The fishbone paradigm and the beam plasma system*

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Outline

I. Introduction: summary of 2017 activities

- Physics basis of the fishbone paradigm and theory
- Numerical studies of the beam-plasma system

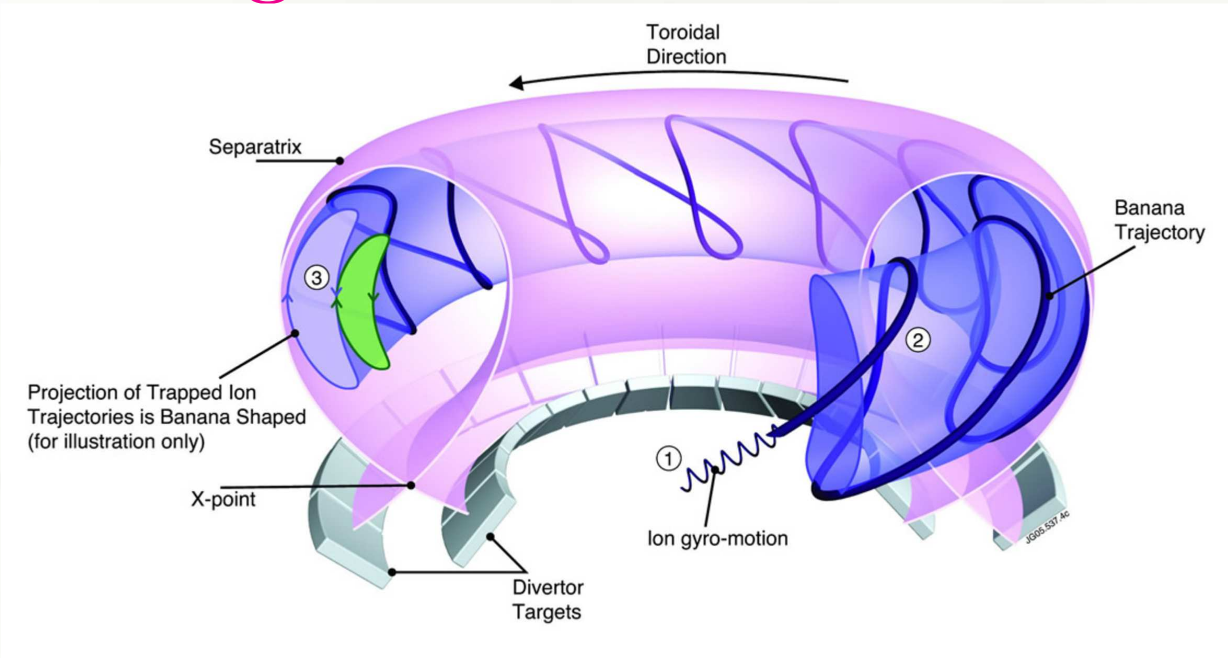
II. Ongoing work: 2018 activities

- Characterization of velocity spread and nonlinear saturation in the beam-plasma system
- Progress in analytic theory of the fishbone paradigm

III. Summary and Discussion

Summary of 2017 activities

Fishbone Paradigm for SAW-EP nonlinear interplay



- Consider $|\omega| \sim |\bar{\omega}_d| \ll |\omega_b| \Rightarrow$ 2 integrals of motion: μ and $J = \oint v_{\parallel} d\ell$.
- The system behaves as non-autonomous, **non-uniform** system with one degree of freedom. Reminiscence of 3D equilibrium system.
- Crucial difference with the beam plasma system: non-autonomous, **uniform** system with one degree of freedom.

[Zonca et al. NJP 2015]

Theoretical approach based on NL GKE

- The fluctuating particle distribution functions are decomposed in adiabatic and nonadiabatic responses as [Frieman and Chen 1982].

$$\delta f = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\left[\delta g - \frac{e}{m}\frac{1}{B_0}\frac{\partial \bar{F}_0}{\partial \mu}\langle \delta L_g\rangle\right] + \frac{e}{m}\left[\frac{\partial \bar{F}_0}{\partial \mathcal{E}}\delta\phi + \frac{1}{B_0}\frac{\partial \bar{F}_0}{\partial \mu}\delta L\right] \quad .$$

Here, $\bar{F}_0$ is the equilibrium guiding-center particle distribution function,

$$\delta L_g = \delta\phi_g - \frac{v_\parallel}{c}\delta A_{\parallel g} = e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\delta L = e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\left(\delta\phi - \frac{v_\parallel}{c}\delta A_\parallel\right) \quad ,$$

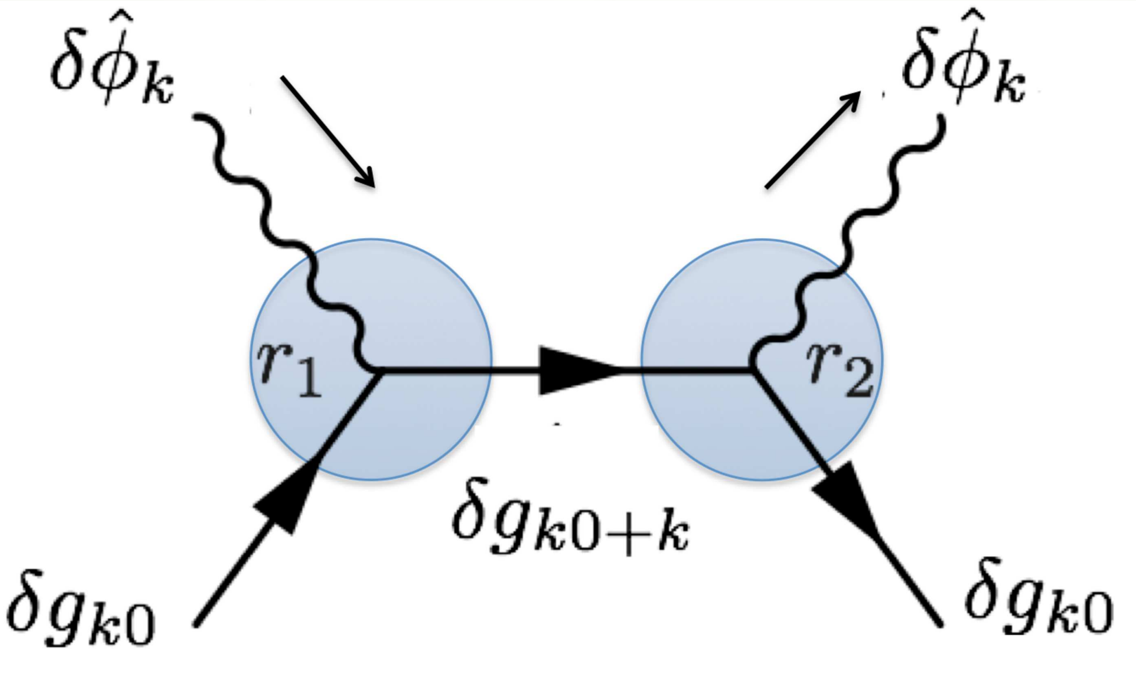
and $\langle\cdots\rangle$ denotes gyrophase averaging and δg satisfies the NL GKE

$$\left(\frac{\partial}{\partial t} + v_\parallel\nabla_\parallel + \boldsymbol{v}_d\cdot\boldsymbol{\nabla}_\perp\right)\delta g = \left(i\frac{e}{m}Q\bar{F}_0\langle\delta L_g\rangle - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\langle\delta L_g\rangle\cdot\boldsymbol{\nabla}\delta g\right)$$

$$Q\bar{F}_0 = i\frac{\partial \bar{F}_0}{\partial \mathcal{E}}\frac{\partial}{\partial t} - i\frac{\boldsymbol{b}\times\boldsymbol{\nabla}\bar{F}_0}{\Omega}\cdot\boldsymbol{\nabla} \quad .$$

- Introducing the generator of coordinate transformation to banana centers, $\delta g_k = e^{-iQ_k}\delta\bar{G}_k$, with $v_\parallel \nabla_\parallel Q_k - \tilde{\boldsymbol{v}}_d \cdot \boldsymbol{k}_\perp \equiv 0$ and denoting bounce averaging as $\overline{[\ldots]} = \left(\oint d\ell/v_\parallel\right)^{-1} \oint [\ldots] d\ell/v_\parallel$, [neglect $\mathcal{O}(1/nq)$]

$$(\bar{\omega}_d - \omega)_k \delta\bar{G}_k = \overline{\left[e^{iQ_k} \left(\frac{e}{m} Q\bar{F}_0 \left\langle \delta L_g \right\rangle_k + \frac{ic}{B_0} \boldsymbol{b} \times \boldsymbol{\nabla} \left\langle \delta L_g \right\rangle \cdot \boldsymbol{\nabla} \delta g \right) \right]}$$



[Dupree 66; Laval & Pesme 84,99]

- Focus on a test mode and diagonal nonlinear interactions. Furthermore, note

$$\frac{\boldsymbol{b}}{B_0}\cdot\boldsymbol{\nabla}A\times\boldsymbol{\nabla}G=\frac{1}{d\psi/dr}\left(\frac{\partial A}{\partial\zeta}\frac{\partial G}{\partial r}-\frac{\partial G}{\partial\zeta}\frac{\partial A}{\partial r}\right)+\mathcal{O}\left(\frac{1}{|nq|}\right).$$

- This yields

$$\begin{aligned}(\bar{\omega}_d-\omega)_{k0}\delta\bar{G}_{k0} &= \overline{\left[e^{iQ_{k0}}\left(\frac{e}{m}Q\bar{F}_0\left\langle\delta L_g\right\rangle_{k0}\right)\right]}\\ &+\frac{c}{d\psi/dr}\frac{\partial}{\partial r}\sum_{\boldsymbol{k}}n\left[\overline{e^{iQ_{k0}}\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\delta\bar{G}_{k+k0}\right]\\ &+\frac{c}{d\psi/dr}\sum_{\boldsymbol{k}}n_0\left[\overline{e^{iQ_{k0}}\partial_r\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\delta\bar{G}_{k+k0}\right]\end{aligned}$$

- Note the peculiarity with respect to the beam plasma system: bounce averaging and finite orbit width bring in nonlocality and integral plasma response.

- Isolating the diagonal nonlinear response

$$(\bar{\omega}_d - \omega)_{k+k0}\delta\bar{G}_{k+k0} \sim -\frac{nc}{d\psi/dr}\left[\overline{e^{iQ_{k+k0}}\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}\partial_r\delta\bar{G}_{k0}\right] + \frac{n_0 c}{d\psi/dr}\left[\overline{e^{iQ_{k+k0}}\partial_r\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}\delta\bar{G}_{k0}\right]$$

- Note the role of the symmetric part of the spectrum vs. the anti-symmetric part in determining the structure of the diagonal nonlinear response.
- This is crucial as it impacts the momentum transport (Z. Lu contribution to NAT).
- Simplified treatment is possible, e.g., assuming a symmetric fluctuation spectrum (NAT contribution 2017, and NLED-NAT overview paper to be submitted to JPP, Athens invited).
- More complete analysis is in progress. This is for illustrating the qualitative physics

- Formally, this equation can be written as (geometry effect through $\bar{\omega}_d$, $\rho = r - r_0$, r_0 radial localization region of symmetric fluctuation spectrum)

$$\left\{ i \left[\bar{\omega}'_{d0}\rho - (\omega_0 - \bar{\omega}_{d0}) \right] - \Delta - \frac{\partial}{\partial \rho} D \frac{\partial}{\partial \rho} \right\} \delta \bar{G}_{k0} = \mathcal{L}_{k0} + [\text{NL OFF DIAGONAL}]$$

  - Resonant particle response (Δ, D real): resonance broadening
  - Non-resonant particle response (Δ, D imaginary): non-linear frequency shift

- Both effects are crucial for the nonlinear dynamics.

- Difference of the present approach with others:
  - Others: various kinds of models
  - Present approach: based on first principle calculation

- Question: is it possible to calculate fluctuation induced modification of wave-particle resonances in cases of practical interest? (ongoing work)

Phase space zonal structures and transport in tokamaks

- Specialize equations above to phase space zonal structures (PSZS), which are defined as those unaffected by fast collisionless damping ⇒ important on transport time scale [Falessi, ArXiV16, POP18].

- Considering $\partial_\mu \bar{F}_0 = 0$ and since PSZS are undamped by (fast) collisionless dissipation mechanisms, $\delta g_z = e^{-iQ_z}\delta\bar{G}_z$ and

$$\delta f_z \;=\; e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\delta g_z + \frac{e}{m}\delta\phi_{0,0}\frac{\partial \bar{F}_0}{\partial \mathcal{E}} = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} e^{-iQ_z}\delta\bar{G}_z + \frac{e}{m}\delta\phi_{0,0}\frac{\partial \bar{F}_0}{\partial \mathcal{E}} \;.$$

- Here, $0,0$ subscript to $\delta\phi$ indicates the $m = n = 0$ component; and, given $k_z \equiv (-i\partial_r)$, e^{iQ_z} controls transformation to banana center frame; with

$$Q_z = F(\psi)\left[\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega}\right)}\right]\frac{k_z}{d\psi/dr}$$

- The collisionless evolution equation for phase space zonal structures is [Falessi, ArXiV16]

$$\partial_t \delta \bar{G}_z = \overline{\left[e^{iQ_z} \left(-\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \langle \delta L_g \rangle_{0,0} \right) \right]} + \frac{c}{d\psi/dr} \frac{\partial}{\partial r} \sum_{\boldsymbol{k}} in \left[\overline{e^{iQ_z} \langle \delta L_g \rangle_{-k} e^{-iQ_k} \delta \bar{G}_k} \right]$$

- The evolution equation for phase space zonal structures is valid on a time scale up to $\mathcal{O}(\delta^{-3})\Omega^{-1}$, $\delta \sim \rho/L$, consistent with [Hinton and Hazeltine 76; Frieman and Chen 1982].

- Collisions can be included by suitable gyro- and bounce-averaged collision operator [Brizard et al 2010].

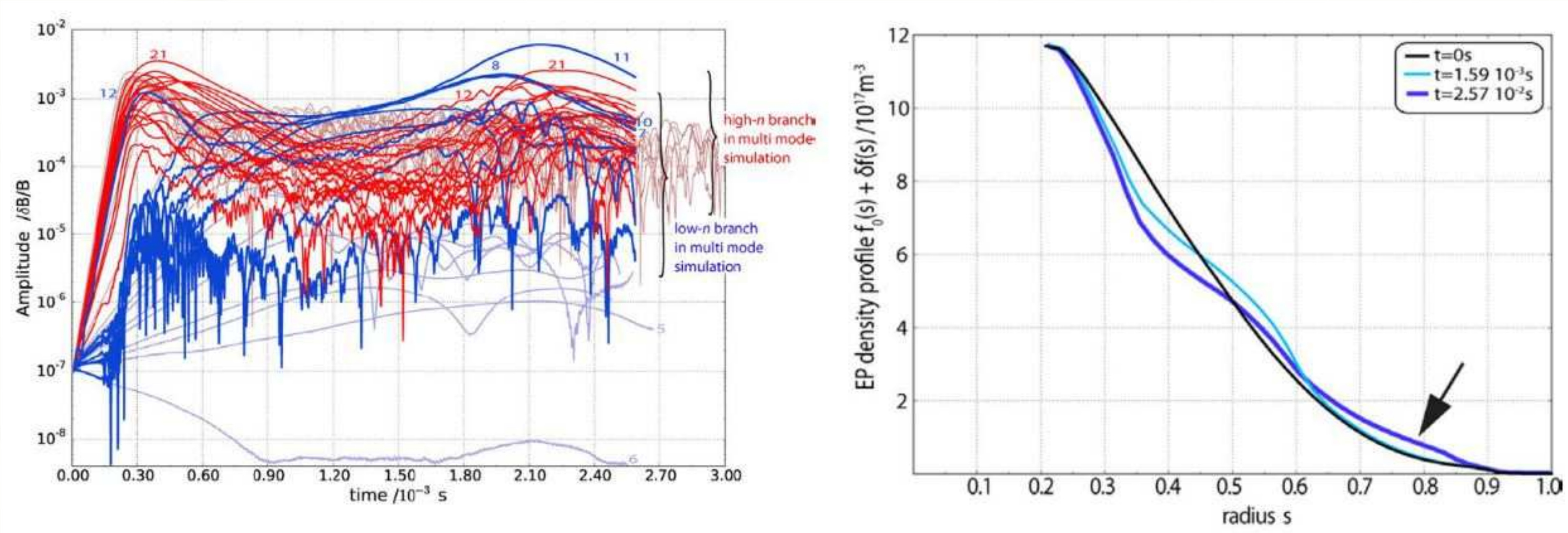
- Adding collisions, the density transport equation can be written, given the radial particle flux $\boldsymbol{\Gamma} \equiv n\boldsymbol{V}$:

$$\langle \langle \partial_t f \rangle_v \rangle_\psi = -\frac{1}{V'} \frac{\partial}{\partial \psi} \left[V' \langle n\boldsymbol{V} \cdot \nabla \psi \rangle_{\psi c} + V' \langle n\boldsymbol{V} \cdot \nabla \psi \rangle_{\psi NC} + V' \langle n\boldsymbol{V} \cdot \nabla \psi \rangle_{\psi gk} \right]$$

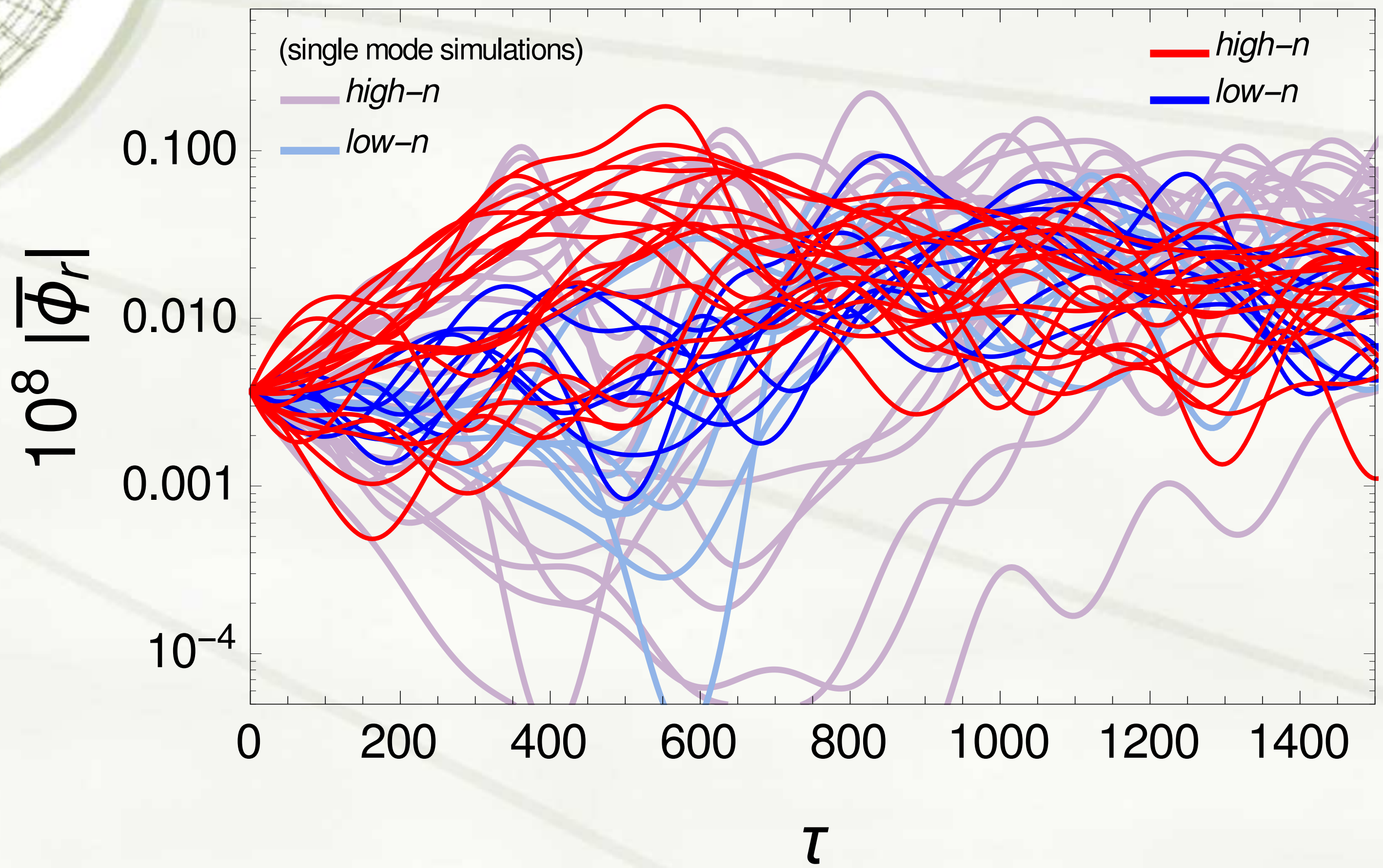
- The contributions from classical, neo-classical, and fluctuation-induced (gyrokinetic) fluxes (transport) is additive up to the $\mathcal{O}(\delta^{-3})\Omega^{-1}$ time scale.
- This result is obtained within the transport ordering [Hinton and Hazeltine 76] and the gyrokinetic ordering [Frieman and Chen 1982]
  $\Rightarrow$ On longer time scales these processes influence each other and cannot be considered mutually independent.
- Interesting interplay of collisional and fluctuation-induced transports are expected where transport ordering and gyrokinetic ordering are stretched.
  $\Rightarrow$ edge transport? phase transitions? (transport barriers) ...
  $\Rightarrow$ Consistent with [Sugama et al., 1996].

Analysis of the beam-plasma system: simulation results

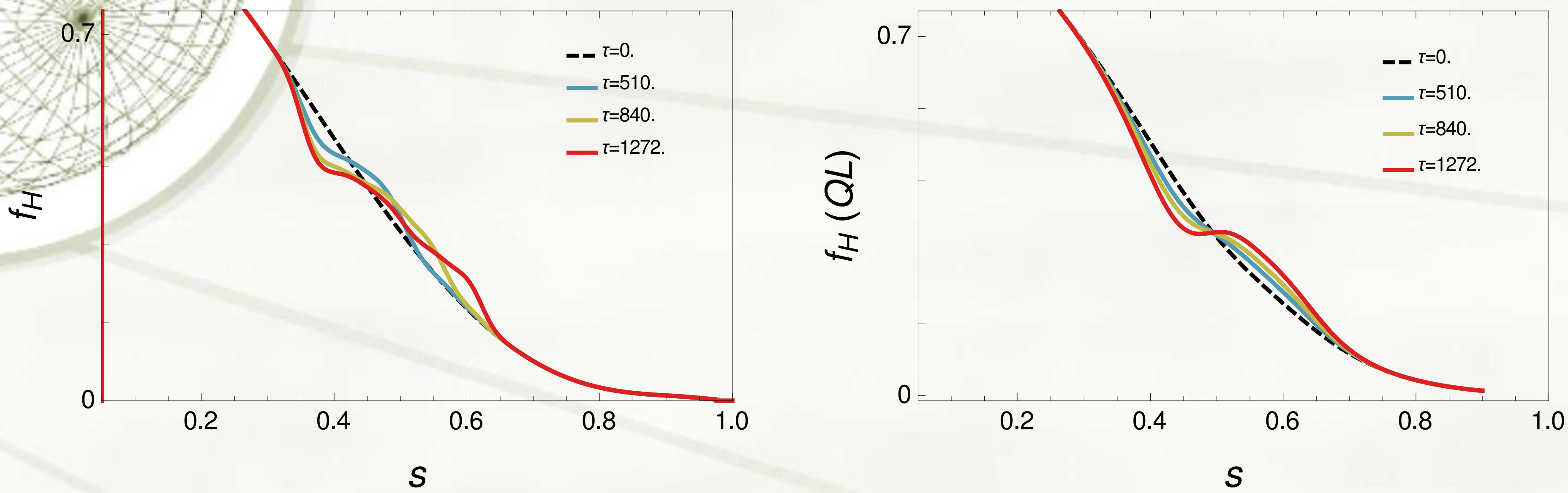
- Comparative study with the beam-plasma system: $r \leftrightarrow v$ mapping [Carlevaro & Montani 2017].
- Similarities and difference with [Schneller et al. 2016] for AE induced transport in ITER reference scenario.



- Evolution of fluctuating fields (single-mode vs. multi-mode) of the equivalent beam-plasma system:

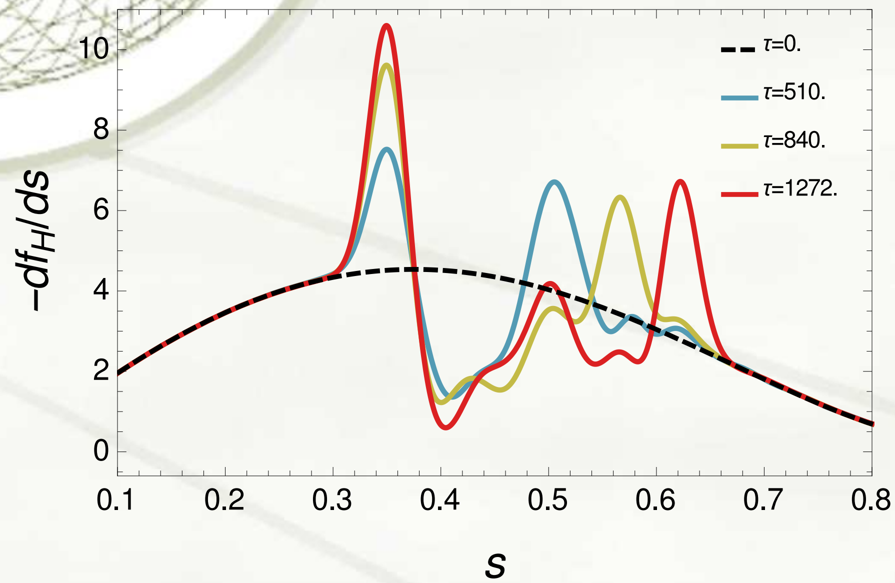


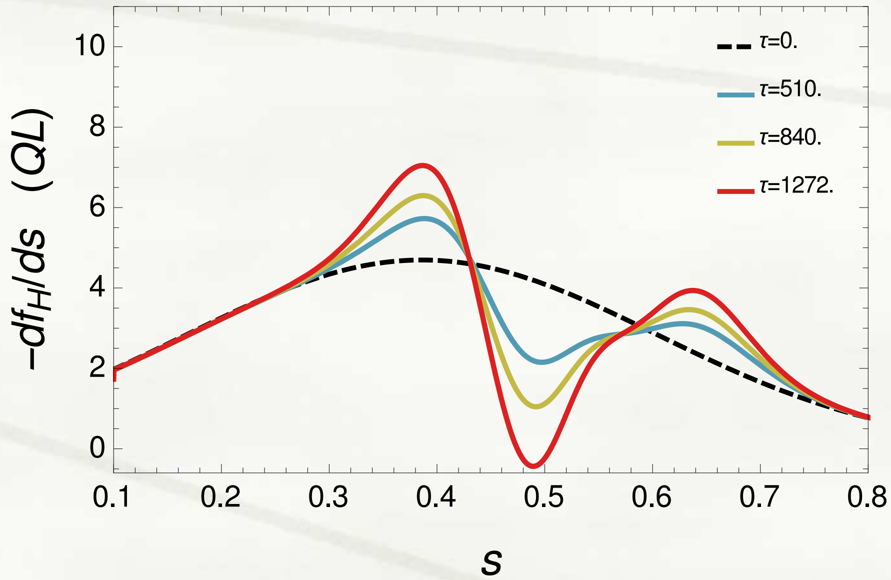
- Corresponding evolution of EP profiles (nonlinear vs. quasilinear):



- Difference is due to the modified nonlinear evolution of the fluctuation spectrum (low mode numbers).

- This is emphasized in the evolution of EP gradient profiles (nonlinear vs. quasilinear):



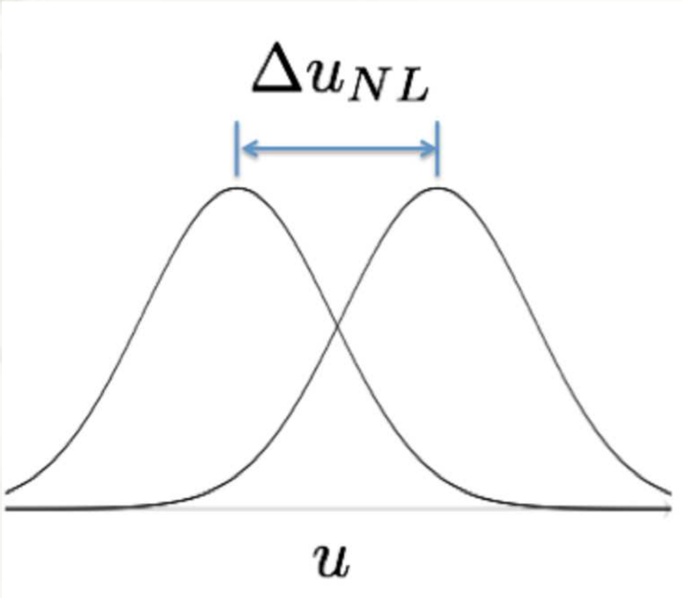


- Evidence of avalanches and spectral transfers.

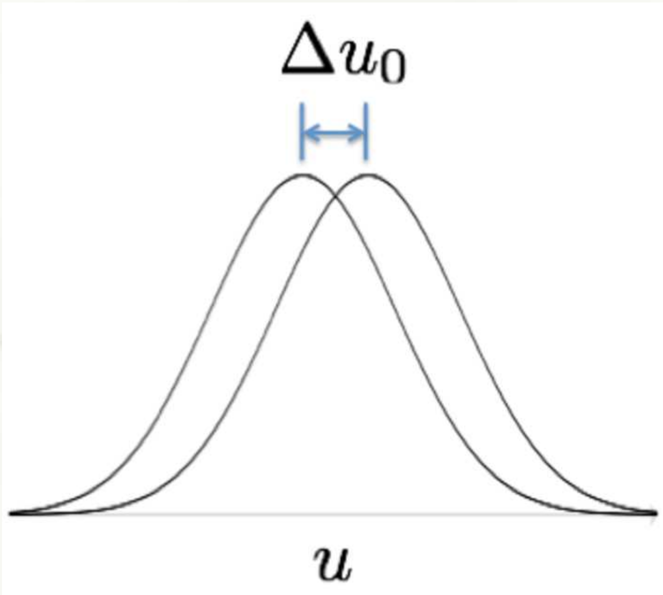
Ongoing work: 2018 activities

Analysis of the beam-plasma system: NL velocity spread and saturation amplitude

- Simulation results by [Carlevaro & Montani 2017] [submitted to PRE 2018].
  ⇒ Intuitive picture of NL velocity spread.



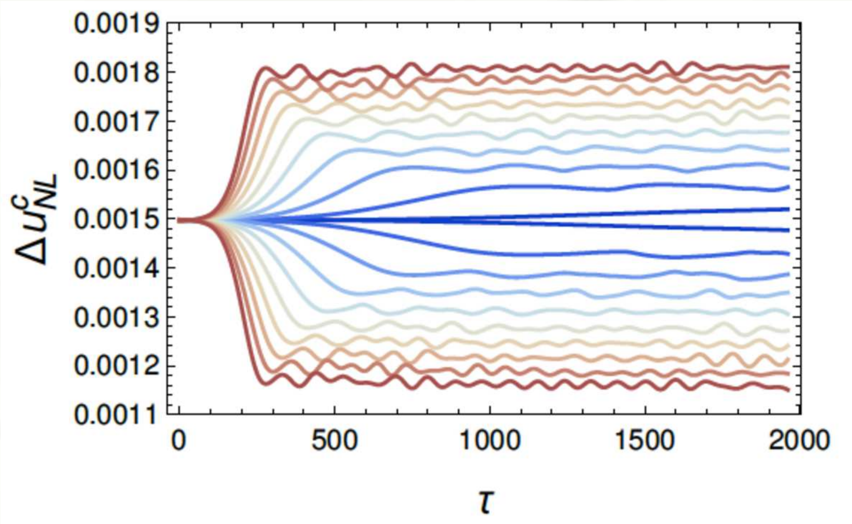
Δu_{NL}: Marginally overlapped resonances

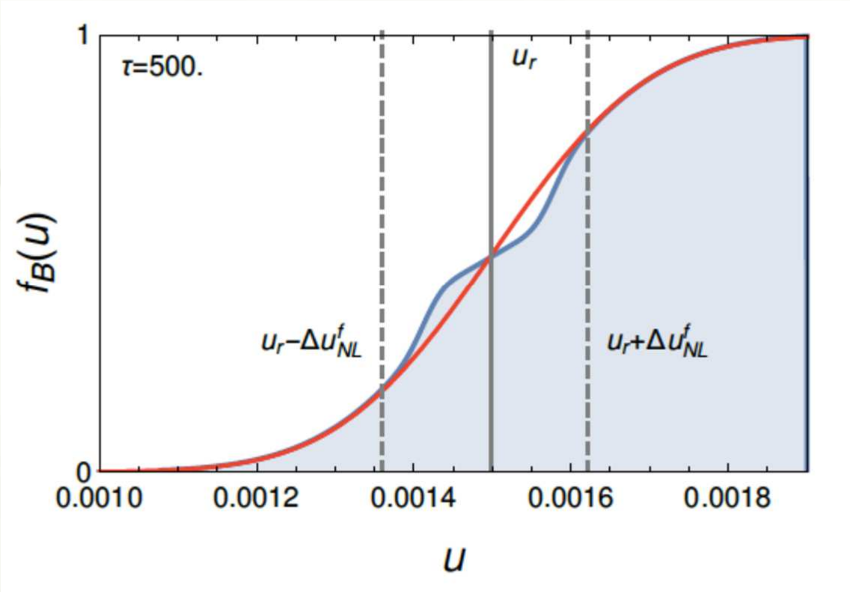


Δu_0: Strongly overlapped resonances

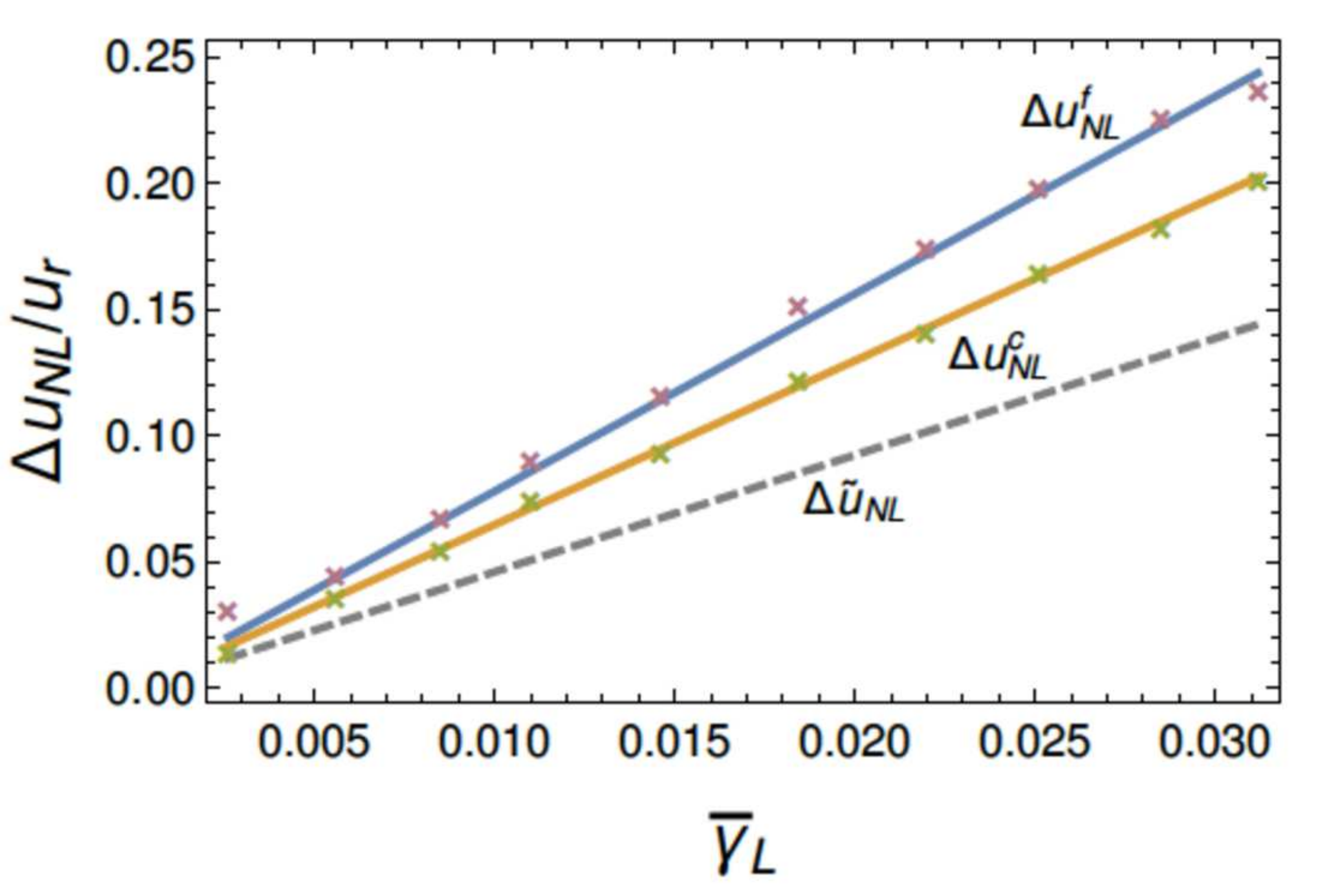
- Introduce three definitions of Δu_{NL} (normalized to $\omega_p L/(2\pi)$, with L the system size)
  - $\Delta \tilde{u}_{NL}$: connected with the average energy conservation
  - Δu^c_{NL}: defined as the clump width
  - Δu^f_{NL}: defined based on the distortion of particle distribution function

Typically $\Delta \tilde{u}_{NL} \lesssim \Delta u^c_{NL} \lesssim \Delta u^f_{NL}$

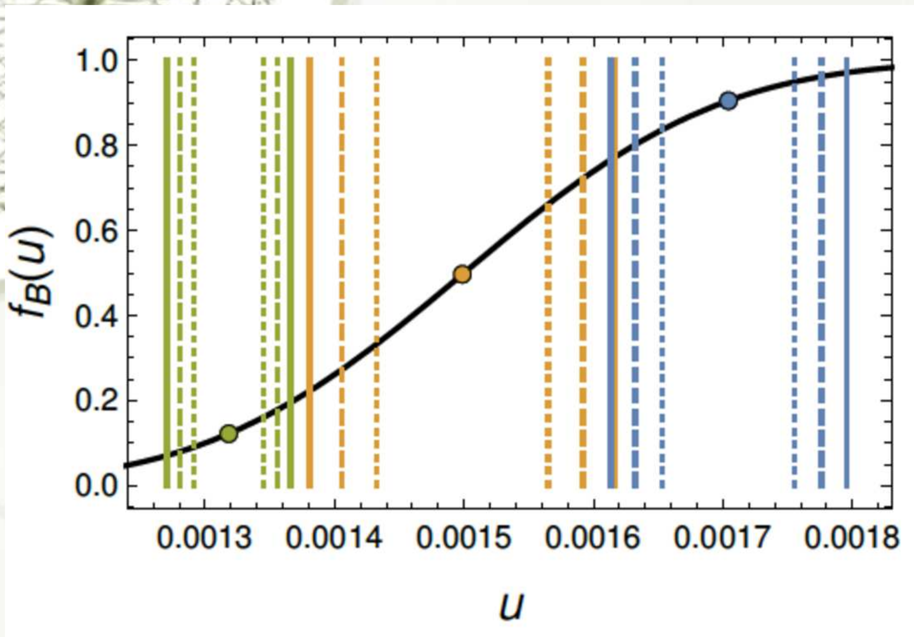


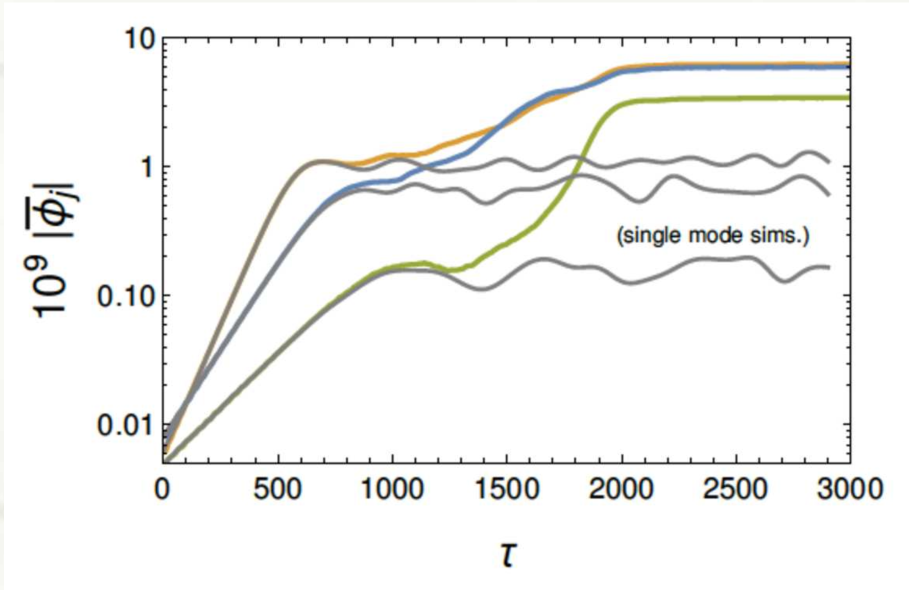


- Evidence of linear scaling of Δu_{NL} with mode growth rate:



Three resonances (two overlapped): predictive relevance of Δu^f_{NL}





- For $\Delta u_{NL}^{f} > \Delta u_{SEP} > \Delta u_0$ the fluctuation intensity is larger than the incoherent superposition of intensities of individual modes:
  - For $\Delta u_0 > \Delta u_{SEP}$: resonances are strongly overlapped (essentially the same mode)
  - For $\Delta u_{SEP} > \Delta u_{NL}^{f}$: resonances are isolated
  - $\Delta u_{NL}^{f} > \Delta u_{SEP} > \Delta u_0$: condition for synergistic tapping of power from particle phase space

- Quantitative model for predicting effective range of synergistic nonlinear interaction and corresponding enhanced fluctuation level: [Carlevaro & Montani, PRE 2018 submitted].

Future work: simulations of the beam-plasma system

- Construct the PSZS analysis for the beam-plasma system (Dyson equation):

$$\partial_t f_0 = -i\frac{e}{m}\sum_k k\left[\delta\phi_k \frac{\partial}{\partial v}\delta f_{-k} - \delta\phi_{-k}\frac{\partial}{\partial v}\delta f_k\right] \quad .$$

- Note similarities and differences with:

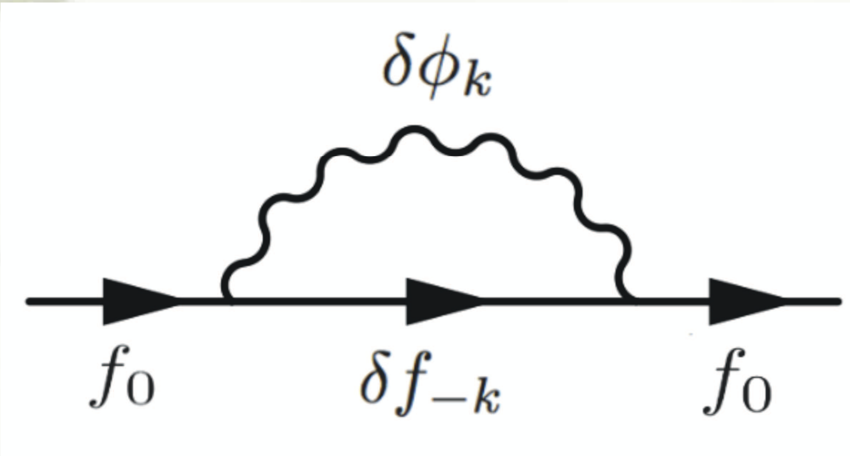
$$\partial_t \delta\bar{G}_z = \overline{\left[e^{iQ_z}\left(-\frac{e}{m}\frac{\partial \bar{F}_0}{\partial \mathcal{E}}\frac{\partial}{\partial t}\left\langle\delta L_g\right\rangle_z\right)\right]} + \frac{c}{d\psi/dr}\frac{\partial}{\partial r}\sum_{\boldsymbol{k}} in\left[\overline{e^{iQ_z}\left\langle\delta L_g\right\rangle_{-k} e^{-iQ_k}}\delta\bar{G}_k\right]$$

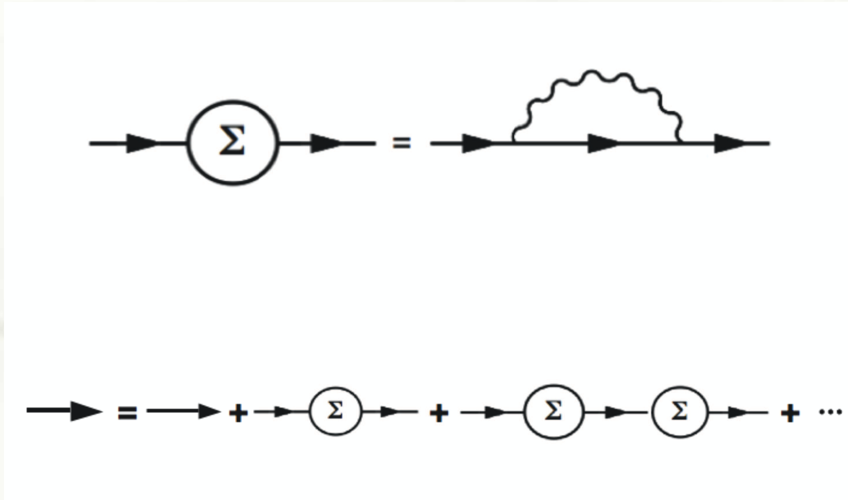
- Based on weak turbulence expansion, construct models of increasing simplification:

  - Weak turbulence expansion $\longrightarrow$ Quasi-linear description

- Dyson equation approach solving in Laplace space:

$$\hat{f}_0(\omega) = \frac{i}{2\pi\omega} F_0 + \frac{e}{m}\frac{k_0}{\omega}\int_{-\infty}^{+\infty}\left[\delta\hat{\phi}_{k_0}(\omega')\frac{\partial}{\partial v}\delta\hat{f}_{-k_0}(\omega-\omega') - \delta\hat{\phi}_{-k_0}(\omega')\frac{\partial}{\partial v}\delta\hat{f}_{k_0}(\omega-\omega')\right] d\omega' \ .$$





- Paradigm problem for understanding fundamental processes underlying transport in the beam-plasma system.

The fishbone paradigm: progress in theory

- A. Generalized resonance broadening theory:

$$\left\{ i\left[\bar{\omega}'_{d0}\rho - (\omega_0 - \bar{\omega}_{d0})\right] - \Delta - F\frac{\partial}{\partial\rho} - \frac{\partial}{\partial\rho}D\frac{\partial}{\partial\rho}\right\}\delta\bar{G}_{k0} = \mathcal{L}_{k0} + [\text{NL OFF DIAGONAL}]$$

$$\Delta = -\sum_{\boldsymbol{k}}\left(1+\frac{n}{n_0}\right)\left(\frac{n_0 c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k0}}\partial_r\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k0}}\overline{e^{iQ_{k+k0}}\partial_r\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}$$

$$F = \sum_{\boldsymbol{k}} nn_0\left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k0}}\partial_r\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k0}}\overline{e^{iQ_{k+k0}}\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}$$

$$-\sum_{\boldsymbol{k}} nn_0\left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k0}}\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k0}}\overline{e^{iQ_{k+k0}}\partial_r\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}$$

$$D = \sum_{\boldsymbol{k}}\left(\frac{nc}{d\psi/dr}\right)^2 \overline{e^{iQ_{k0}}\left\langle\delta L_g\right\rangle_{-k}e^{-iQ_{k+k0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k0}}\overline{e^{iQ_{k+k0}}\left\langle\delta L_g\right\rangle_k e^{-iQ_{k0}}}$$

□ Physical interpretation:

- Δ: nonlinear complex frequency shift
- F: nonlinear anti-symmetric resonance distortion (in radius)
- D: nonlinear resonance broadening (in radius)

□ Issues arising:

- Connection with momentum transport
- Creation of nonlinear length-scales
- Explicit calculation in cases of practical interest (BAE?)

- B. Including explicitly the effect of ZS and PSZS (isolated from other off-diagonal interactions):

$$\left\{ i\left[\bar{\omega}'_{d0}\rho - (\omega_0 - \bar{\omega}_{d0})\right] - i\frac{cn_0}{d\psi/dr}\overline{e^{iQ_{k0}}\partial_r \left\langle \delta L_g \right\rangle_z e^{-iQ_{k0}}} - \Delta - F\frac{\partial}{\partial\rho} - \frac{\partial}{\partial\rho}D\frac{\partial}{\partial\rho} \right\} \delta\bar{G}_{k0}$$

$$= i\overline{\left[e^{iQ_{k0}}\left(\frac{e}{m}Q\bar{F}_0 \left\langle \delta L_g \right\rangle_{k0}\right)\right]} - i\frac{cn_0}{d\psi/dr}\overline{e^{iQ_{k0}}\left\langle \delta L_g \right\rangle_{k0} e^{-iQ_z}}\frac{\partial\delta\bar{G}_z}{\partial r}$$

$$+ \left[\text{NL OFF DIAGONAL}\right]$$

- Issues arising:
  - Still valid for strong turbulence
  - Proper tool for general description of quasi-coherent SAW spectra
  - Test bench for advanced theoretical descriptions?

- C. PSZS description and transport theory:

$$\partial_t \delta \bar{G}_z = \overline{\left[e^{iQ_z} \left(-\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \left\langle \delta L_g \right\rangle_z \right) \right]} + \frac{c}{d\psi/dr} \frac{\partial}{\partial r} \sum_{\boldsymbol{k}} in \left[\overline{e^{iQ_z} \left\langle \delta L_g \right\rangle_{-k} e^{-iQ_k}} \delta \bar{G}_k \right]$$

- Issues arising:
  - This approach includes quasi-linear description but is not limited to it: general description
  - Progressively introduce approximations by similarity with the beam-plasma problem
  - Weak turbulence expansion $\longrightarrow$ Quasi-linear description
  - Accurate test for reduced models

Summary and discussion

- Consistent with NAT milestones:
  - Theoretical framework is complete, using the fishbone paradigm.
    ⇒ applications to problems of practical interest underway
  - Numerical simulations of the beam-plasma system, constructed by mapping from the AE case in ITER are almost complete.
- Two journal papers are expected to be completed by early 2018 (shared with NLED): one ready for submission, another one being written.
- Work package 1 is progressing as expected and should converge on the expected objectives by end of 2018.