

Linear and non-linear characterization of transient waves and their interactions

Kick-off meeting for EnR project on Nonlinear interaction of Alfvénic and turbulent fluctuations in burning plasmas

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ASDEX Upgrade

19 January 2017

WP6 Deliverables

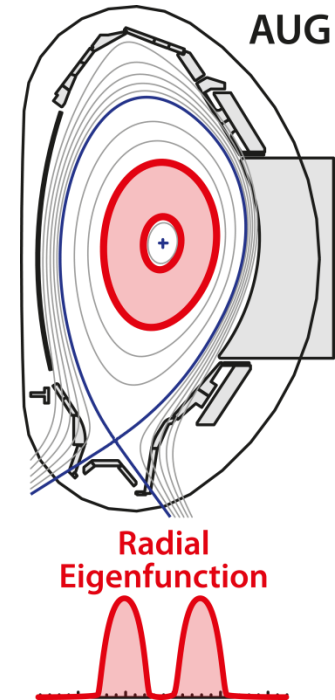
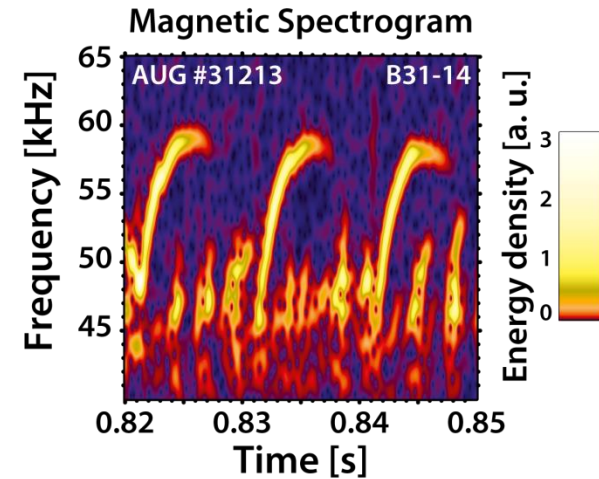
- **Develop a standard set of tools for the linear characterisation of chirping modes, comparison to simulations; linear characterization of chirping modes demonstrated on EGAMs, BAEs and bursting TAEs at the ASDEX Upgrade tokamak (2017)**
- **Characterise the non-linear interactions by higher order spectra and band-power correlation with careful consideration of error propagation and significance levels, comparison to simulations; quantitative study of non-linear interactions of various fast particle-related transient modes (e.g. EGAMs, BAEs and/or bursting TAEs) with special emphasis on detecting wave-wave coupling (2018)**

Justification

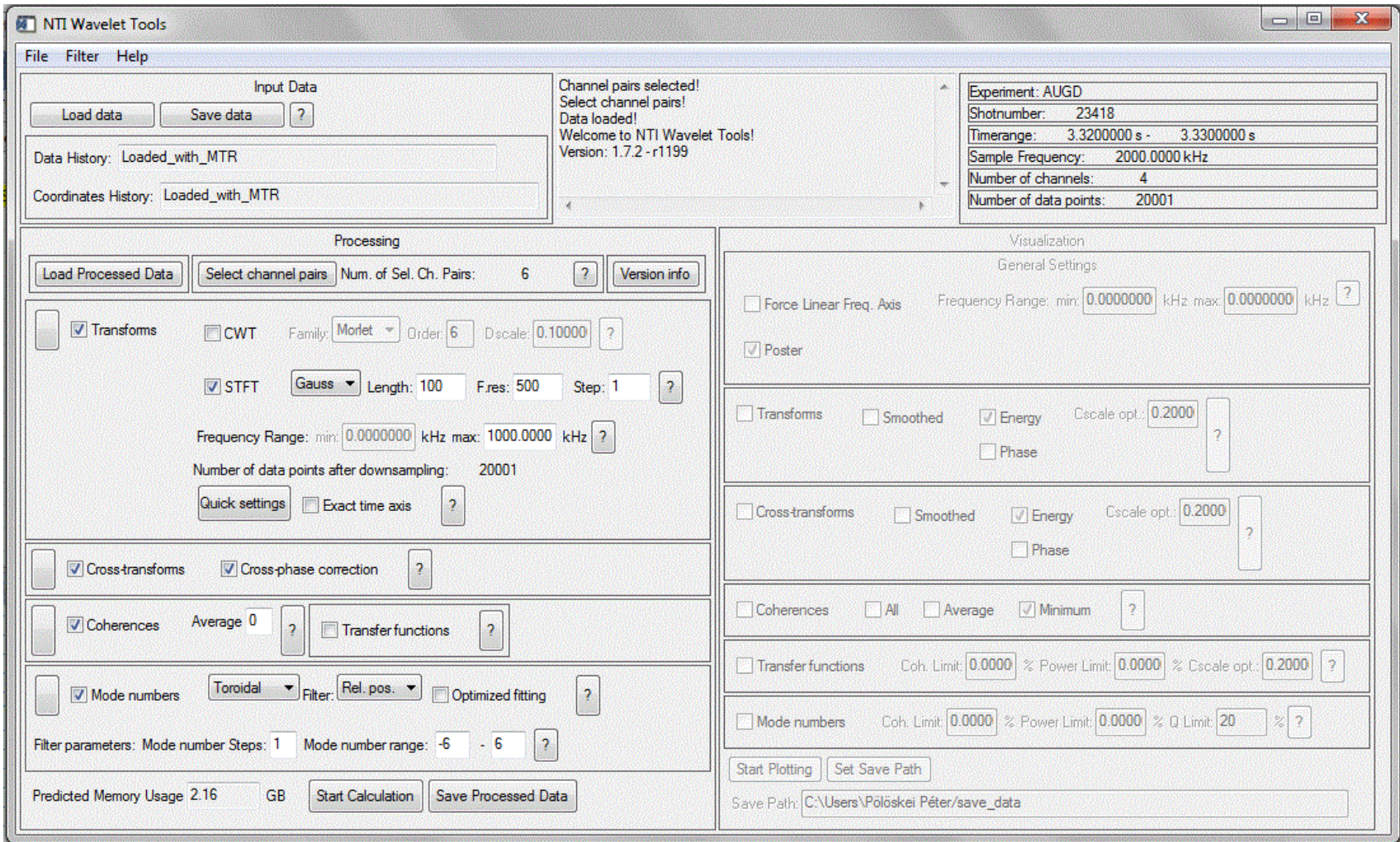
- Data processing aimed at focus issues identified by theory research
- Custom developed data analysis methods aimed at characterizing fast transient phenomena in noisy environment
- Quantitative analysis with thorough evaluation of significance levels

History

- 2002 Toolbox development for time-frequency analysis of plasma transients starts
- 2005 First applications at ASDEX Upgrade (mode structure)
- 2011 NTI Wavelet Tools GUI deployed to ASDEX Upgrade
- 2012 Move to quantitative analysis with error bars
- 2013 Start theory-motivated data analysis



NTI Wavelet Tools



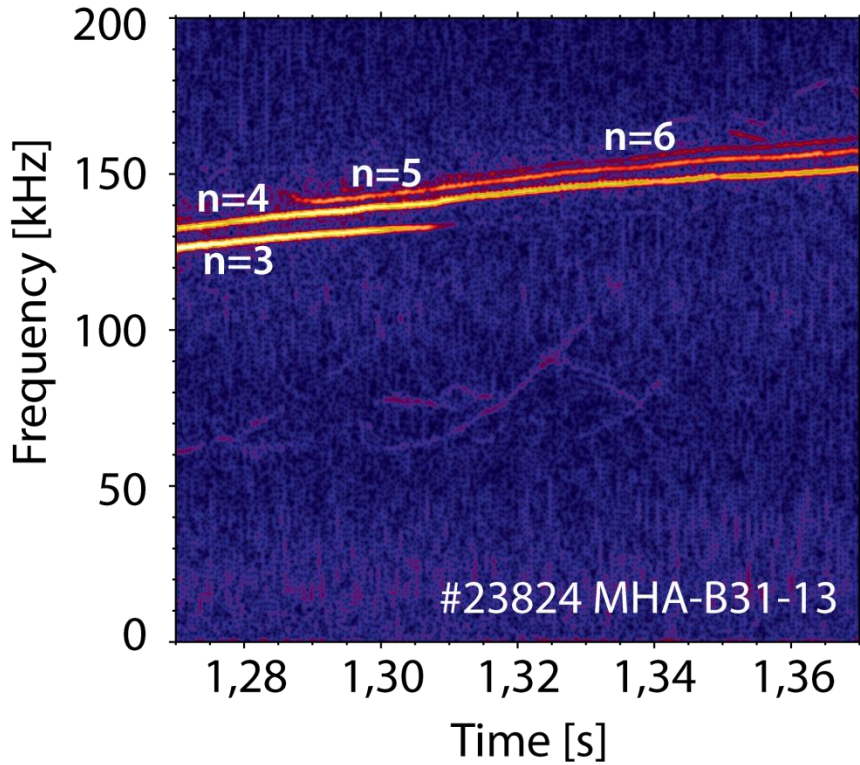
The screenshot shows the NTI Wavelet Tools software interface. The window title is "NTI Wavelet Tools". The interface is divided into several sections:

- Input Data:** Contains "Load data" and "Save data" buttons. Below are "Data History: Loaded_with_MTR" and "Coordinates History: Loaded_with_MTR" text boxes.
- Channel Pairs:** A status box on the right says "Channel pairs selected! Select channel pairs! Data loaded! Welcome to NTI Wavelet Tools! Version: 1.7.2 -r1199".
- Experiment Info:** A table on the right shows:

Experiment:	AUGD
Shotnumber:	23418
Timerange:	3.3200000 s - 3.3300000 s
Sample Frequency:	2000.0000 kHz
Number of channels:	4
Number of data points:	20001
- Processing:**
 - Buttons: "Load Processed Data", "Select channel pairs", "Num. of Sel. Ch. Pairs: 6", "Version info".
 - Transforms:** Transforms, CWT, Family: Morlet, Order: 6, Dscale: 0.10000. STFT, Gauss, Length: 100, F.res: 500, Step: 1. Frequency Range: min: 0.0000000 kHz max: 1000.0000 kHz. Number of data points after downsampling: 20001. Quick settings, Exact time axis.
 - Cross-transforms:** Cross-transforms, Cross-phase correction.
 - Coherences:** Coherences, Average: 0, Transfer functions.
 - Mode numbers:** Mode numbers, Toroidal, Filter: Rel. pos., Optimized fitting. Filter parameters: Mode number Steps: 1, Mode number range: -6 - 6.
- Visualization:**
 - General Settings: Force Linear Freq. Axis, Frequency Range: min: 0.0000000 kHz max: 0.0000000 kHz. Poster.
 - Transforms:** Transforms, Smoothed, Energy, Cscale opt.: 0.2000. Phase.
 - Cross-transforms:** Cross-transforms, Smoothed, Energy, Cscale opt.: 0.2000. Phase.
 - Coherences:** Coherences, All, Average, Minimum.
 - Transfer functions:** Transfer functions, Coh. Limit: 0.0000, % Power Limit: 0.0000, % Cscale opt.: 0.2000.
 - Mode numbers:** Mode numbers, Coh. Limit: 0.0000, % Power Limit: 0.0000, % Q Limit: 20.
- Bottom Panel:** Predicted Memory Usage: 2.16 GB. Buttons: "Start Calculation", "Save Processed Data". "Start Plotting" and "Set Save Path" buttons. Save Path: C:\Users\Pölöskei Péter\save_data.

Spectrogram – energy density:

Magnetic coil – plasma waves



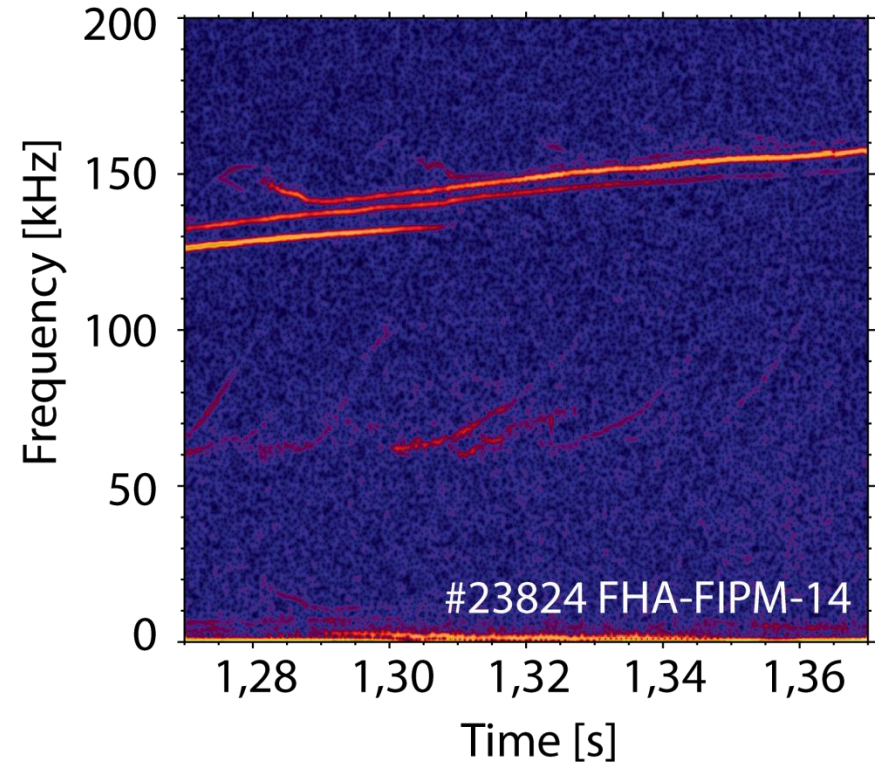
Energy density



0 0,125 0,25
a. u.

Window size:
0,0005 sec

FILD – Fast-ion losses



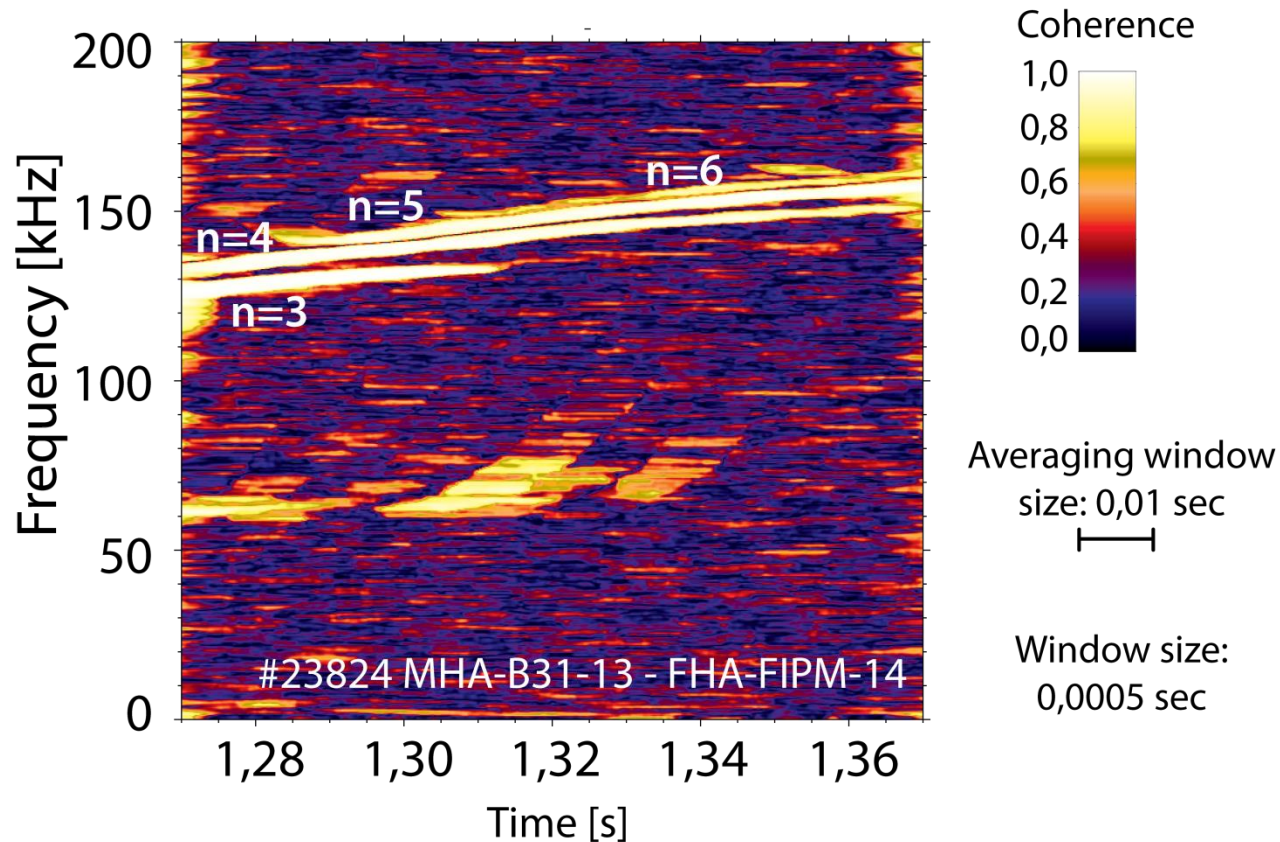
Energy density



0 5E-5 1E-4 1,5E-4
a. u.

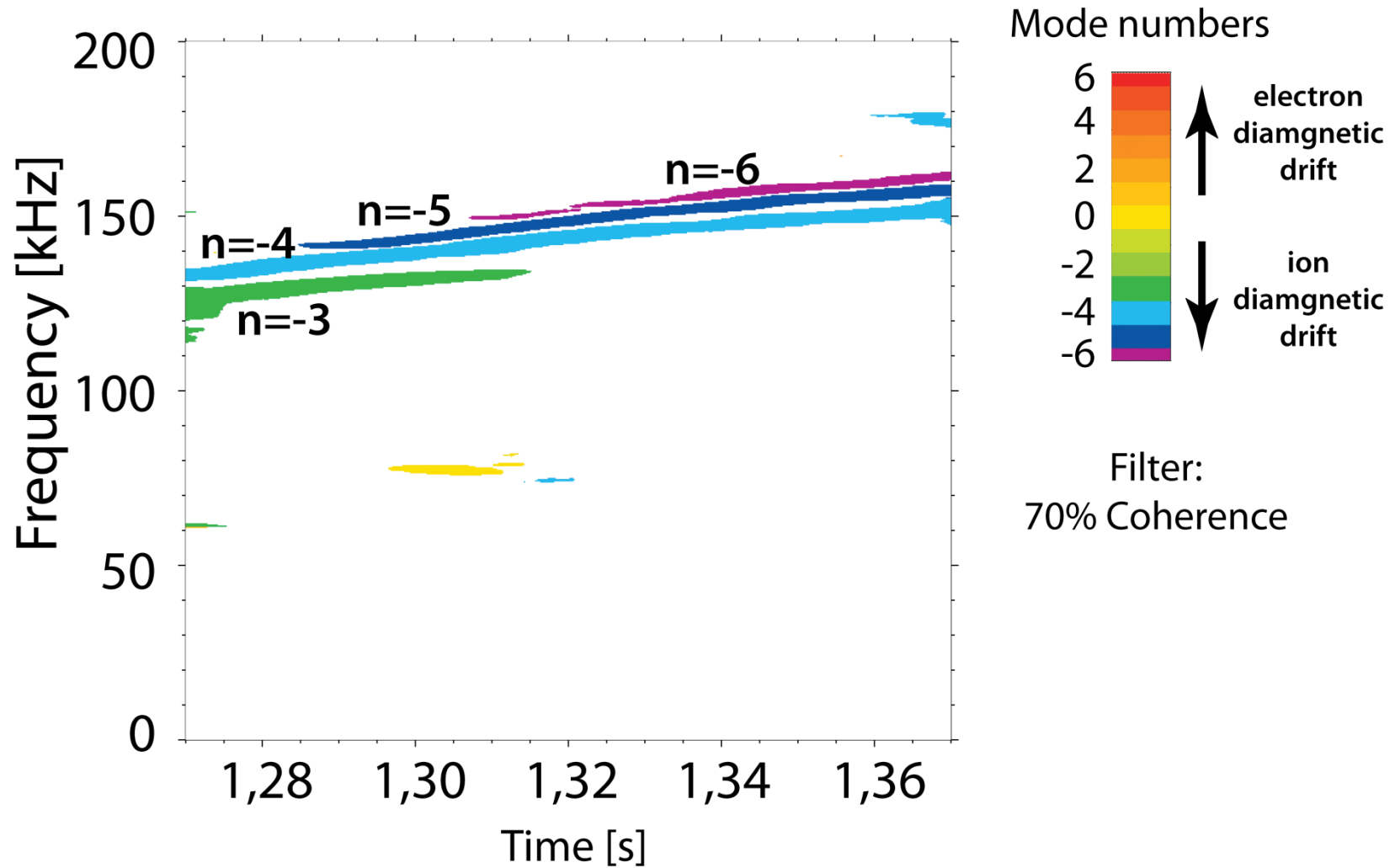
Window size:
0,0005 sec

Coherence: plasma waves – fast-ion losses



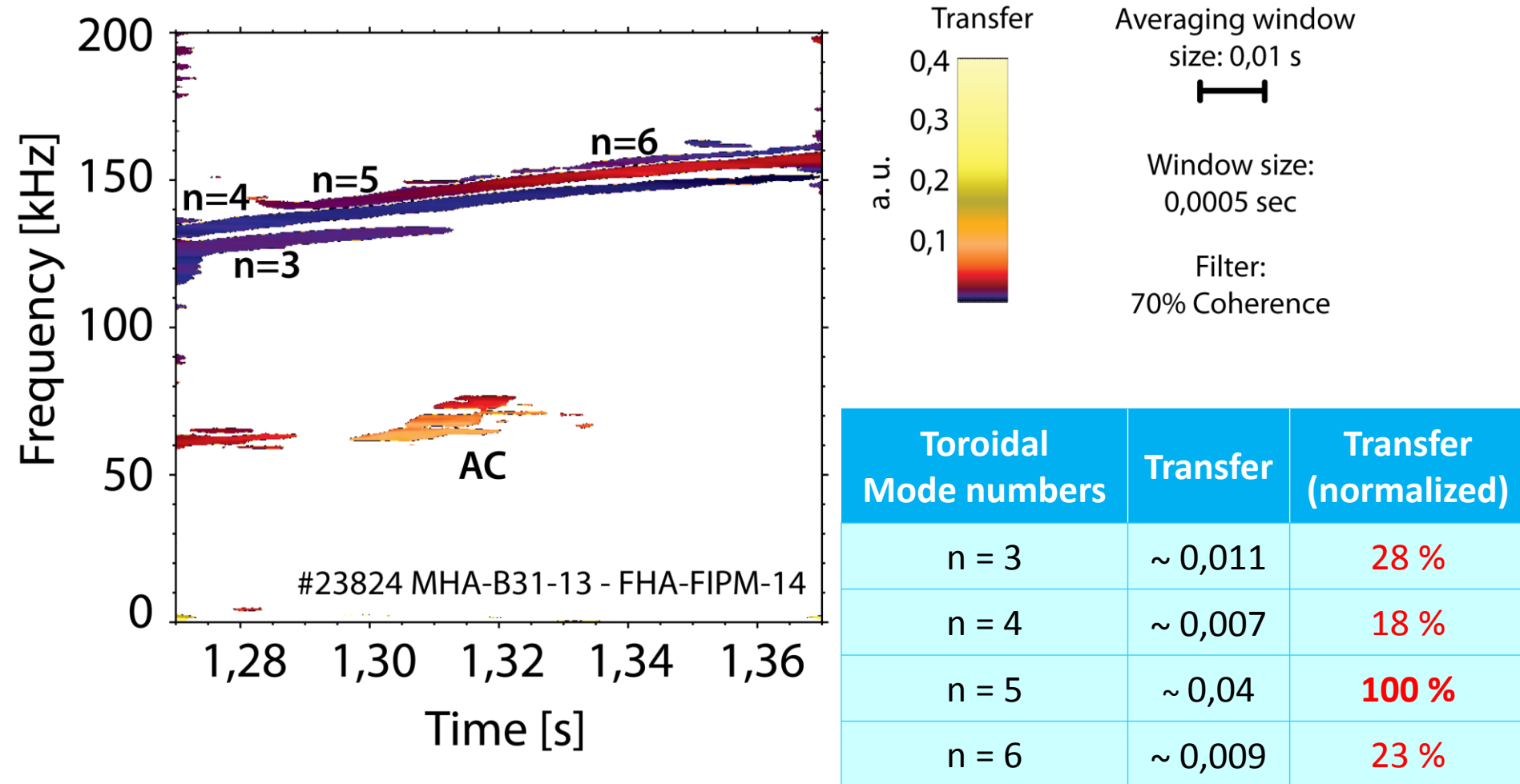
In spite of the high value of the number of averages ($N = 20$), the coherence is near to 1, which suggests **strong linear coupling** between **TAEs** and **fast-ion losses**

Toroidal mode numbers of TAEs



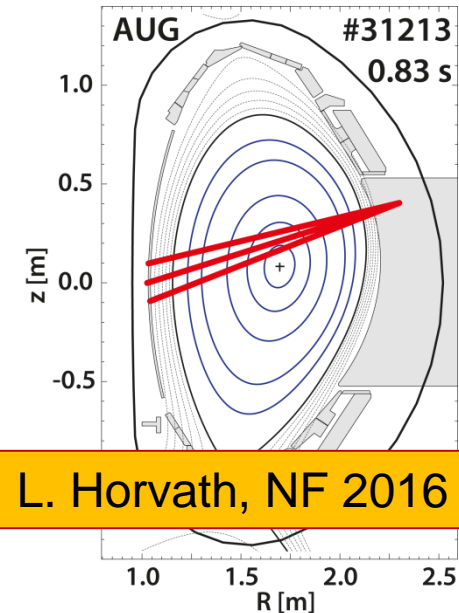
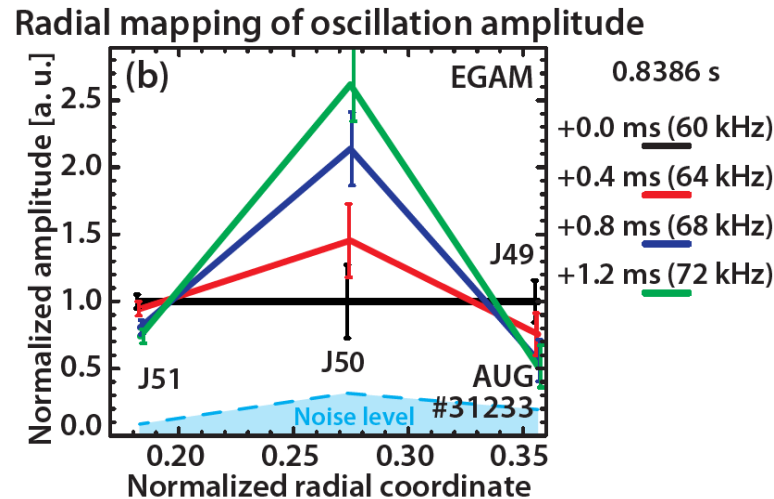
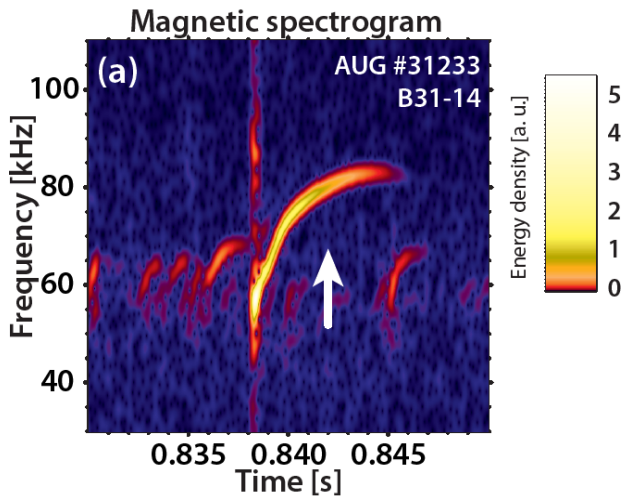
Coherence filtered Transfer function

plasma waves – fast-ion losses



Radial eigenfunction evolution of EPs

- **Downchirping BAEs** and **upchirping EGAMs** have been analysed in a number of discharges.
- **No significant change in the radial structure of BAEs** has been observed.
- In the **EGAM case the results show a slight shrinkage in the radial mode structure** which is consistent with the physical picture.



L. Horvath, NF 2016

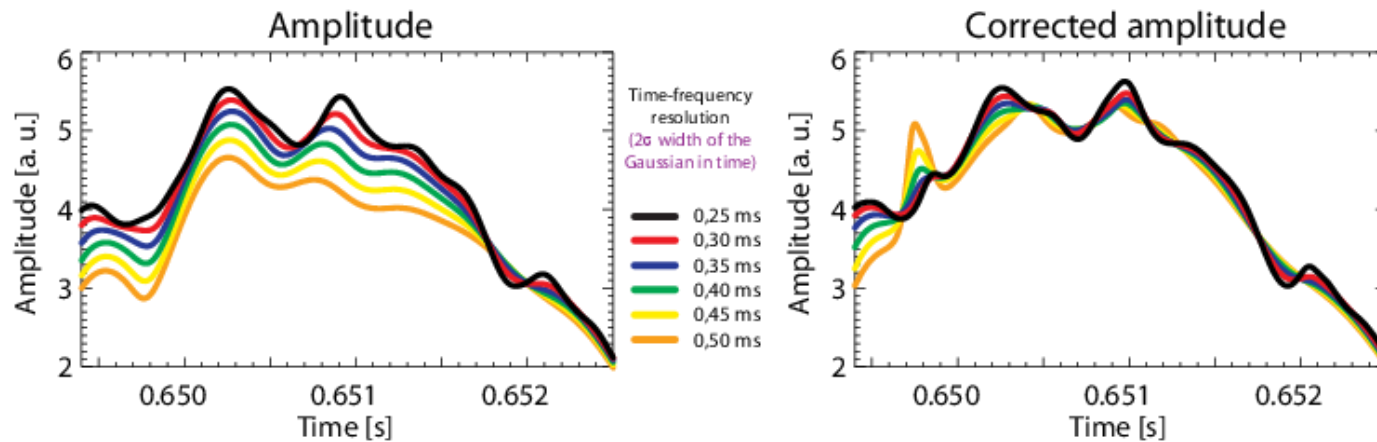
Mode amplitude of chirping waves

- A chirping waves are modelled as: $f(t) = a(t) \cos[\phi(t)]$
- Constant amplitude, constant frequency approximation:

$$a_0(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}} \left| \text{STFT}(t, \omega = \omega_{\text{ridge}}) \right|$$

- Linear chirps:

$$a_1(t) = \sqrt{\frac{2}{\sqrt{\pi}\sigma}} \sqrt[4]{1 + 4\phi''(u)^2\sigma^4} \left| \text{STFT}(t, \omega = \omega_{\text{ridge}}) \right|$$

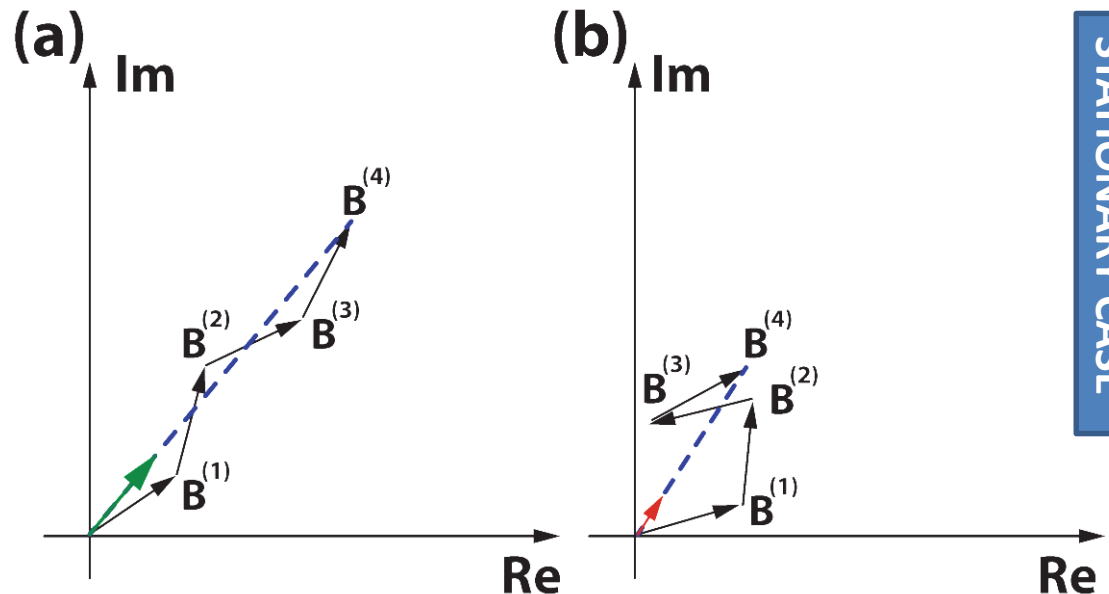


Bicoherence for stationary processes

- Used for detecting quadratic nonlinearities

$$B(f_1, f_2) = \mathbb{E} [X(f_1)X(f_2)X^*(f_1 + f_2)]$$

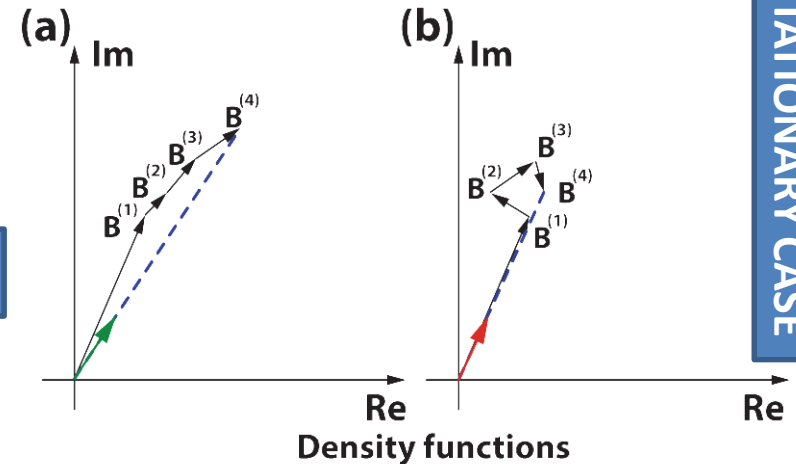
- Bispectrum calculation as random walk on the complex plane



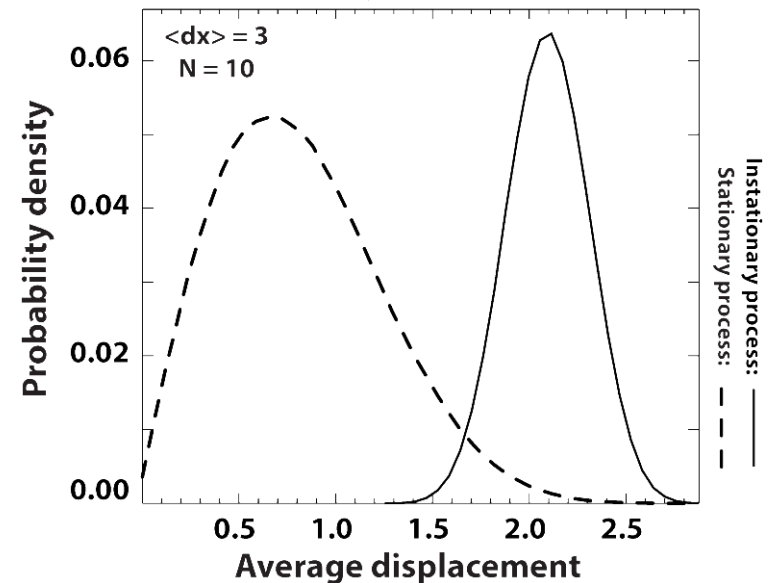
STATIONARY CASE

Bicoherence for instationary case

- Due to instationarity „false high” bicoherence
- Random walk with same total length, but different step length
- Significant differences in the probability density functions of bispectrum



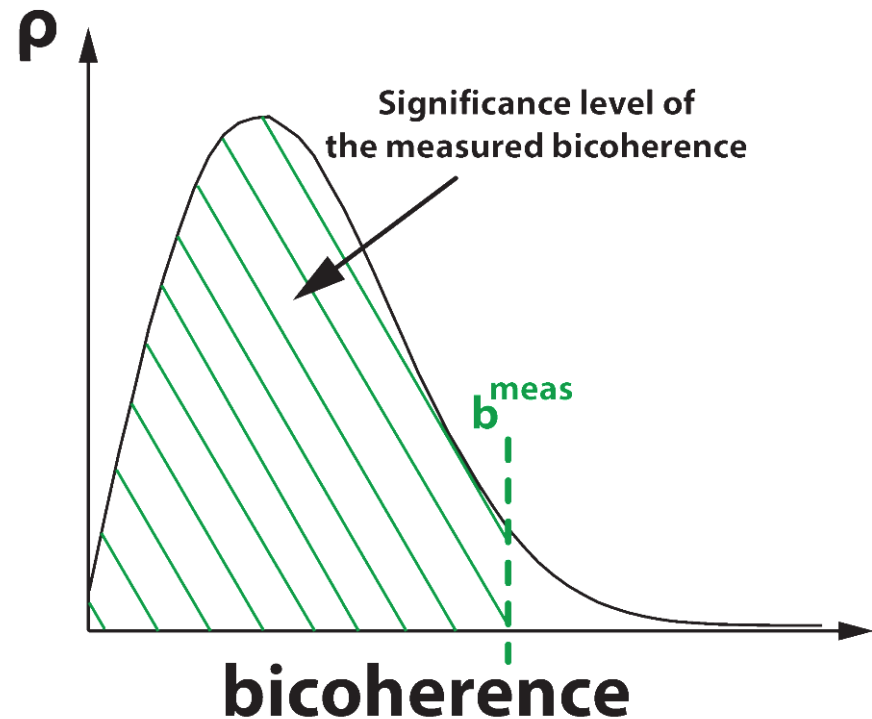
INSTATIONARY CASE



Estimation of bicoherence significance

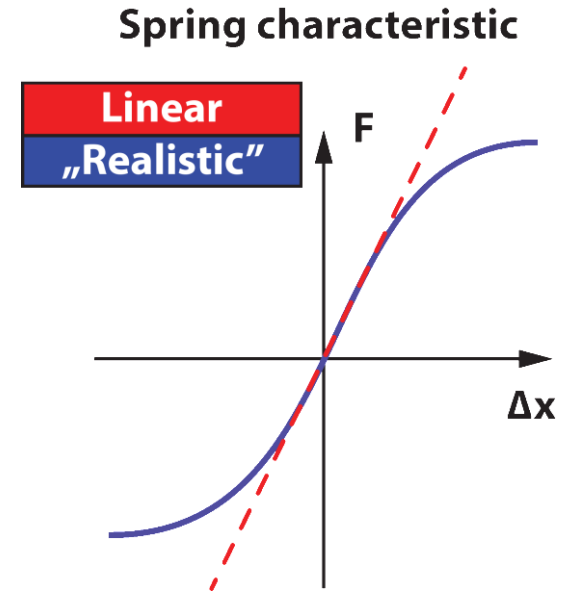
- Randomized bicoherence density function generated from instationary amplitudes
- Significance level can be estimated for measured value

$$\alpha = \int_0^{b^m} \rho(b) db$$



Method testing with different model-systems

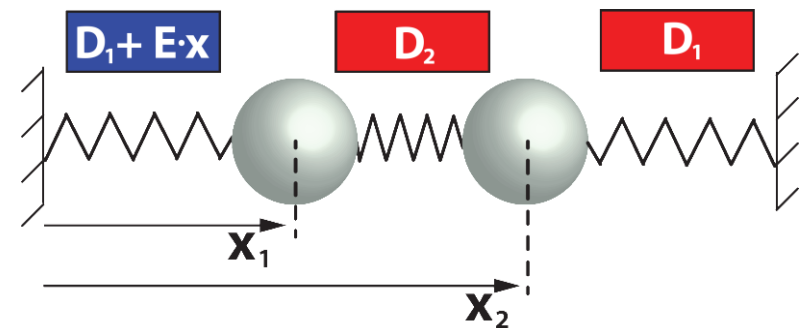
- Parameters of the model-system adjusted to later investigation
 - 30 ms simulated
 - 2 MHz sampling frequency
 - 10-30% additive white noise



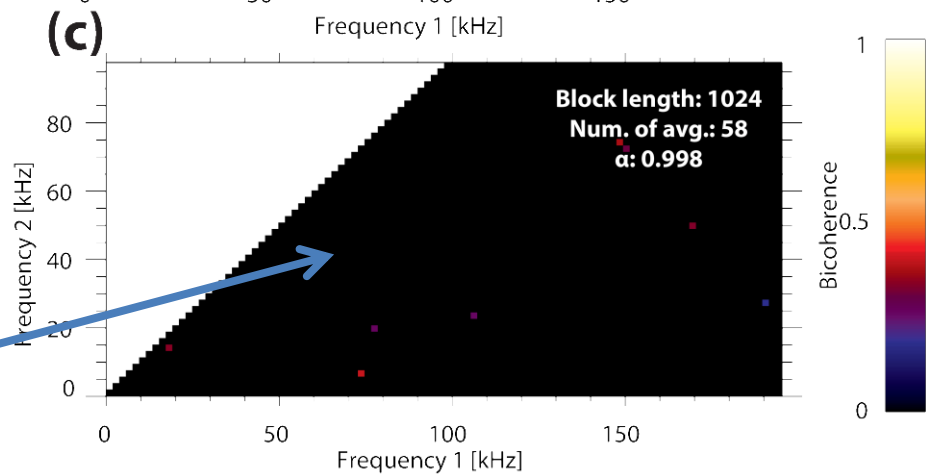
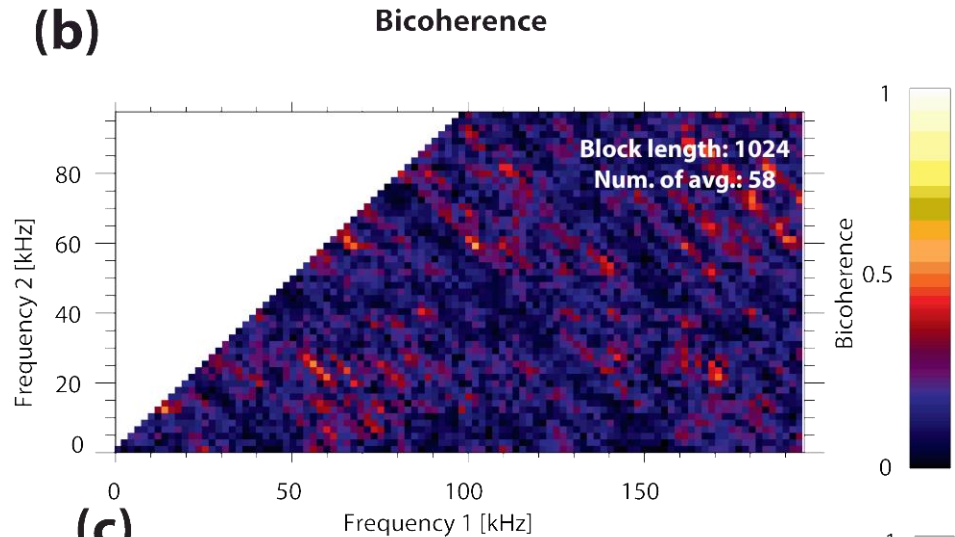
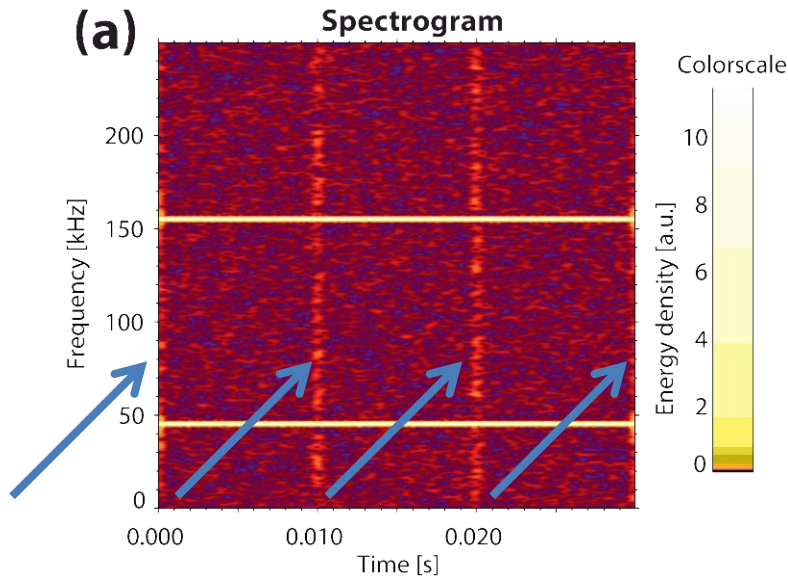
$$m \equiv 1$$

$$\ddot{x}_1 = -(D_1 + Ex_1)x_1 + D_2(x_2 - x_1)$$

$$\ddot{x}_2 = -D_1x_2 - D_2(x_2 - x_1)$$



Stationary modes with additive broadband perturbations – linear case

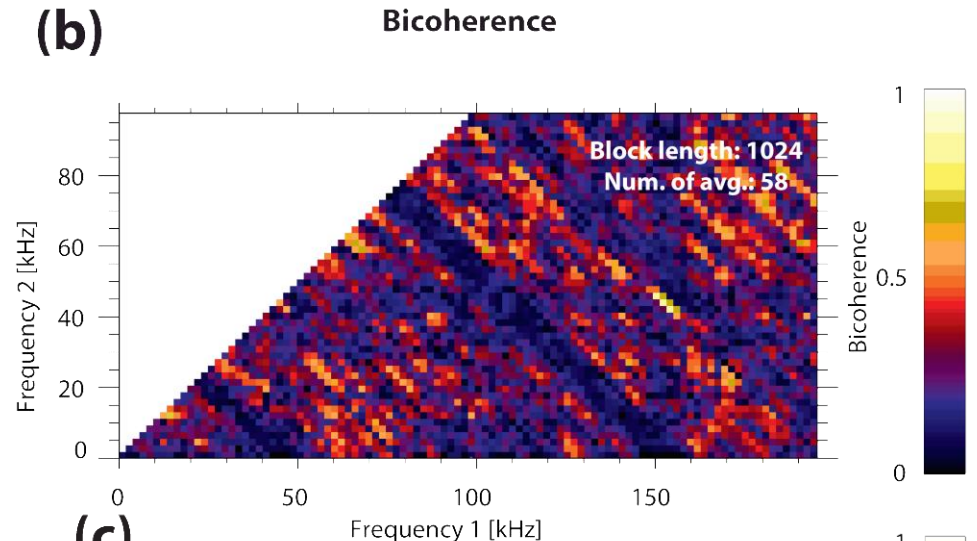
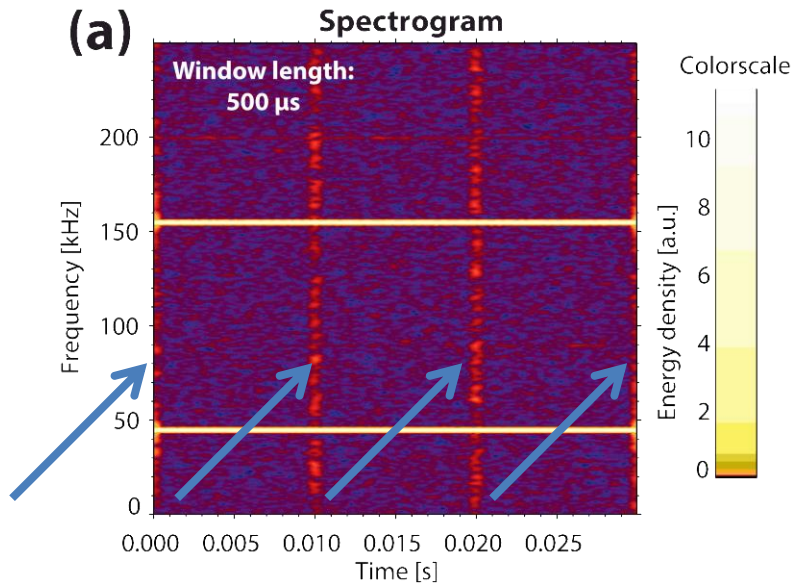


$$Tf(t, \omega) = \langle f, g_{t, \omega} \rangle$$

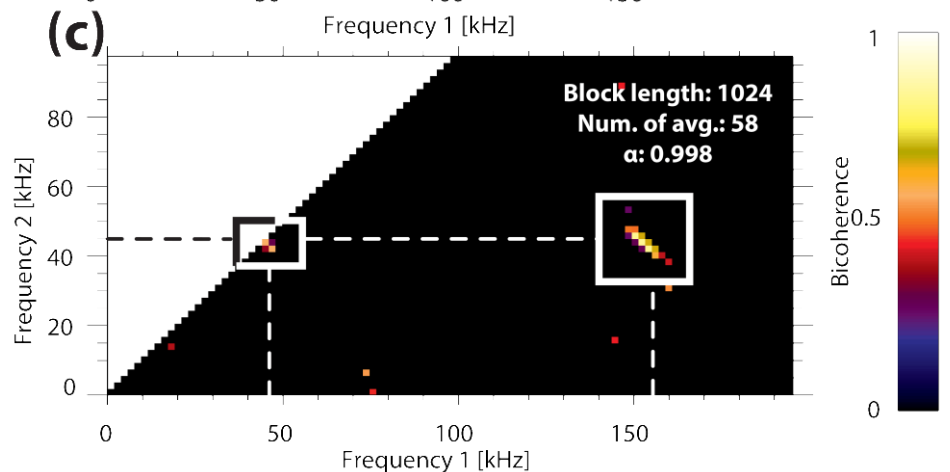
- 45- ,155 kHz modes
- Broadband perturbations (4)

No significant high bicoherence

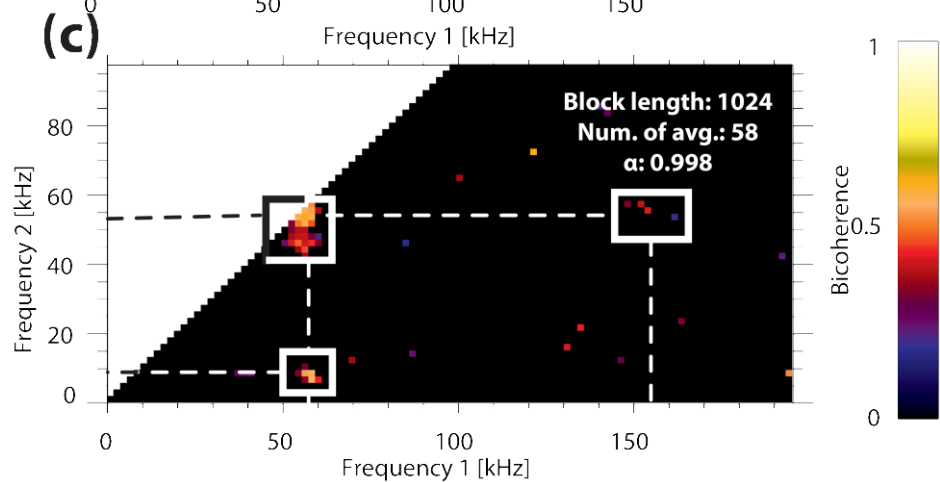
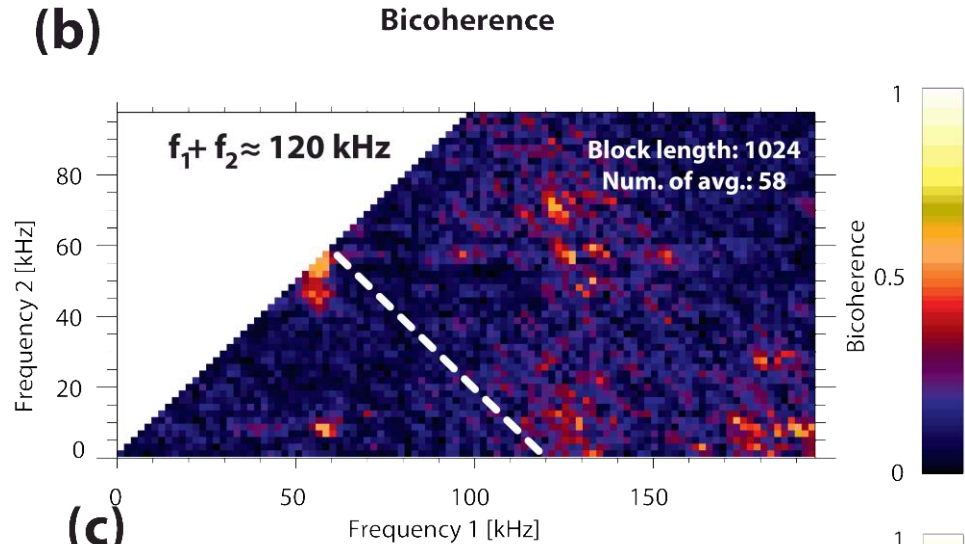
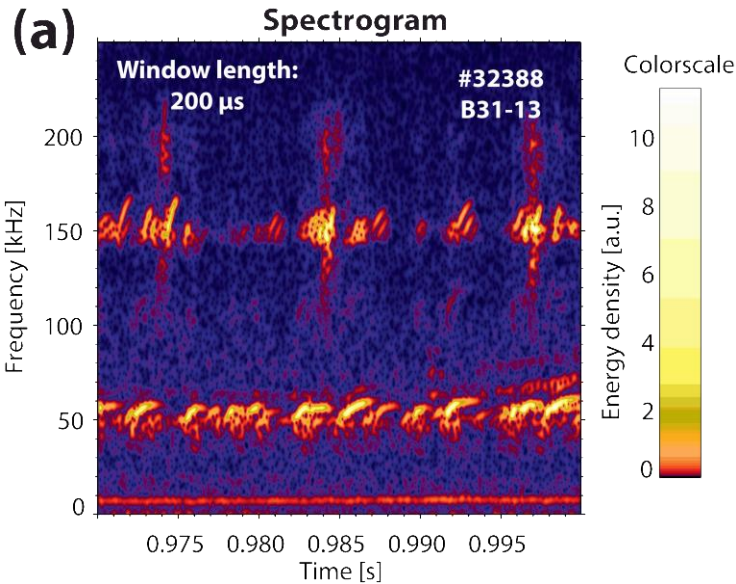
Stationary modes with additive broadband perturbations – nonlinear case



- Strongly nonlinear 45 kHz
- Significant high bicoherence between attached frequencies (45;155) kHz



Application on real measurements



- 51 < f < 55 kHz
high bicoherence

(55;150) kHz significant nonlinear interaction ?

WP6 Status and plans

- **Develop a standard set of tools for the linear characterisation of chirping modes, comparison to simulations; linear characterization of chirping modes demonstrated on EGAMs, BAEs and bursting TAEs at the ASDEX Upgrade tokamak (2017)**
Development in progress: 2D mode number + radial eigenfunction identification
- **Characterise the non-linear interactions by higher order spectra and band-power correlation with careful consideration of error propagation and significance levels, comparison to simulations; quantitative study of non-linear interactions of various fast particle-related transient modes (e.g. EGAMs, BAEs and/or bursting TAEs) with special emphasis on detecting wave-wave coupling (2018)**
Development in progress: statistics of higher order spectra for non-stationary signals, testing on simple non-linear model systems