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Linear and non-linear characterization of transient waves and their interactions

Kick-off meeting for EnR project on Nonlinear interaction of Alfvénic and turbulent fluctuations in burning plasmas

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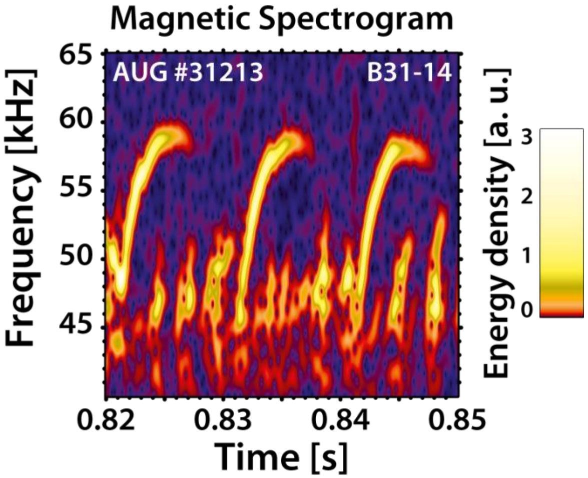
WP6 Deliverables

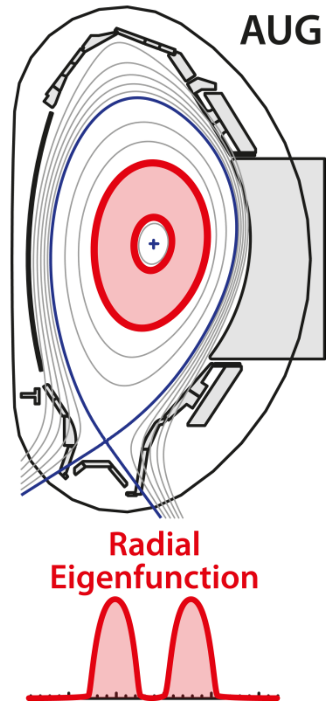
- **Develop a standard set of tools for the linear characterisation of chirping modes, comparison to simulations; linear characterization of chirping modes demonstrated on EGAMs, BAEs and bursting TAEs at the ASDEX Upgrade tokamak (2017)**

- **Characterise the non-linear interactions by higher order spectra and band-power correlation with careful consideration of error propagation and significance levels, comparison to simulations; quantitative study of non-linear interactions of various fast particle-related transient modes (e.g. EGAMs, BAEs and/or bursting TAEs) with special emphasis on detecting wave-wave coupling (2018)**

Justification

- Data processing aimed at focus issues identified by theory research
- Custom developed data analysis methods aimed at characterizing fast transient phenomena in noisy environment
- Quantitative analysis with thorough evaluation of significance levels

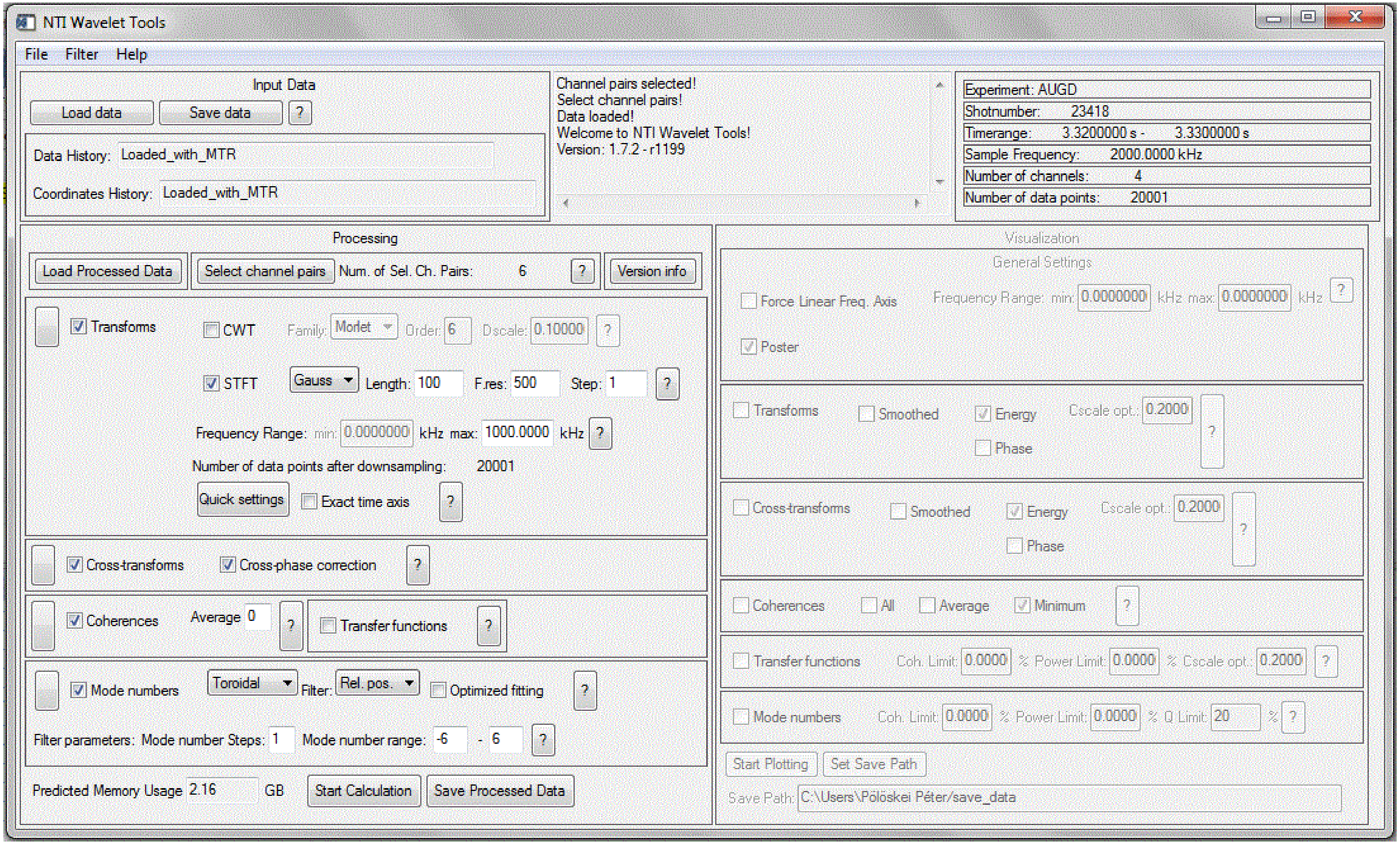




History

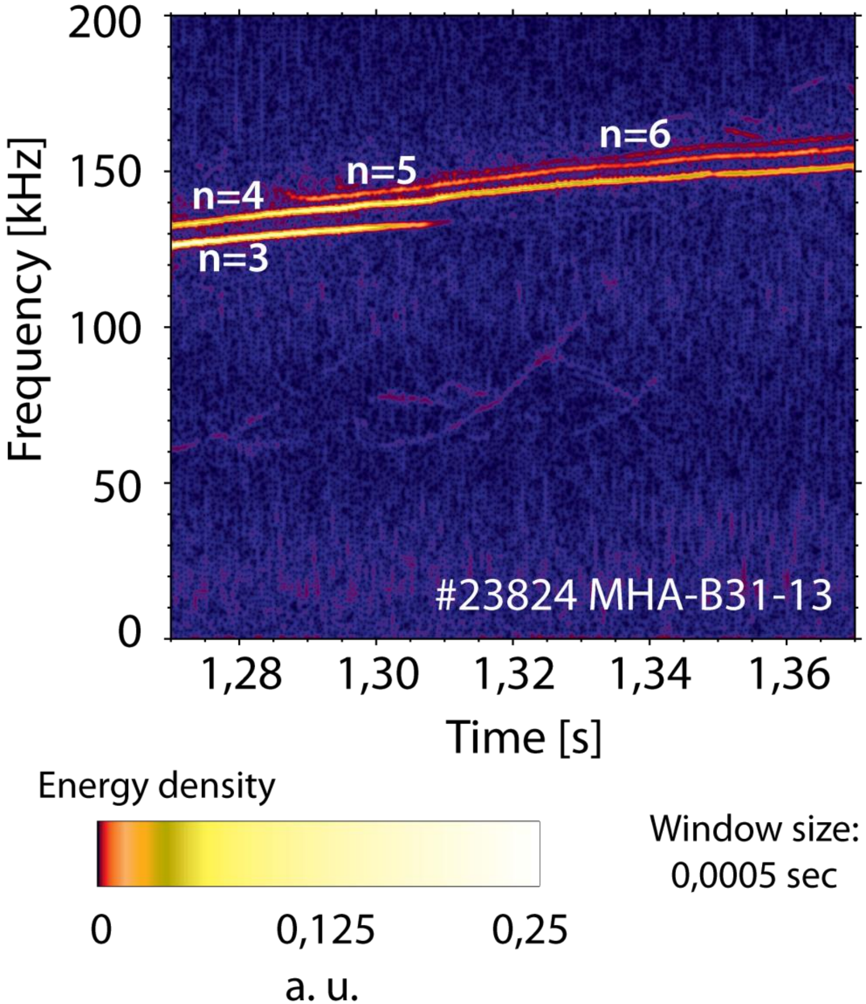
2002	Toolbox development for time-frequency analysis of plasma transients starts
2005	First applications at ASDEX Upgrade (mode structure)
2011	NTI Wavelet Tools GUI deployed to ASDEX Upgrade
2012	Move to quantitative analysis with error bars
2013	Start theory-motivated data analysis

NTI Wavelet Tools

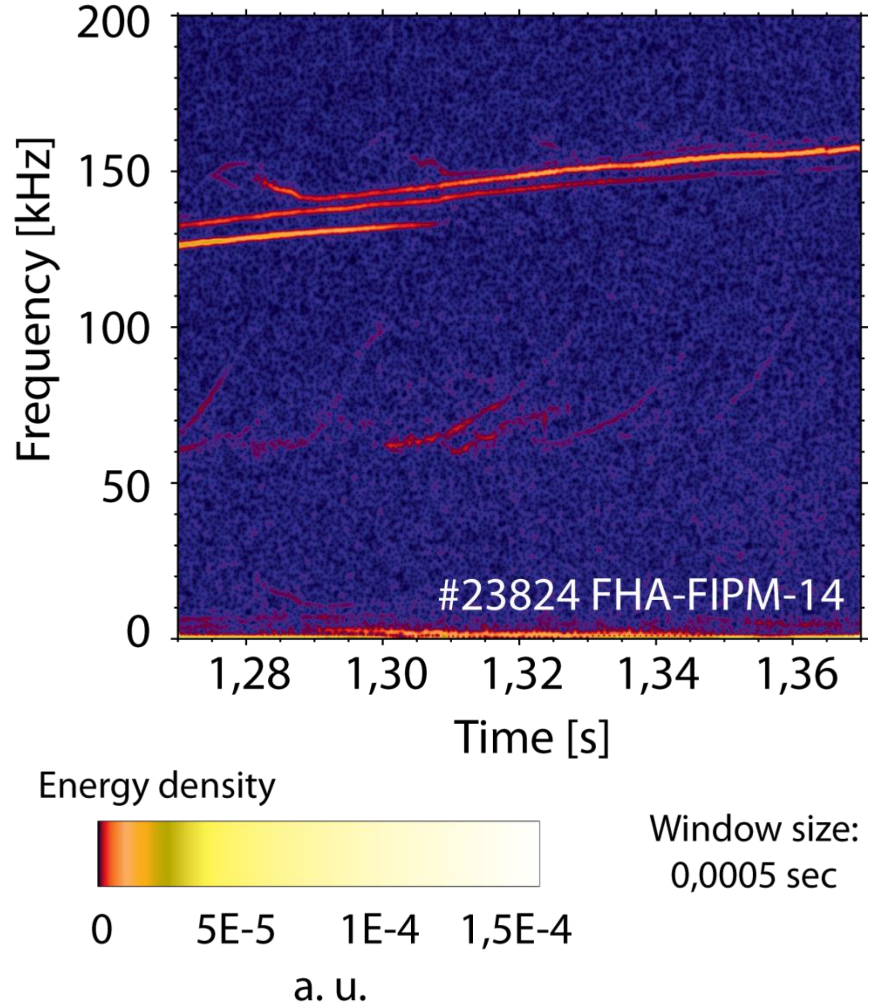


Spectrogram – energy density:

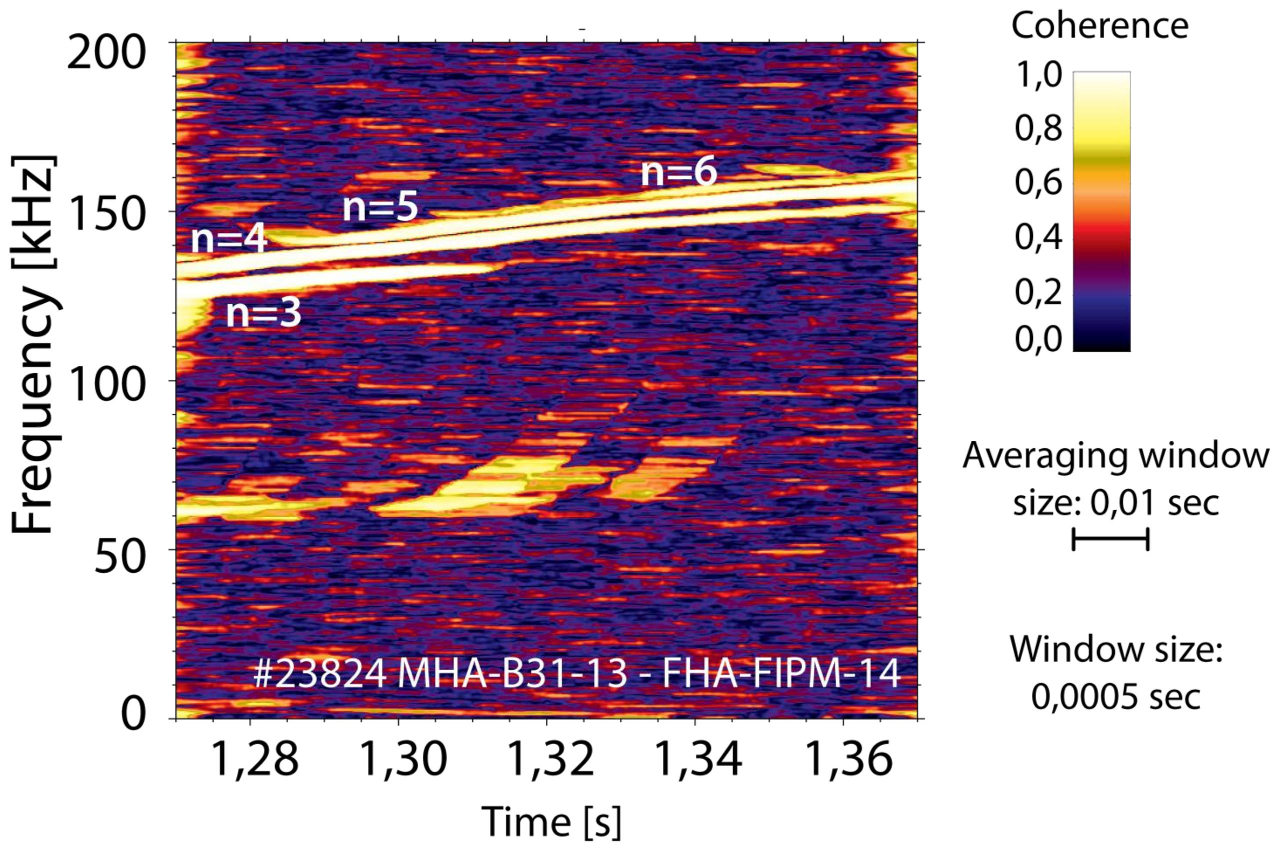
Magnetic coil – plasma waves



FILD – Fast-ion losses

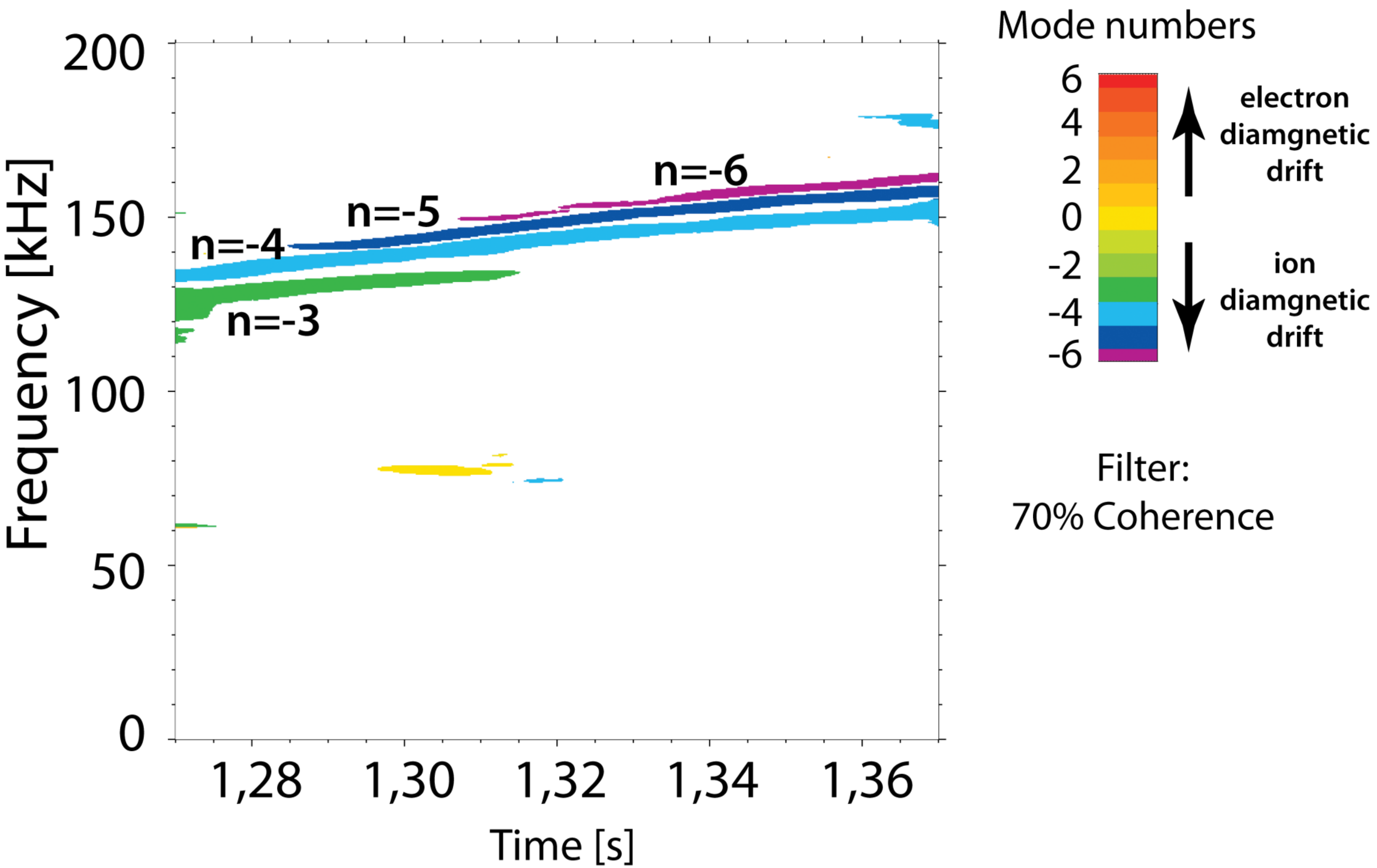


Coherence: plasma waves – fast-ion losses



In spite of the high value of the number of averages (N = 20), the coherence is near to 1, which suggests **strong linear coupling** between **TAEs** and **fast-ion losses**

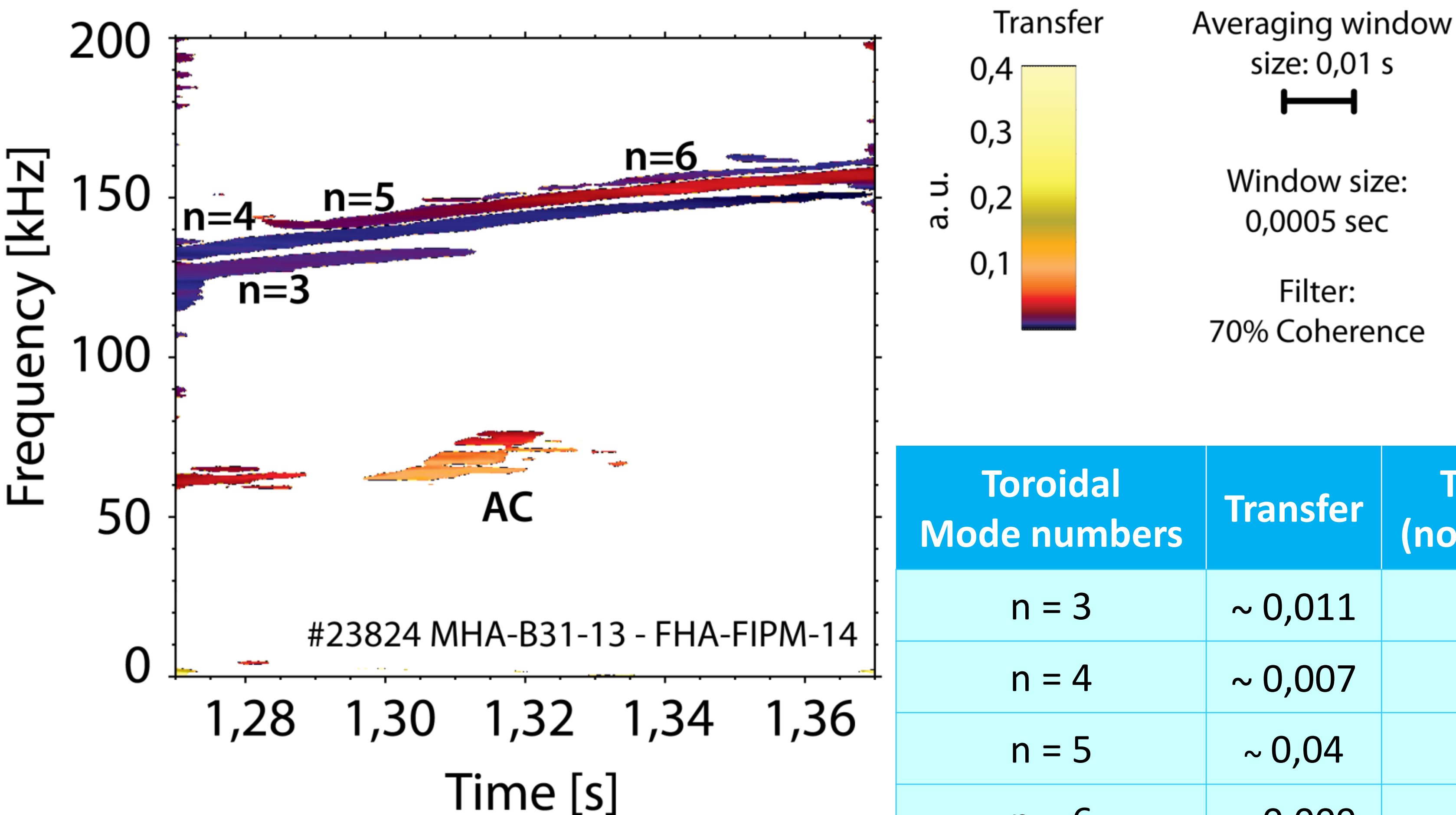
Toroidal mode numbers of TAEs



Filter:
70% Coherence

Coherence filtered **Transfer function**

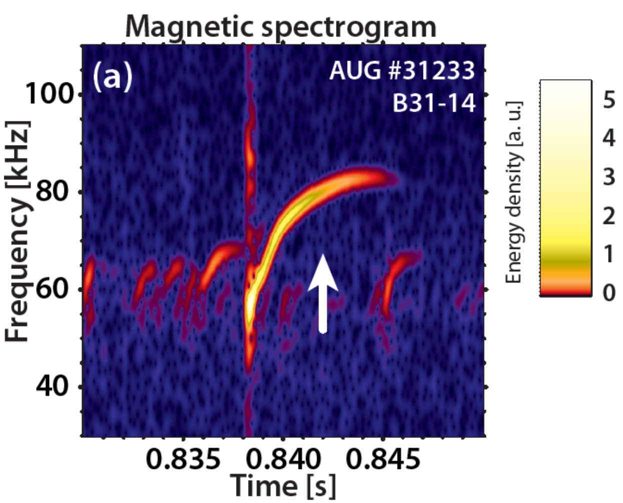
plasma waves – fast-ion losses

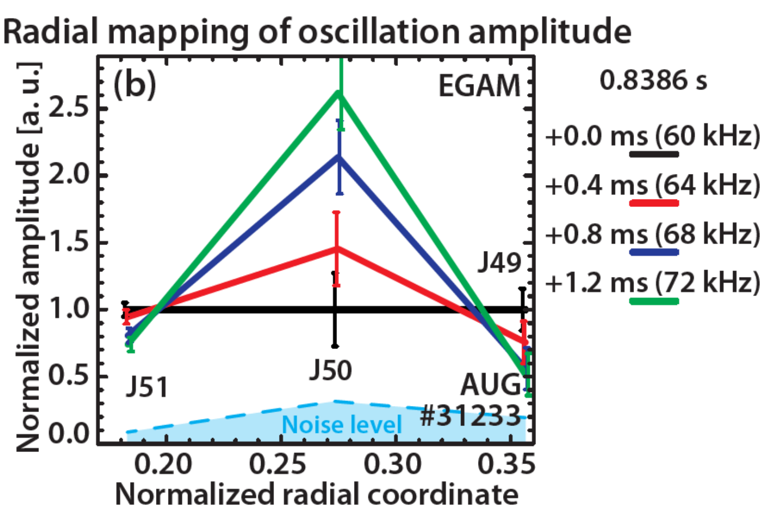


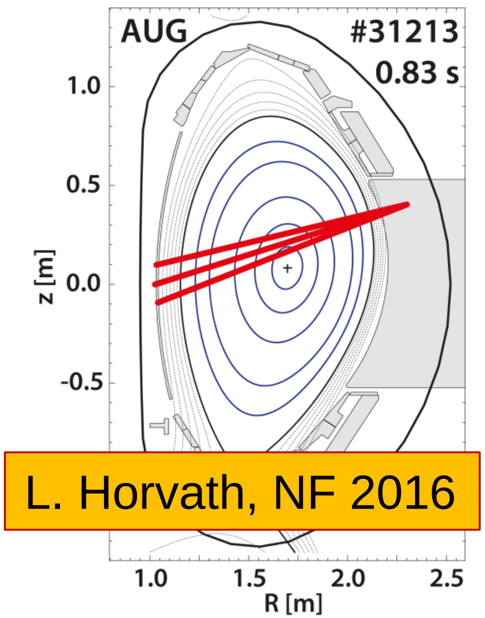
Toroidal Mode numbers	Transfer	Transfer (normalized)
n = 3	~ 0,011	28 %
n = 4	~ 0,007	18 %
n = 5	~ 0,04	**100 %**
n = 6	~ 0,009	23 %

Radial eigenfunction evolution of EPMs

- **Downchirping BAEs** and **upchirping EGAMs** have been analysed in a number of discharges.
- **No significant change in the radial structure of BAEs** has been observed.
- In the **EGAM case the results show a slight shrinkage in the radial mode structure** which is consistent with the physical picture.







L. Horvath, NF 2016

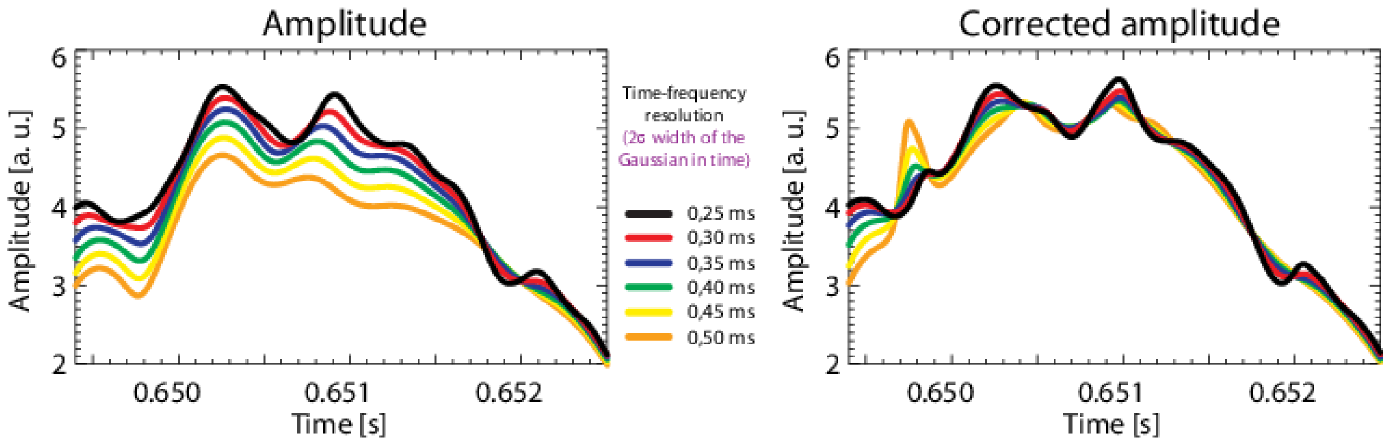
Mode amplitude of chirping waves

- A chirping waves are modelled as: $f(t) = a(t)\cos[\phi(t)]$
- Constant amplitude, constant frequency approximation:

$$a_0(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}}\left|\mathrm{STFT}(t, \omega = \omega_{\mathrm{ridge}})\right|$$

- Linear chirps:

$$a_1(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}}\sqrt[4]{1 + 4\phi''(u)^2\sigma^4}\left|\mathrm{STFT}(t, \omega = \omega_{\mathrm{ridge}})\right|$$

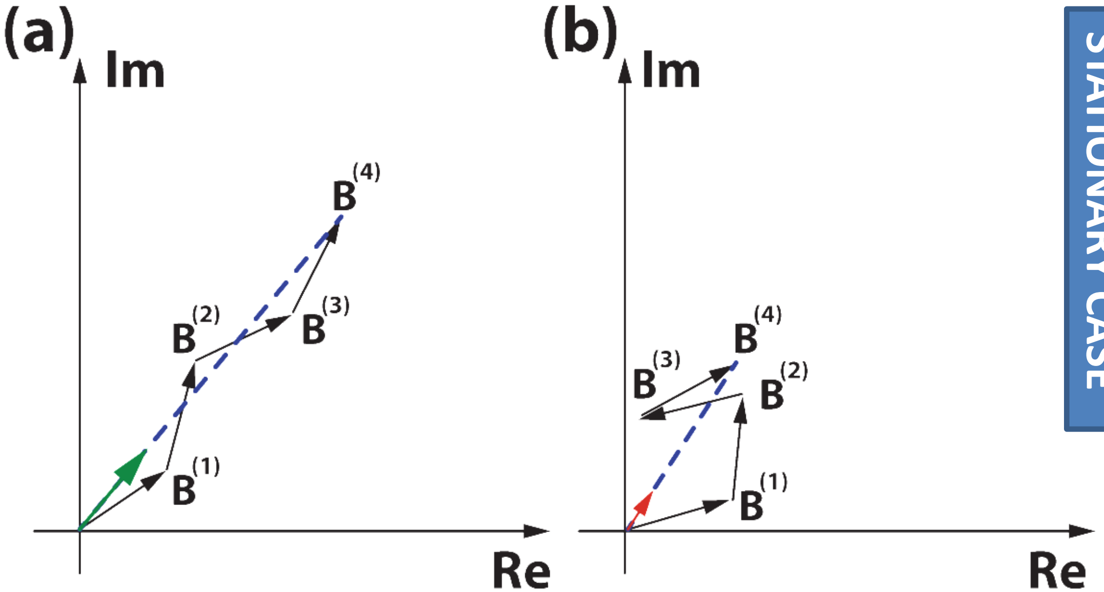


Bicoherence for stationary processes

- Used for detecting quadratic nonlinearities

$$B(f_1, f_2) = \mathbb{E}\left[X(f_1)X(f_2)X^*(f_1+f_2)\right]$$

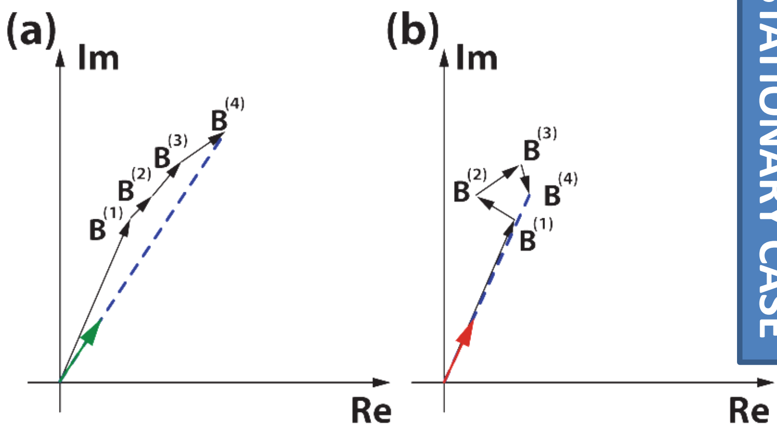
- Bispectrum calculation as random walk on the complex plane

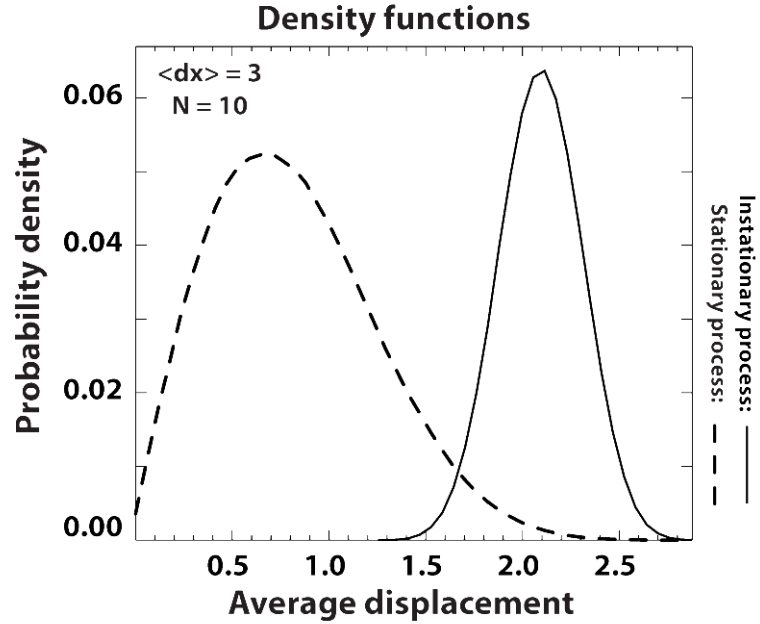


STATIONARY CASE

Bicoherence for instationary case

- Due to instationarity
  „false high” bicoherence
- Random walk with same total length, but different step length
- Significant differences in the probability density functions of bispectrum

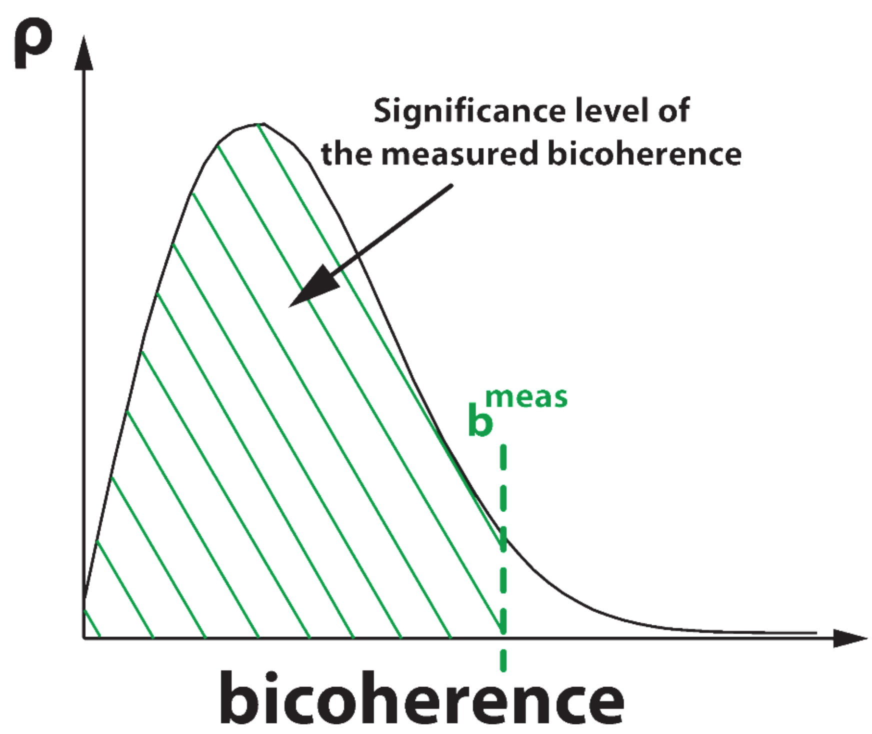




Estimation of bicoherence significance

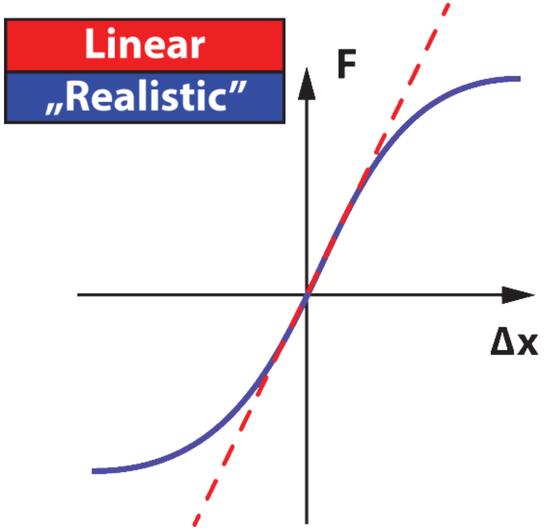
- Randomized bicoherence density function generated from instationary amplitudes
- Significance level can be estimated for measured value

$$\alpha = \int_0^{b^m} \rho(b)db$$



Method testing with different model-systems

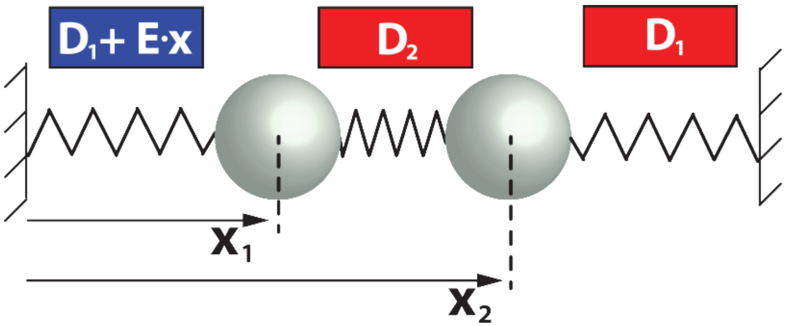
- Parameters of the model-system adjusted to later investigation
  - 30 ms simulated
  - 2 MHz sampling frequency
  - 10-30% additive white noise



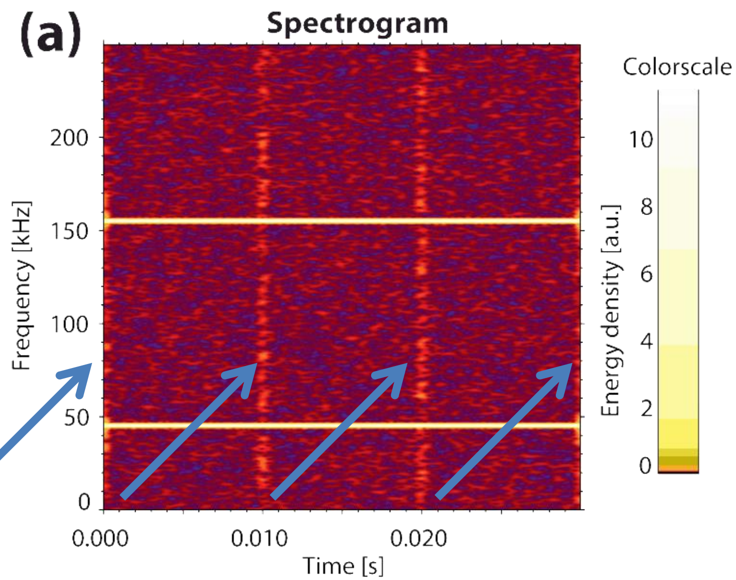
$$m \equiv 1$$

$$\ddot{x}_1 = -(D_1 + E x_1) x_1 + D_2 (x_2 - x_1)$$

$$\ddot{x}_2 = -D_1 x_2 - D_2 (x_2 - x_1)$$

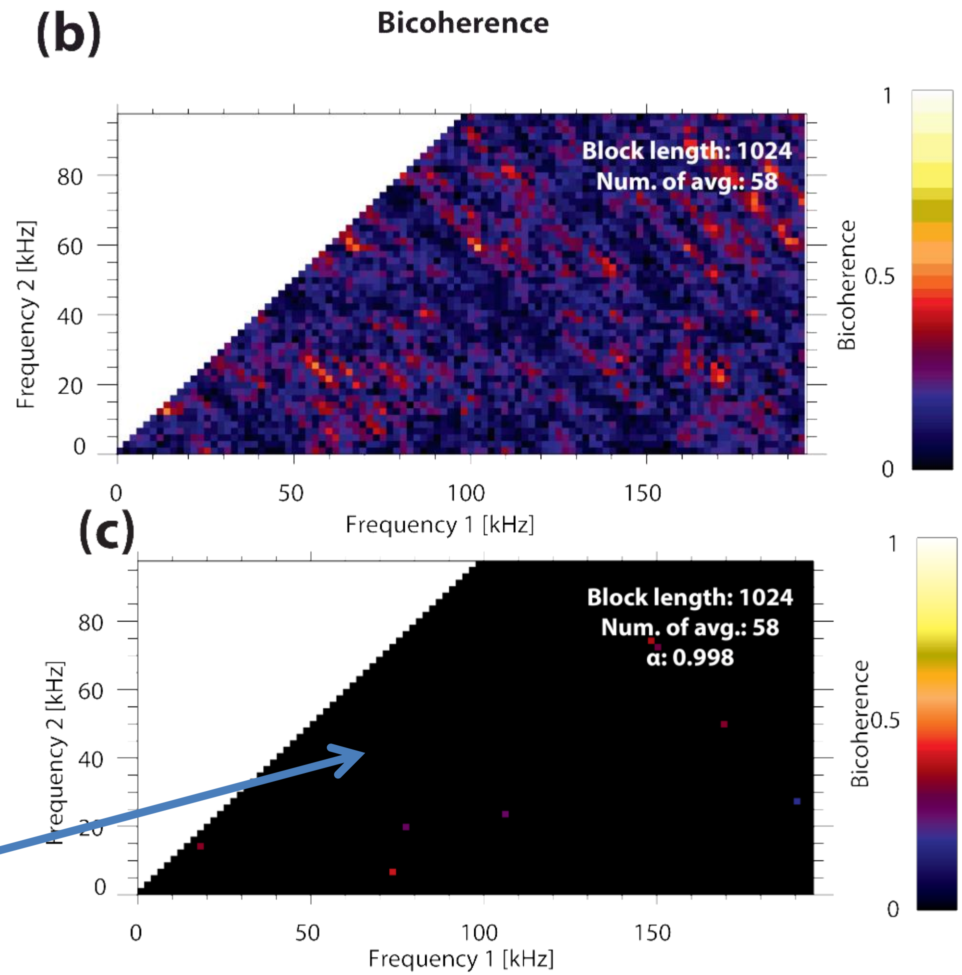


Stationary modes with additive broadband perturbations – linear case

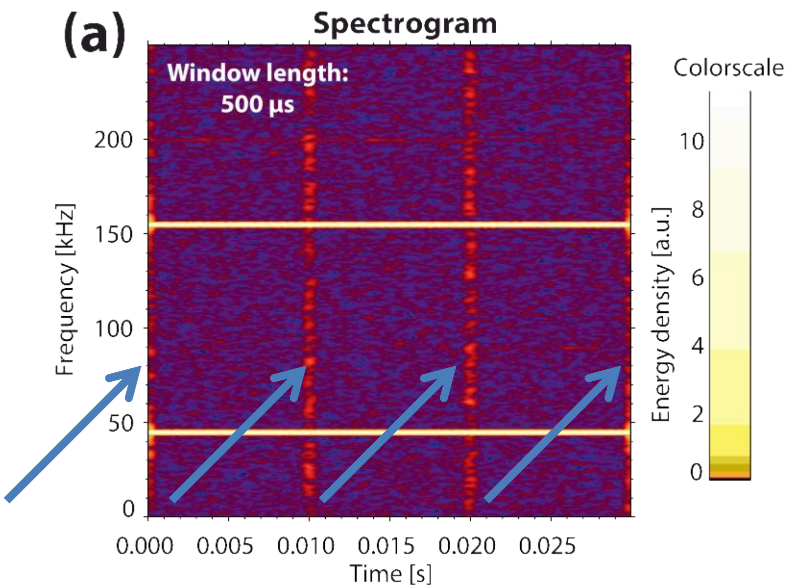


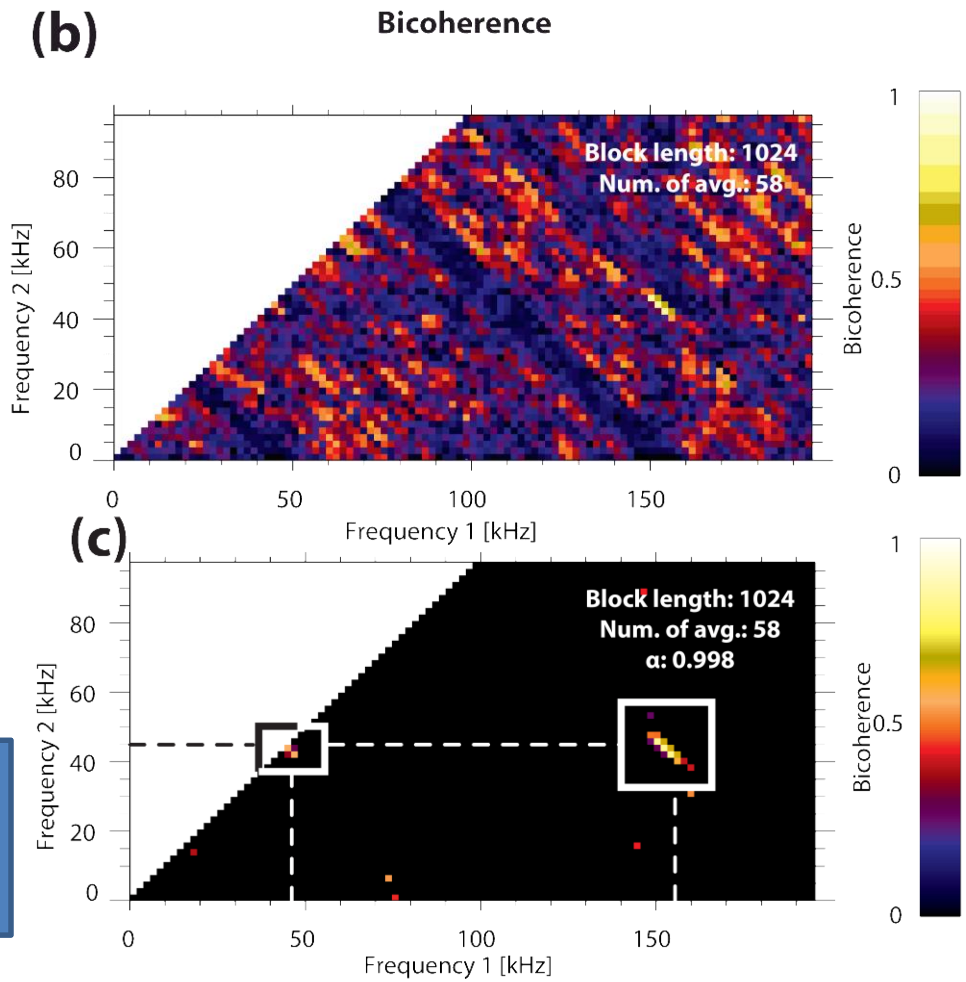
$$Tf(t,\omega) = \langle f, g_{t,\omega} \rangle$$

- 45- ,155 kHz modes
- Broadband perturbations (4)
- No significante high bicoherence



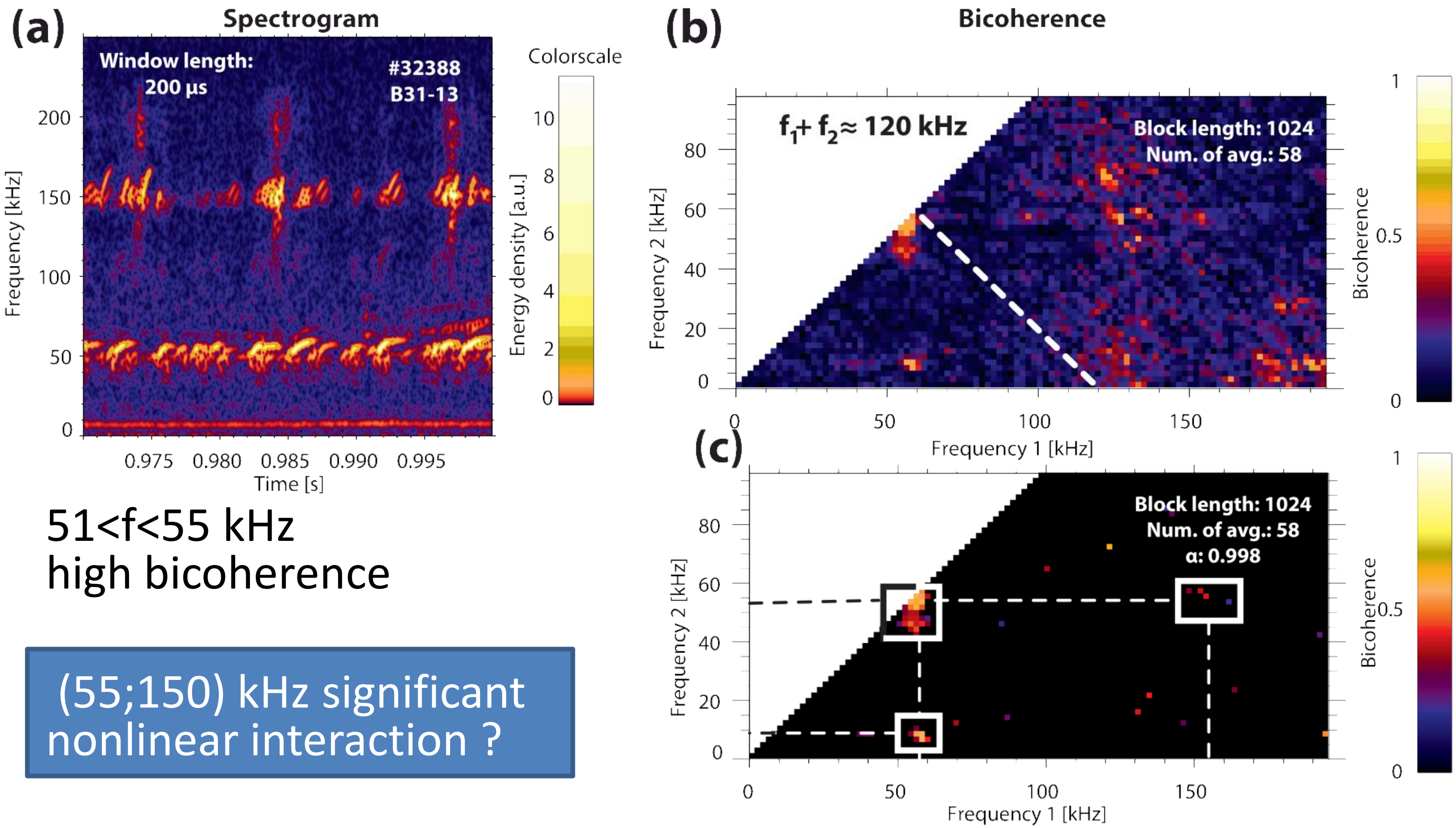
Stationary modes with additive broadband perturbations – nonlinear case





- Strongly nonlinear 45 kHz
- Significante high bicoherence between attached frequencies (45;155) kHz

Application on real measurements



- 51<f<55 kHz
  high bicoherence
- (55;150) kHz significant nonlinear interaction ?

WP6 Status and plans

- **Develop a standard set of tools for the linear characterisation of chirping modes, comparison to simulations; linear characterization of chirping modes demonstrated on EGAMs, BAEs and bursting TAEs at the ASDEX Upgrade tokamak (2017)**
  Development in progress: 2D mode number + radial eigenfunction identification

- **Characterise the non-linear interactions by higher order spectra and band-power correlation with careful consideration of error propagation and significance levels, comparison to simulations; quantitative study of non-linear interactions of various fast particle-related transient modes (e.g. EGAMs, BAEs and/or bursting TAEs) with special emphasis on detecting wave-wave coupling (2018)**
  Development in progress: statistics of higher order spectra for non-stationary signals, testing on simple non-linear model systems