WP1: Weak turbulence analysis of energetic particles due to SAW/DAW*

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Description of NAT WP1

- Nonlinear interaction of energetic particles (EP) with Alfvén fluctuations: Alfvén eigenmodes (AEs), EP modes (EPM) and shear/drift Alfvén waves (SAW/DAW)
- Approach based on (singular) perturbation expansion is well-known for Langmuir turbulence in uniform plasmas: weak turbulence theory
- SAW/DAW fluctuations in fusion plasmas are characterized by both broad-band (turbulent) feature as well as nearly-periodic (coherent) behavior.
- Need for a general theoretical framework for a self-consistent description
  - Gyrokinetic transport theory, phase space zonal structures (PSZS): long-lived nonlinear equilibria consistent with fluctuation spectrum
  - Need to go beyond the local description of fluctuation-induced fluxes, extending the diffusive transport paradigm and accounting for modes of the linear stable spectrum

Gyrokinetic transport theory and time scales

- Fluctuation induced transport in fusion plasmas is due to low frequency fluctuations ($|\omega| \ll |\Omega|$) $\Rightarrow$ gyrokinetic transport theory.

- Particle dynamics independent of the gyrophase $\Rightarrow$ reduced phase-space description in terms of an invariant of motion: magnetic moment μ. Gyrokinetic transport theory deals with transport in non-uniform, non-autonomous system with 2 degrees of freedom.

- Reduced dynamic description of a time dependent non-uniform plasma with one degree of freedom in the corresponding reduced phase space
  - identification of additional (nonlinear) invariant of motion $|\omega| \sim |n\bar{\omega}_{dk}| \ll \omega_b \Rightarrow J = \text{const} \Rightarrow$ fishbone paradigm $\Rightarrow$ neglect finite Larmor and magnetic drift orbit width
  - Advantage of simplicity and of reducing to the bump-on-tail paradigm in the uniform plasma limit
  - Breaks down on long time scales: collisions? Arnold diffusion?

Milestones and Deliverables 2017/2018

- Address the effect of interaction of SAW/DAW spectrum with ZS and PSZS effect of fluctuation spectrum on wave-particle resonances (2017)
  - Derivation of nonlinear model equations for the self-consistent evolution of SAW/DAW and ZS/PSZS for the "fishbone paradigm"
  - generalization of resonance broadening theory [Dupree 66]: non-Gaussianity of fluctuation spectrum, non-diffusive transport

- Numerical solution of model equations and applications (2018)
  - Uniform plasma: numerical solution of model equations and V&V against Hamiltonian formulation of the bump-on-tail paradigm
  - Non-uniform plasmas: numerical solution of model equations for the "fishbone paradigm" and applications to ITER and DEMO

Phase space zonal structures (ongoing; NAT 2017)

- The fluctuating particle distribution functions are decomposed in adiabatic and nonadiabatic responses as [Frieman and Chen 1982]

$$\delta f = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} \left[\delta g - \frac{e}{m} \frac{1}{B_0} \frac{\partial \bar{F}_0}{\partial \mu} \langle \delta L_g \rangle \right] + \frac{e}{m} \left[\frac{\partial \bar{F}_0}{\partial \mathcal{E}} \delta\phi + \frac{1}{B_0} \frac{\partial \bar{F}_0}{\partial \mu} \delta L \right] ,$$

$$\delta L_g = \delta\phi_g - \frac{v_\parallel}{c} \delta A_{\parallel g} = e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} \delta L = e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} \left(\delta\phi - \frac{v_\parallel}{c} \delta A_\parallel \right) .$$

- Using mode structure decomposition in toroidal geometry [Zonca et al. NJP15], the representation of phase-space zonal structures is $(n = \ell = 0)$

$$\delta f_z = \sum_m \left\{ \mathcal{P}_{m,0,0} \circ [J_0(\lambda)\delta g]_{m,0} \right\} - \left[J_0(\lambda) \left(\frac{e}{m} \frac{1}{B_0} \frac{\partial \bar{F}_0}{\partial \mu} \langle \delta L_g \rangle \right) \right]_{0,0}$$
$$+ \frac{e}{m} \left[\frac{\partial \bar{F}_0}{\partial \mathcal{E}} \delta\phi + \frac{1}{B_0} \frac{\partial \bar{F}_0}{\partial \mu} \delta L \right]_{0,0} .$$

- Using the nonlinear gyrokinetic equation [Frieman and Chen 1982]; and assuming that $|k_\parallel| \ll |\boldsymbol{k}_\perp|$ [Zonca et al NJP15, Chen RMP16]

$$\frac{\partial \delta g_z}{\partial t} = -\mathcal{P}_{0,0,0}\circ\left(\frac{e}{m}\frac{\partial}{\partial t}\langle\delta L_g\rangle_z \frac{\partial \bar{F}_0}{\partial \mathcal{E}}\right)_{0,0} + i\sum_m \mathcal{P}_{m,0,0}\circ\frac{c}{d\psi/dr}\frac{\partial}{\partial r}\sum_n n\left(\delta g_n \langle\delta L_g\rangle_{-n}\right)_{m,0} ,$$

where $d\psi/dr$ is the derivative of the equilibrium magnetic flux.

- This equation contains both zonal flows and fields (the first term on the RHS) as well as the nonlinear effect of EP on equilibrium via wave-EP interactions, dominated by wave-particle resonances.

- In turn, the feedback of phase space zonal structures onto δg_n $(n \neq 0)$ is

$$\left(\frac{\partial}{\partial t} - \frac{inc}{d\psi/dr}\langle\delta L_g\rangle_z\frac{\partial}{\partial r} + v_\parallel\nabla_\parallel + \boldsymbol{v}_d\cdot\boldsymbol{\nabla}_\perp\right)\delta g_n = i\frac{e}{m}\left(Q\bar{F}_0 - \frac{nB_0}{\Omega d\psi/dr}\mathcal{P}_{0,0,0}\circ\frac{\partial \delta g_z}{\partial r}\right)\langle\delta L_g\rangle_n .$$

- Accounts for zonal flows/fields as well as corrugation of radial profiles.

- These equations for δg_z and δg_n are closed by the $(\delta\phi_n, \delta A_{\parallel n})$ DAW field equations for the dynamic evolution of Alfvénic fluctuations and by the equations for the zonal flows/fields and $(\delta\phi_z, \delta A_{\parallel z})$.

- Studied so far in simplified limits:
  - Neglecting wave-particle resonances $\Rightarrow$ dominant zonal flows/fields [Chen POP00; Chen NF01; Guo PRL09; Kosuga POP12]
  - Neglecting effect of zonal flows/fields $\Rightarrow$ dominant EP wave-particle resonances [Zonca Th.Fus.Pl.00; NF05; PPCF06]

Connection with other projects

- NLED Project (WP15_ENEA-03, Theory and simulation of energetic particle dynamics and ensuing collective behaviors in fusion plasmas):
  - NC, GM and FZ (PI), involved in NLED
  - Predominant focus on bump-on-tail paradigm and its applicability to reduced models for EP transport by SAW/DAW in fusion plasmas

- Complete weak-turbulence description of EP transport on long time scales [M. Falessi PhD Thesis 2016]: connection with the WP17_ENEA-10 Project (MF, FZ, AM(PI) participation)
  - Hierarchy of spatiotemporal scales: role of collisions/dissipation, sources/sinks on long time scales; must be included into the bounce averaged evolution of PSZS
  - effect of spectral transfers must be addressed for high mode number and long time scales [Chen RMP16]; connected with resonance broadening [Dupree 66]

Minor manpower change suggested

- Original proposal
  - 2017 N. Carlevaro (0.0); G. Montani (0.4ppy); F. Zonca (0.35ppy). Total: 0.75ppy
  - 2018 N. Carlevaro (1.0 ppy); G. Montani (0.5 ppy); F. Zonca (0.5 ppy). Total: 2.0ppy

- Suggested amendment (at fixed cost): to get NC structurally involved in NAT activities from the start
  - 2017 N. Carlevaro (0.25ppy); G. Montani (0.4ppy); F. Zonca (0.2ppy). Total: 0.85ppy
  - 2018 N. Carlevaro (1.0 ppy); G. Montani (0.5 ppy); F. Zonca (0.5 ppy). Total: 2.0ppy