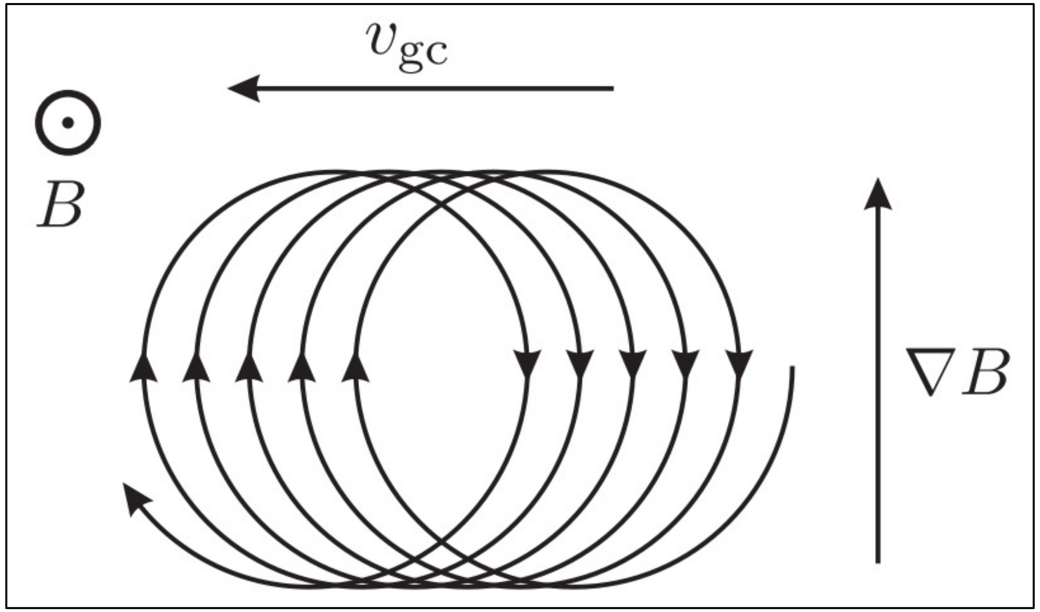
Particle Drifts

∇B-Drift

- μ is the magnetic dipole moment of the charged particle

$$IA = \frac{q\omega_c}{2\pi}\pi r_L^2 = \frac{q^2 B}{2m}\left(\frac{mv_\perp}{qB}\right)^2 = \frac{mv_\perp^2}{2B} = \mu$$

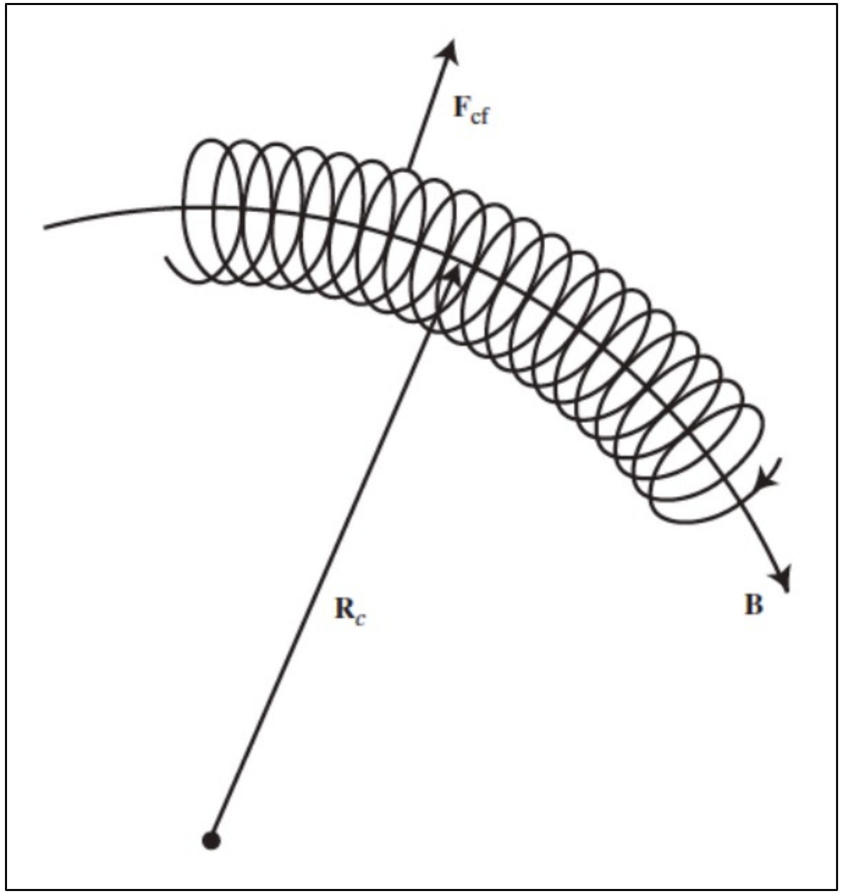
- the magnetic dipole is anti-aligned with the B-field (diamagnetic)
- Potential energy: $E = \mu B$ → $\boldsymbol{F} = -\mu \boldsymbol{\nabla} B$
- ∇B-Drift velocity: $\boldsymbol{v}_D = -\frac{mv_\perp^2}{2qB^3}\, \boldsymbol{\nabla} B \times \boldsymbol{B}$



Source: Simulation of Charged Particle Orbits in Fusion Plasmas

Particle Drifts

Curvature Drift



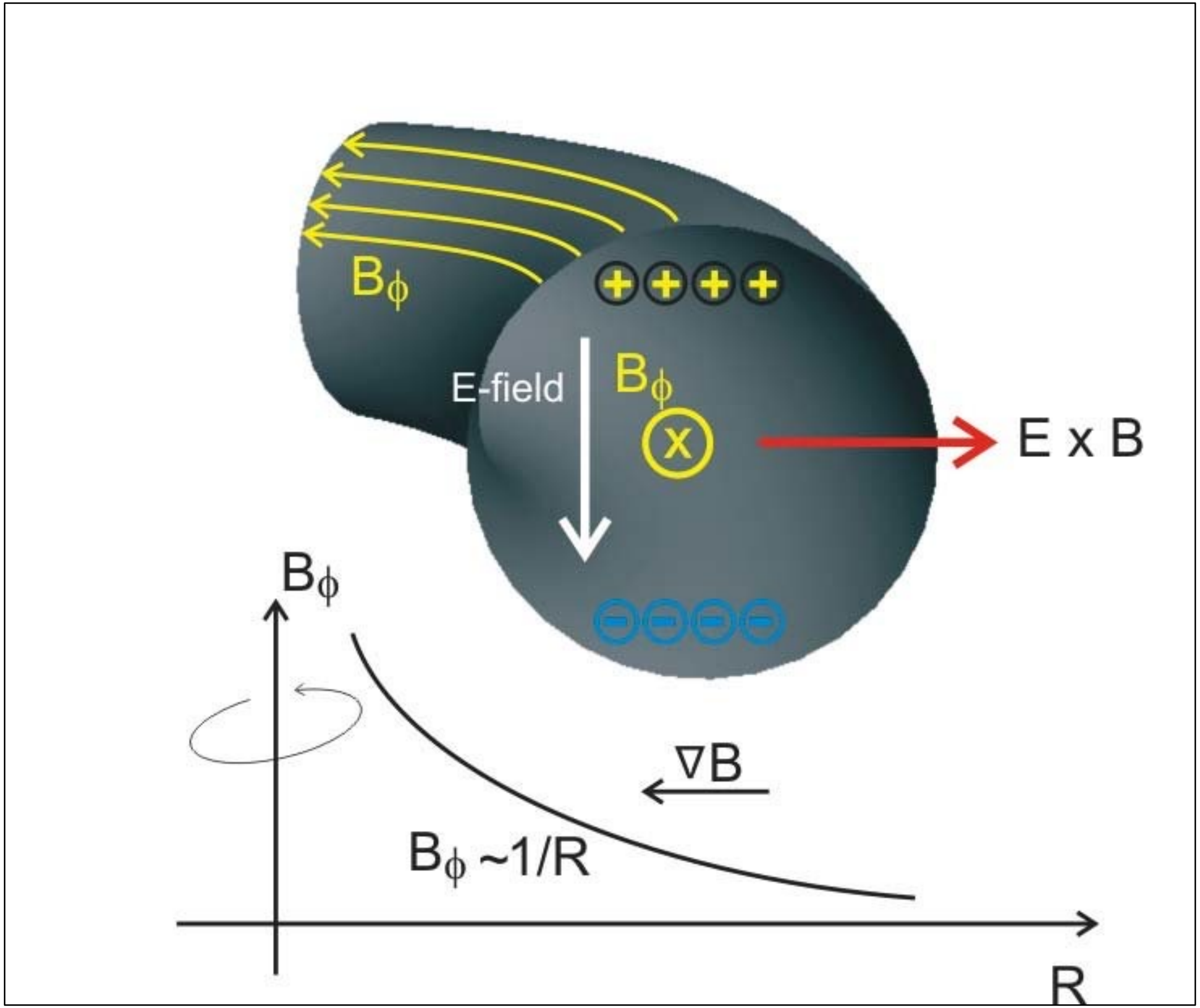
Source: Introduction to Plasma Physics (PY5012) Lecture 4: Single-Particle Motion, Dr. Peter T. Gallagher

- $\nabla \cdot \boldsymbol{B} = 0, \nabla B \neq 0$
  - B-field lines are curved
  - Parallel motion leads to a centrifugal force
- Centrifugal force: $\boldsymbol{F} = \frac{m v_\parallel^2}{R_C} \boldsymbol{e}_{R_C}$
- Drift velocity: $\boldsymbol{v}_D = -\frac{m v_\parallel^2}{q B^3} \boldsymbol{\nabla} B \times \boldsymbol{B}$

One can combine the ∇B- and curvature drift into

$$\boldsymbol{v}_D = \frac{m}{q B^3}\left(v_\parallel^2 + \frac{1}{2} v_\perp^2\right) \boldsymbol{B} \times \boldsymbol{\nabla} B$$

Approximately proportional to E_{kin}



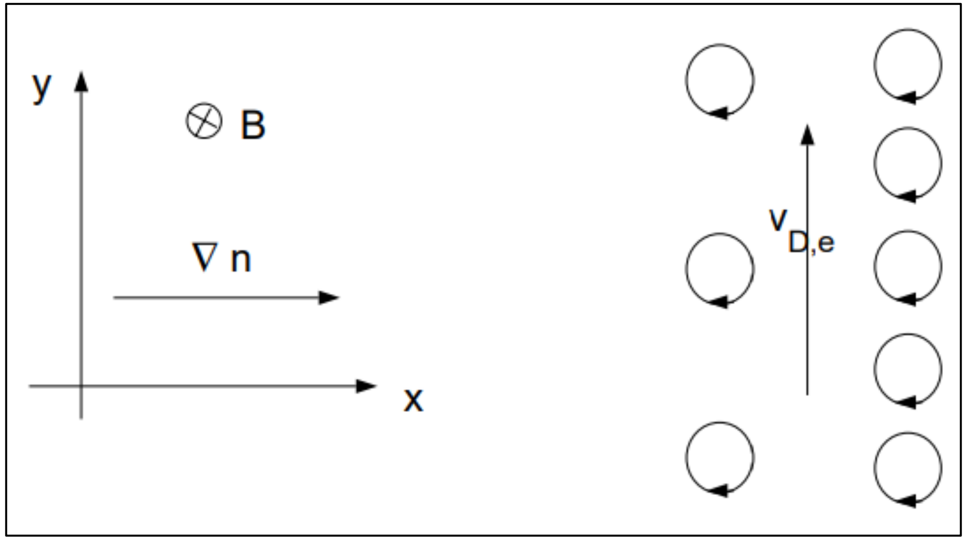
Source: https://www.researchgate.net/figure/Particle-drifts-in-a-toroidal-magnetic-field-configuration_fig1_30049238

Particle Drifts
Diamagnetic Drift

- High pressure and temperature gradients

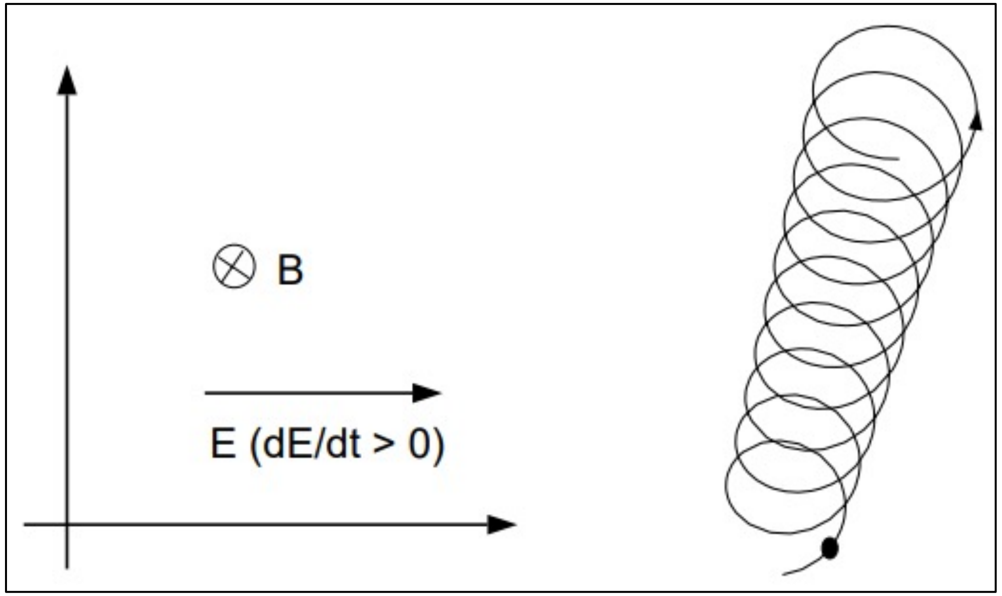
$$\frac{\boldsymbol{F}}{\delta V} = -\nabla p,\ \ \delta N = n\delta V \ \ \rightarrow \boldsymbol{F} = \frac{-\nabla p}{n}$$

- Drift velocity: $\boldsymbol{v}_D = -\frac{\nabla p \times \boldsymbol{B}}{qnB^2}$
- Particles appear to drift in a direction but they don‘t move
- Drift results in a current: $\boldsymbol{j}_D = -\frac{\nabla p \times \boldsymbol{B}}{B^2}$
- Current induced B-field weakens the initial B-field
- Plasma behaves diamagnetically



Source: Einführung in Plasmaphysik I, S.Günter

Particle Drifts
Polarization Drift



Source: Einführung in Plasmaphysik I, S.Günter

- Time dependent electric field
- Equation of motion: $m\ddot{v}_x = -\frac{q^2}{m} B_z^2 v_x + q\dot{E}_x$
- Averaging over many gyrationperiods

→ $$v_x = \frac{m}{qB_z^2}\dot{E}_x$$

- Charge dependency leads to a polarization current:

$$\boldsymbol{j}_{pol} = \frac{nm_i}{B^2}\dot{\boldsymbol{E}}$$

Particle Drifts

Polarization Current

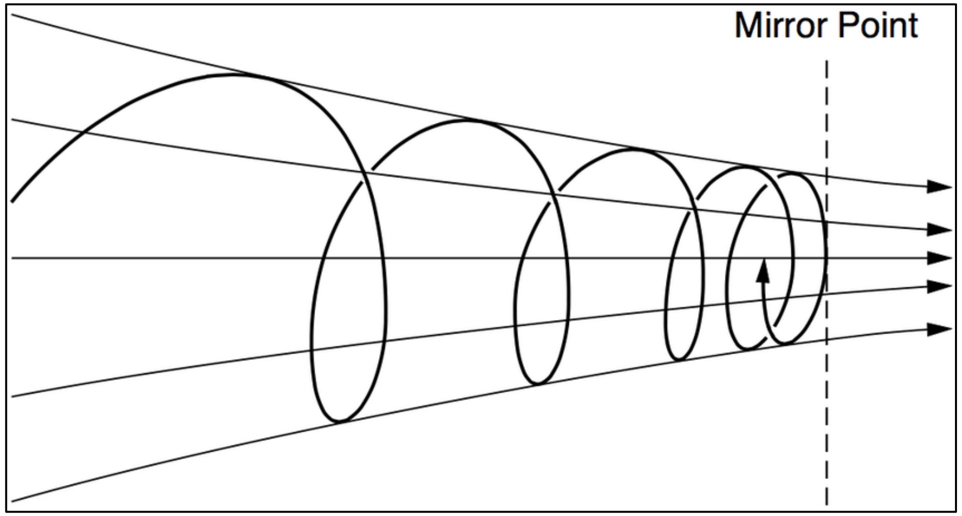
- Important to establish electric fields in a plasma
- Relative permittivity:

$$\varepsilon = 1 + \frac{nm_i}{B^2} = 1 + \frac{c^2}{v_A^2}$$

- $v_A \ll c$ → magnetised plasma effectively shields electric fields

Mirror Confinement

- Mirror Confinement results from the conservation of energy and magnetic momentum
  - $E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_\perp^2 + v_\parallel^2\right) = const.$
  - $\mu = \frac{mv_\perp^2}{2B} = const.$



Source: https://www.physics.smu.edu/fattarus/magbottle.html

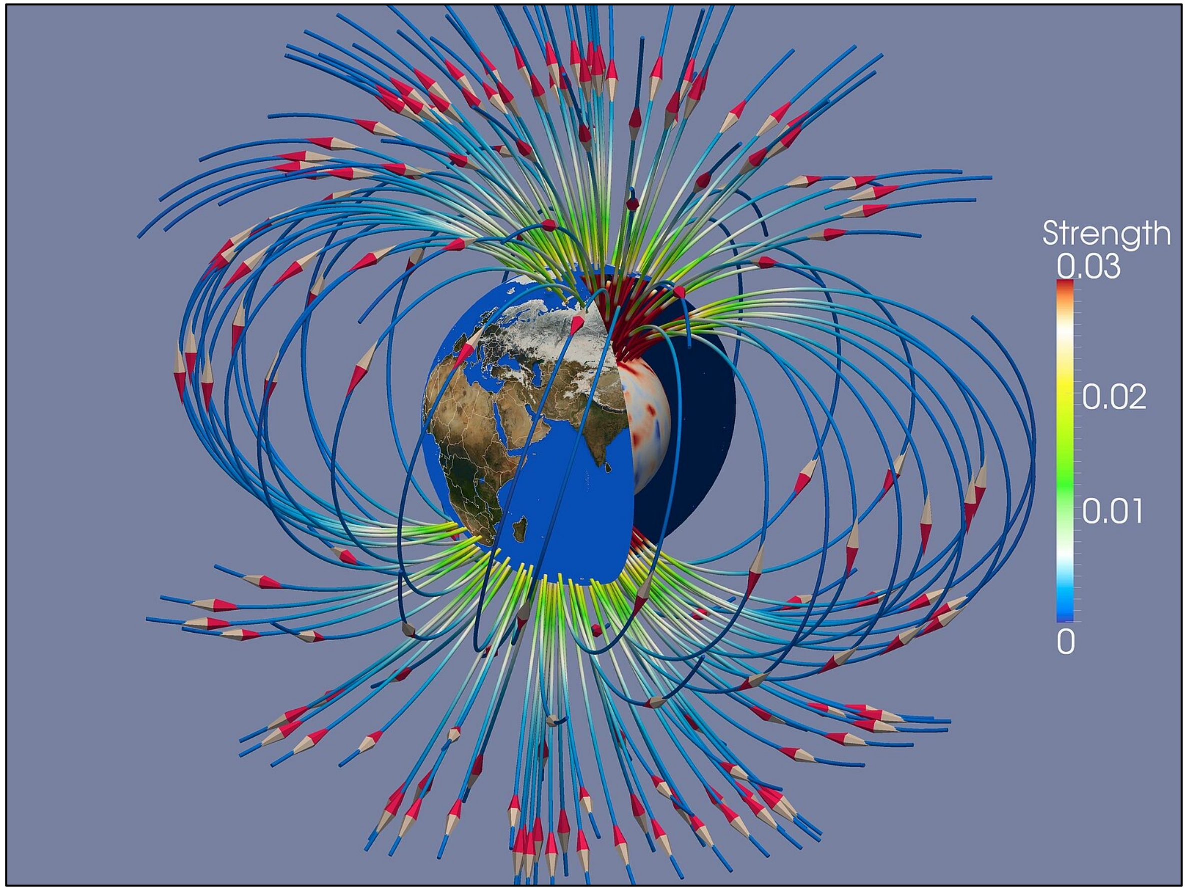
→$E = \mu B + \frac{1}{2}mv_\parallel^2$

- If $\mu B_{max} > E$ then the particle is trapped

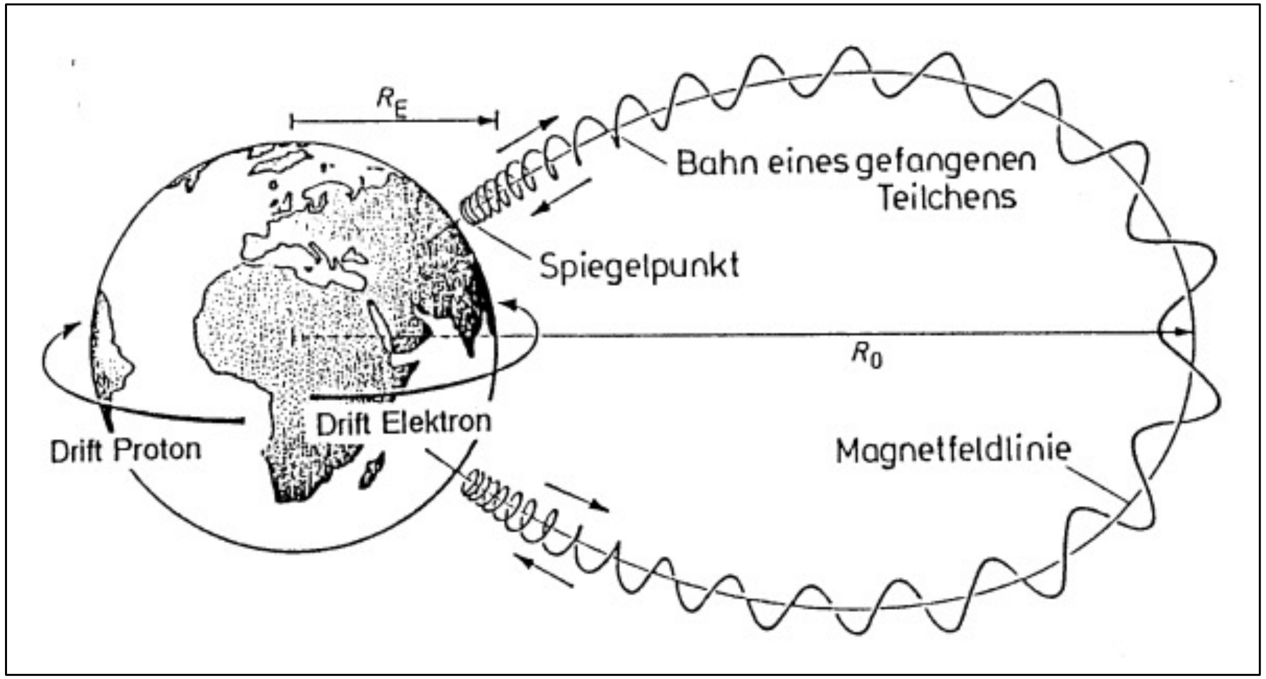
- Mirror condition: $\frac{v_\parallel^2(B_{min})}{v_\perp^2(B_{min})} < \frac{B_{max}}{B_{min}} - 1$

Mirror Confinement

Examples



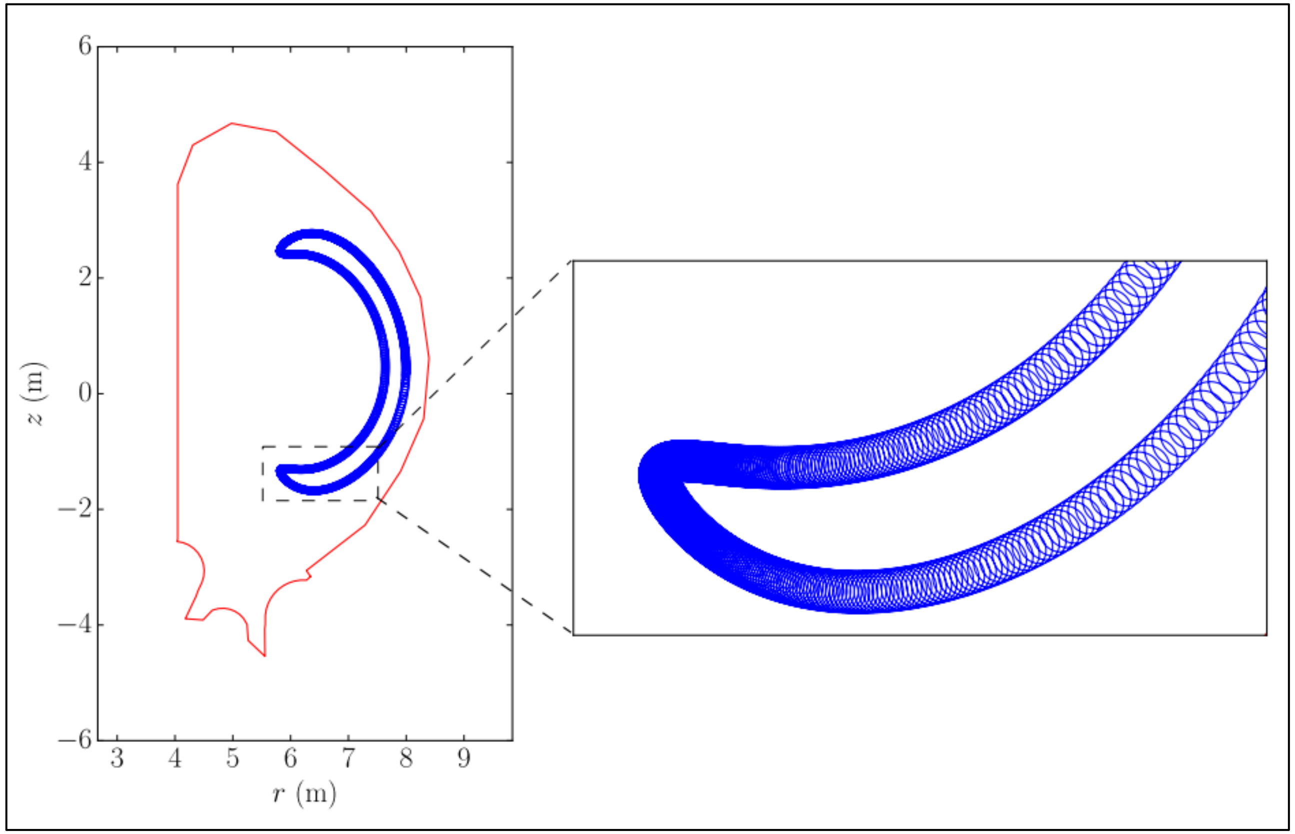
Source:https://www.cscs.ch/science/earth-env-science/2013/new-insight-into-the-earths-deep-interior

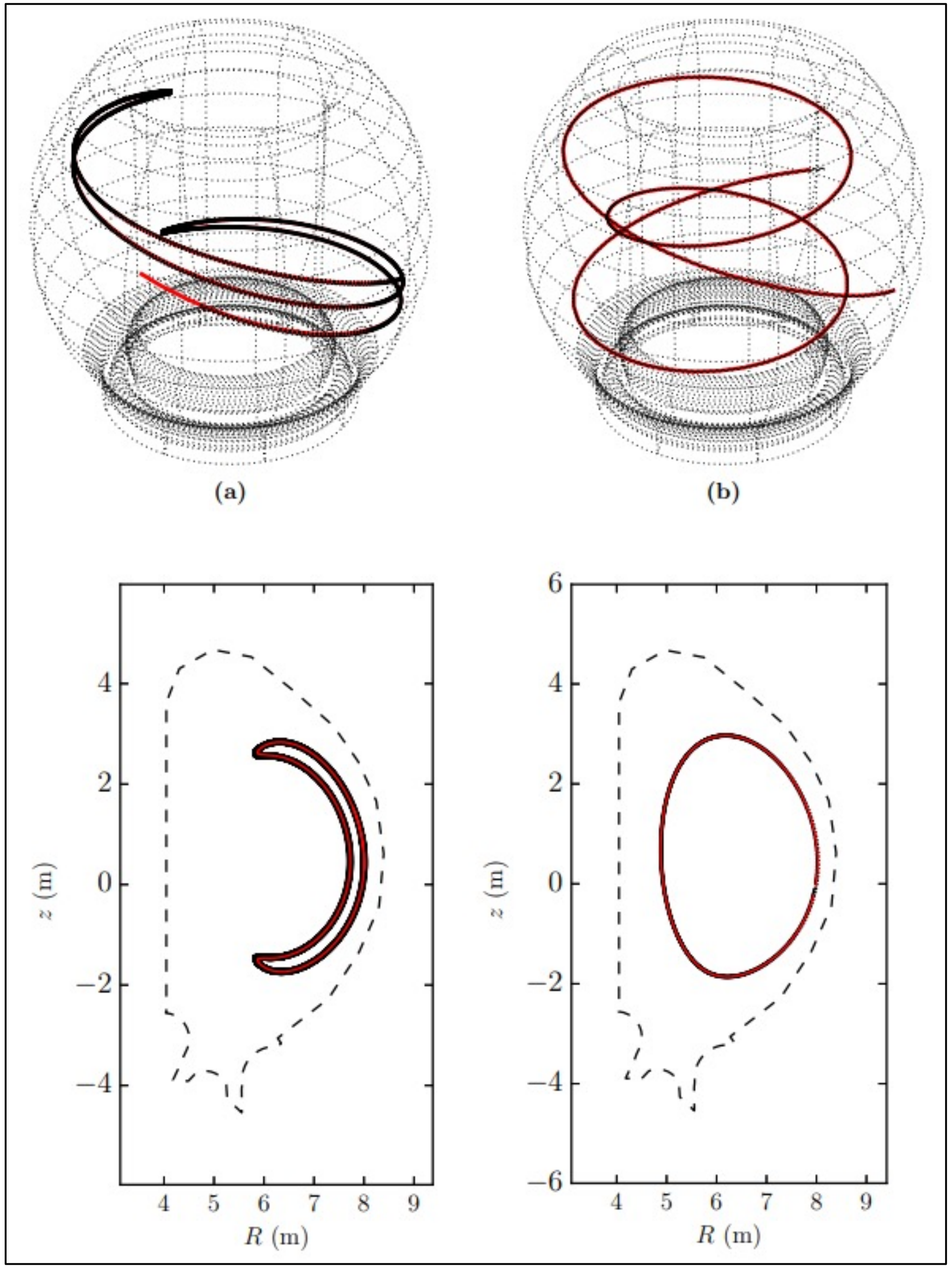


Source: Einführung in Plasmaphysik I, S.Günter

Mirror Confinement

Examples

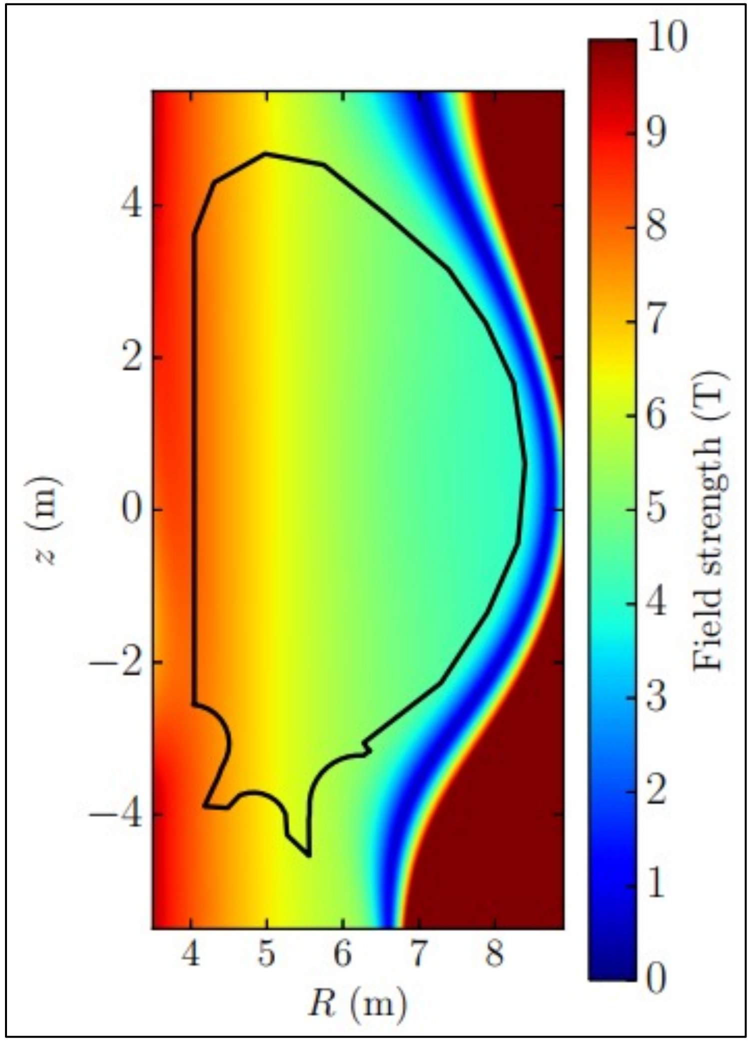


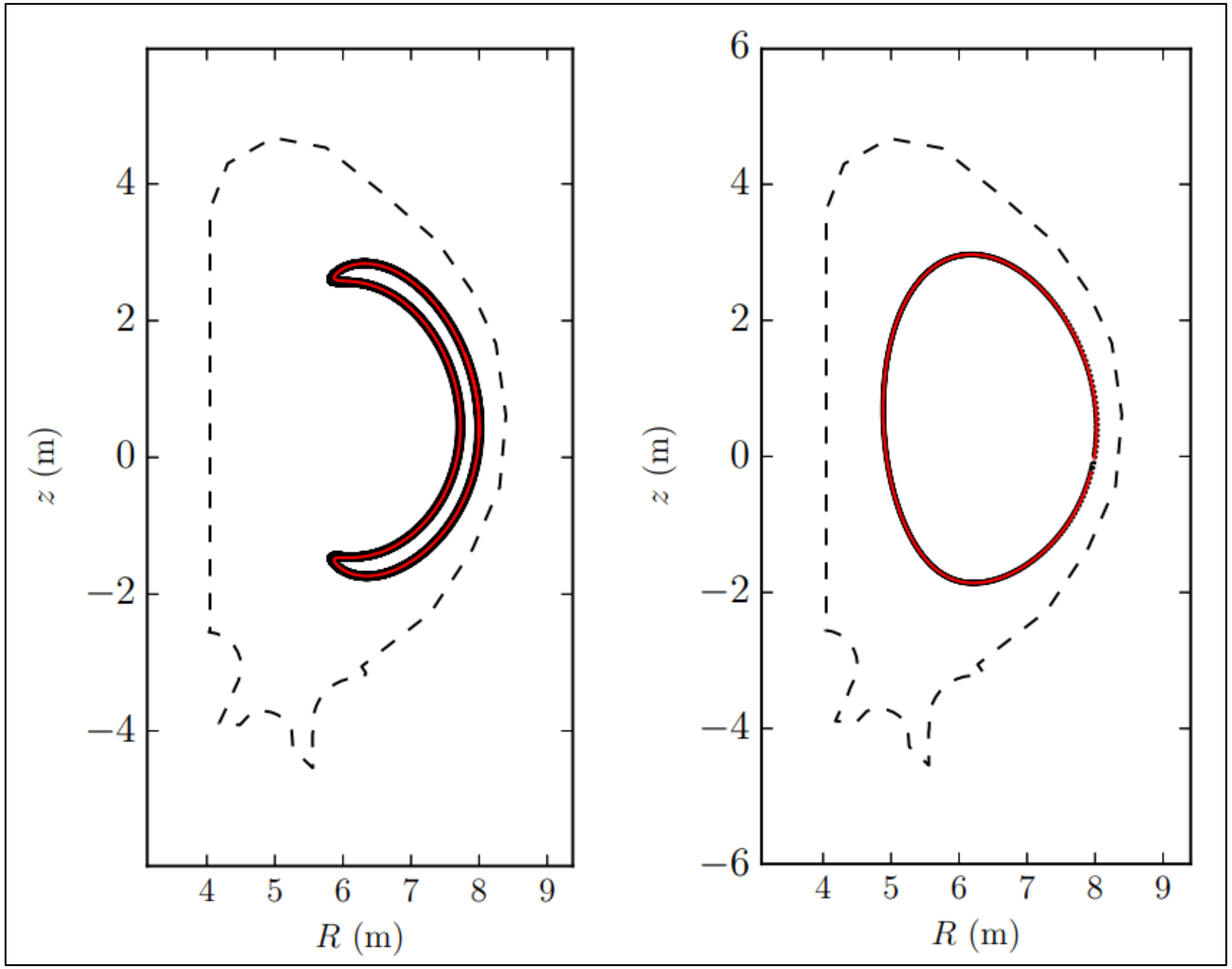


Source: Simulation of Charged Particle Orbits in Fusion Plasmas

Mirror Confinement

Examples





Source: Simulation of Charged Particle Orbits in Fusion Plasmas

Summary

- Confining charged particles in a magnetic field requires small Larmor radius to improve single particle collision count
- Magnetic fields combined with external forces cause various drift motions making full orbit calculations very expensive
- Guiding center approach offers a way to simplify this issue and increase resource and cost efficiency during computation
- Charged particles are approximated as charged current rings with constant anti-aligned magnetic dipole moment a magnetic field lines in the center
- Complicated trajectories as well as slow drifts away from the initial magnetic field lines can easily be calculated analytically