On the nonlinear dynamics of phase space zonal structures*

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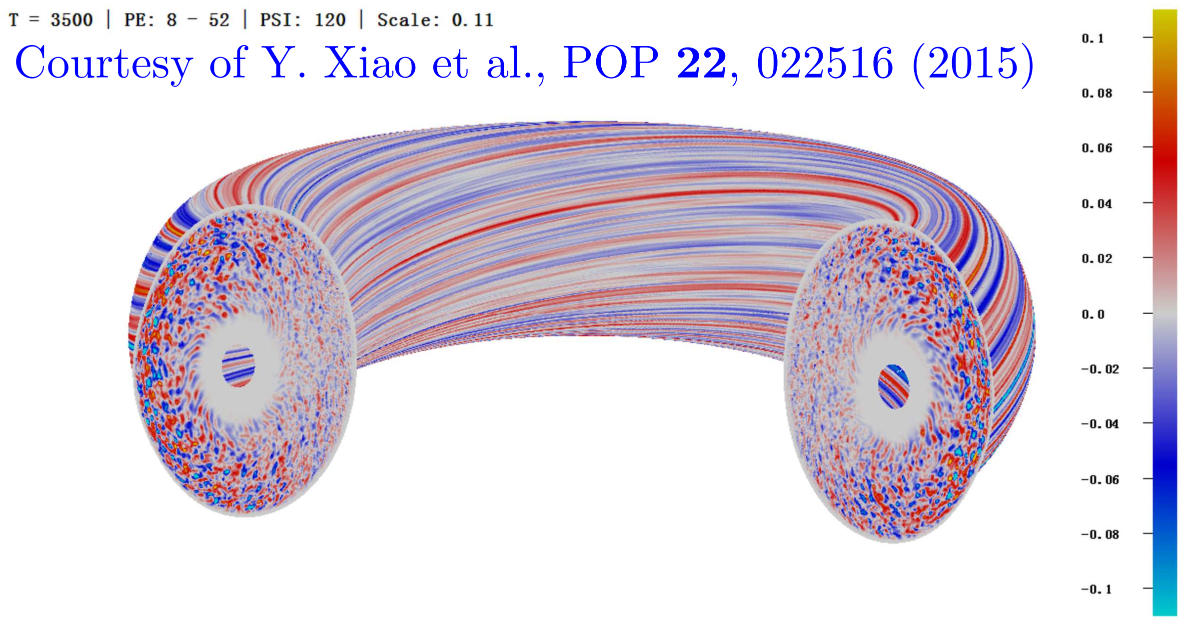
Mode structures and nonlinear interactions in tokamaks



Courtesy of Y. Xiao et al., POP **22**, 022516 (2015)

- ☐ ψ = poloidal magnetic flux

$$\boldsymbol{B}_0 = F(\psi)\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\psi$$
$$\equiv \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\psi$$

$$q \equiv \frac{\boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\zeta}{\boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\theta} = q(\psi)$$

- ☐ Filaments ⇒ Quasi-particles [Zonca et al, PPCF15]
- ☐ Representation based on the Poisson Summation Formula [Z.X. Lu et al., POP12] ⇒ $\sum_m e^{im\theta} = 2\pi \sum_m \delta(\theta - 2\pi m)$.

- Generic fluctuation $\delta\phi(r,\theta,\zeta) = \sum_{m,n} \exp(in\zeta - im\theta)\delta\phi_{m,n}(r)$ can be decomposed as

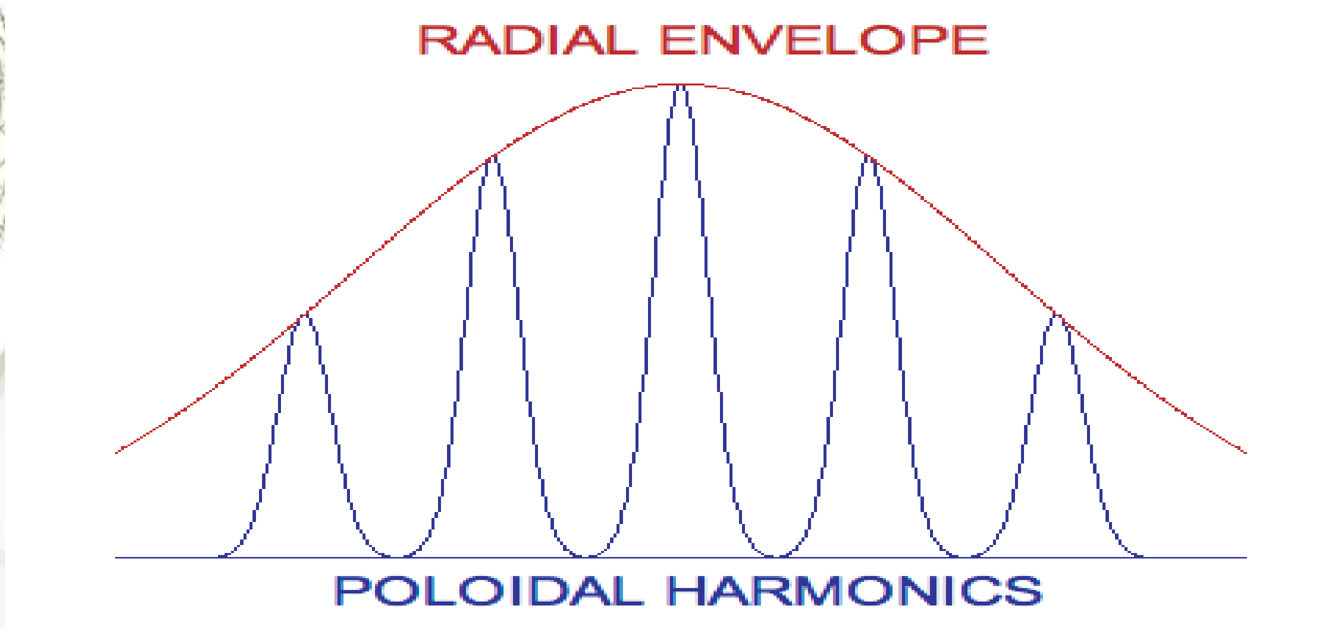
$$\delta\phi(r,\theta,\zeta) = 2\pi \sum_{\ell,n\in\mathbb{Z}} e^{in\zeta - inq(\theta-2\pi\ell)} \delta\hat{\phi}_n(r,\theta-2\pi\ell) = \sum_{m,n\in\mathbb{Z}} e^{in\zeta-im\theta}$$

$$\times \int e^{i(m-nq)\vartheta}\delta\hat{\phi}_n(r,\vartheta)d\vartheta = \sum_{m,n\in\mathbb{Z}} e^{in\zeta-im\theta} \int e^{i(m-nq)\vartheta}\mathcal{P}_{Bn}(r,\vartheta)\left[\delta\phi\right]d\vartheta \ .$$

- Radial envelope (varying on meso-scales) and parallel mode structures (quasi-particles)

$$\delta\hat{\phi}_n(r,\vartheta) = A_n(r)\delta\hat{\phi}_{0n}(r,\vartheta) \simeq A_n(r)\delta\hat{\phi}_{0n}(\vartheta) \ .$$

- Reduces to well-known ballooning formalism when separation of radial scale-length applies $L \gg L_A \gg |nq'|^{-1}$ [Z.X. Lu et al., POP12]

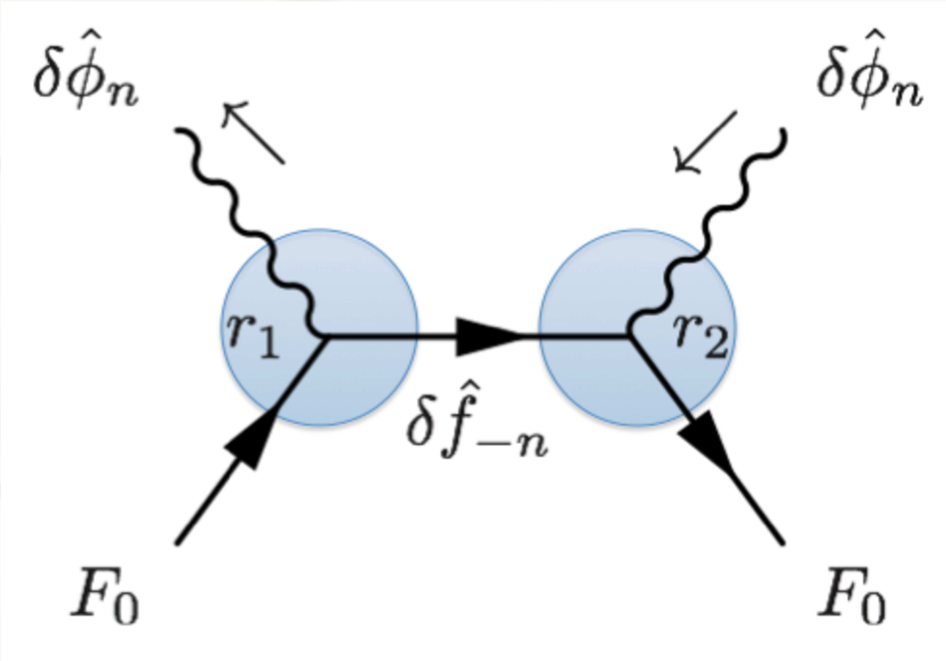


- Mode structures can be represented by three degrees of freedom: the toroidal mode number n, the radial envelope $A_n(r)$ (with scale length L_A); and the parallel (to $\boldsymbol{B}_0$) mode structure $\delta\hat{\phi}_{0n}(r,\vartheta)$, with only a slow radial variation on the equilibrium scale length L.

- Correspondingly, nonlinear interactions can take the following three forms: mode coupling between two ns, modulation of the radial envelope; and distortion of the parallel mode structure [L. Chen et al., PPCF05].

NL Dynamics and fluctuation induced transport

- Description of resonant wave-particle interaction as particles interacting with quasi-particles.
- Quasi-particles carry energy and momentum. But unlike particles, quasi-particles are not conserved in number: occupation number $\propto A_n(r,t)$.
- Fluctuation induced transport due to emission and re-absorption of toroidal symmetry breaking perturbations [Zonca et al., PPCF 2015].

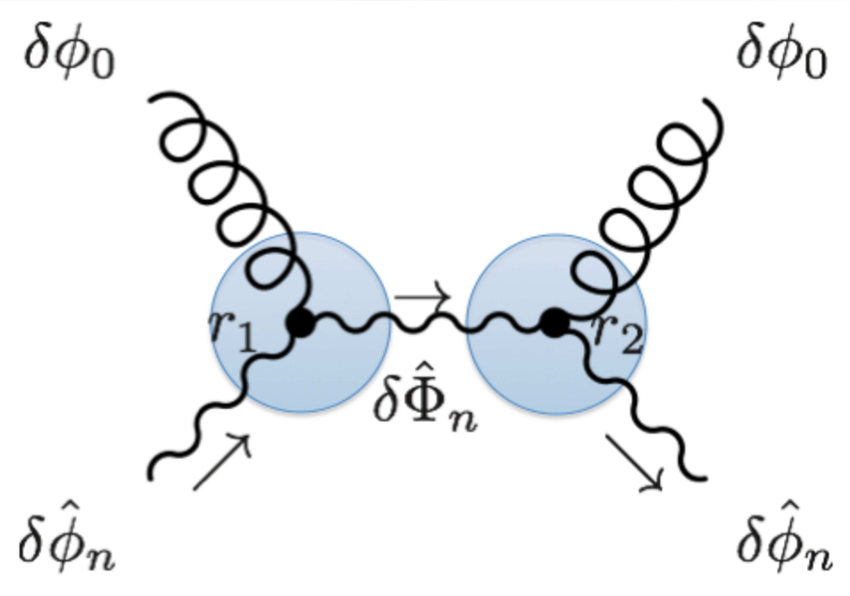


- Characteristic $\delta\hat{\phi}_{0n}(r,\vartheta)$ radial scale is L.
- However, characteristic radial width of filaments $\propto |nq'|^{-1}$ due to magnetic shear.
- Transport may become non-local when $|r_2 - r_1| \gtrsim |nq'|^{-1}$.

The importance of zonal structures

⇒ Zonal structures (ZS) ⇒ coherent micro/meso-scale radial corrugations of equilibrium in toroidal device plasmas [Chen, RMP16].

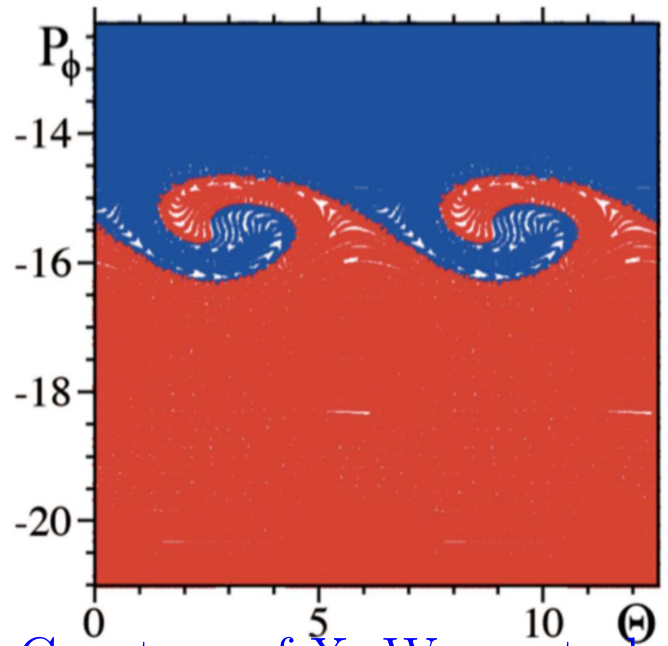
⇒ Zonal structures scatter instability turbulence to shorter-radial wavelength stable domain ⇒ nonlinearly damp the instability
⇒ Self-regulation of plasma instabilities!



- Nonlinear interaction by modulation of the radial envelope $A_n(r,t)$.
- Generation of quasi-particle multiplets $\delta\hat{\Phi}_n$.
- More generally: phase-space zonal structures [Zonca et al., NJP15].

Phase space zonal structures

- ⇒ Phase space zonal structures (PSZS) ⇒ coherent long-lived formations in the particle phase space
- ⇒ PSZS are undamped by (fast) collisionless dissipation mechanisms due to wave-particle interactions [Zonca et al., NJP15]



Courtesy of X. Wang et al. POP **23** 012514 (2016)

- □ ⇒ important roles in transport processes (phase-space)
- □ PSZS describe the deviation from local thermodynamic equilibrium [Falessi, ArXiV16]

- The fluctuating particle distribution functions are decomposed in adiabatic and nonadiabatic responses as [Frieman and Chen 1982].

- Considering $\partial_\mu \bar{F}_0 = 0$ and since PSZS are undamped by (fast) collisionless dissipation mechanisms, $\delta g_z = e^{-iQ_z} \delta \bar{G}_z$ and

$$\delta f_z = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} \delta g_z + \frac{e}{m} \delta\phi_{0,0} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}} e^{-iQ_z} \delta \bar{G}_z + \frac{e}{m} \delta\phi_{0,0} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \; .$$

- Here, $0,0$ subscript to $\delta\phi$ indicates the $m = n = 0$ component; and, given $k_z \equiv (-i\partial_r)$, e^{iQ_z} controls transformation to banana center frame; with

$$Q_z = F(\psi) \left[\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega} \right)} \right] \frac{k_z}{d\psi/dr}$$

and the bounce averaging along unperturbed particle orbits is

$$\overline{[\ldots]} \equiv \left(\oint \frac{d\ell}{v_\parallel} \right)^{-1} \oint \frac{d\ell}{v_\parallel} [\ldots]$$

- The collisionless evolution equation for phase space zonal structures is [Falessi, ArXiV16]

$$\partial_t \delta \bar{G}_z = \overline{\left[e^{iQ_z} \left(-\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \left\langle \delta L_g \right\rangle_{0,0} - \frac{c}{B_0} \boldsymbol{b} \times \boldsymbol{\nabla} \left\langle \delta L_g \right\rangle \cdot \boldsymbol{\nabla} \delta g \right) \right]}$$

- Here,

$$\langle \delta L_g \rangle = \hat{I}_0(\lambda) \left(\delta\phi - \frac{v_\parallel}{c} \delta A_\parallel \right) + \frac{m}{e} \mu \hat{I}_1(\lambda) \delta B_\parallel \quad .$$

and $\hat{I}_n(x) \equiv (2/x)^n J_n(x)$ [Antonsen 80; Catto 81; Brizard 92], $J_n(x)$ are the Bessel functions, $\lambda^2 \equiv 2(\mu B_0/\Omega^2) k_\perp^2$.

- The evolution equation for phase space zonal structures is valid on a time scale up to $\mathcal{O}(\delta^{-3})\Omega^{-1}$, $\delta \sim \rho/L$, consistent with [Hinton and Hazeltine 76; Frieman and Chen 1982].

- Collisions can be included by suitable gyro- and bounce-averaged collision operator [Brizard et al 2010].

- Particle transport equation is obtained as moment from PSZS evolution equation; (similar result for energy transport): $V' d\psi = dV$ plasma volume element

$$\partial_t \langle\langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \partial_t \delta\phi_{0,0} \left\langle \left[1 - \left(e^{-iQ_z} \hat{I}_0 \right) \overline{\left(e^{iQ_z} \hat{I}_0 \right)} \right] \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right\rangle_v$$
$$- \frac{1}{V'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \left(e^{-iQ_z} \hat{I}_0 \right) \overline{\left[c e^{iQ_z} R^2 \boldsymbol{\nabla} \phi \cdot \boldsymbol{\nabla} \langle \delta L_g \rangle \delta g \right]} \right\rangle_v \right\rangle_\psi .$$

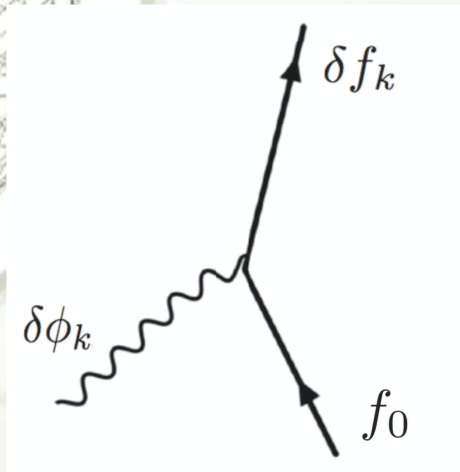
- PSZS bear fundamental information on the nonlinear evolution of plasma equilibria and related transport, and give back expressions of turbulent transport in the long wavelength limit $\left(e^{iQ_z} \hat{I}_0 \right) \to 1$ [Falessi, ArXiV16].

- Adding collisions, the density transport equation can be written, given the radial particle flux $\boldsymbol{\Gamma} \equiv n\boldsymbol{V}$:

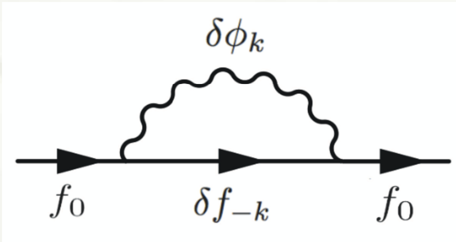
$$\langle\langle \partial_t f \rangle_v \rangle_\psi = -\frac{1}{V'} \frac{\partial}{\partial \psi} \left[V' \langle n\boldsymbol{V} \cdot \nabla\psi \rangle_{\psi c} + V' \langle n\boldsymbol{V} \cdot \nabla\psi \rangle_{\psi NC} + V' \langle n\boldsymbol{V} \cdot \nabla\psi \rangle_{\psi gk} \right]$$

- The contributions from classical, neo-classical, and fluctuation-induced (gyrokinetic) fluxes (trasport) is additive up to the $\mathcal{O}(\delta^{-3})\Omega^{-1}$ time scale.
- This result is obtained within the transport ordering [Hinton and Hazeltine 76] and the gyrokinetic ordering [Frieman and Chen 1982]
  $\Rightarrow$ On longer time scales these processes influence each other and cannot be considered mutually independent.
- Interesting interplay of collisional and fluctuation-induced transports are expected where transport ordering and gyrokinetic ordering are stretched.
  $\Rightarrow$ edge transport? phase transitions? (transport barriers) ...
  $\Rightarrow$ Consistent with [Sugama et al., 1996].

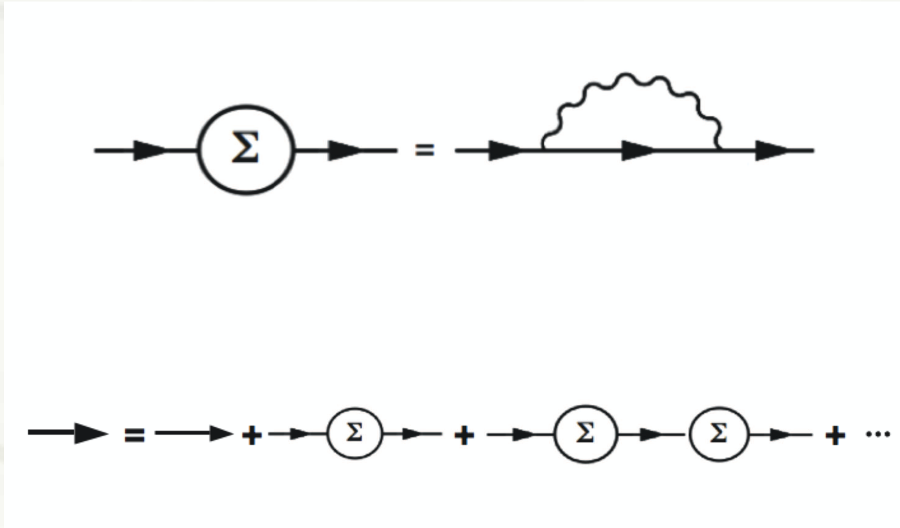
Single-n coherent nonlinear fluctuations



Generation of the distribution δf_k due to the interaction of f_0 with $\delta\phi_k$.



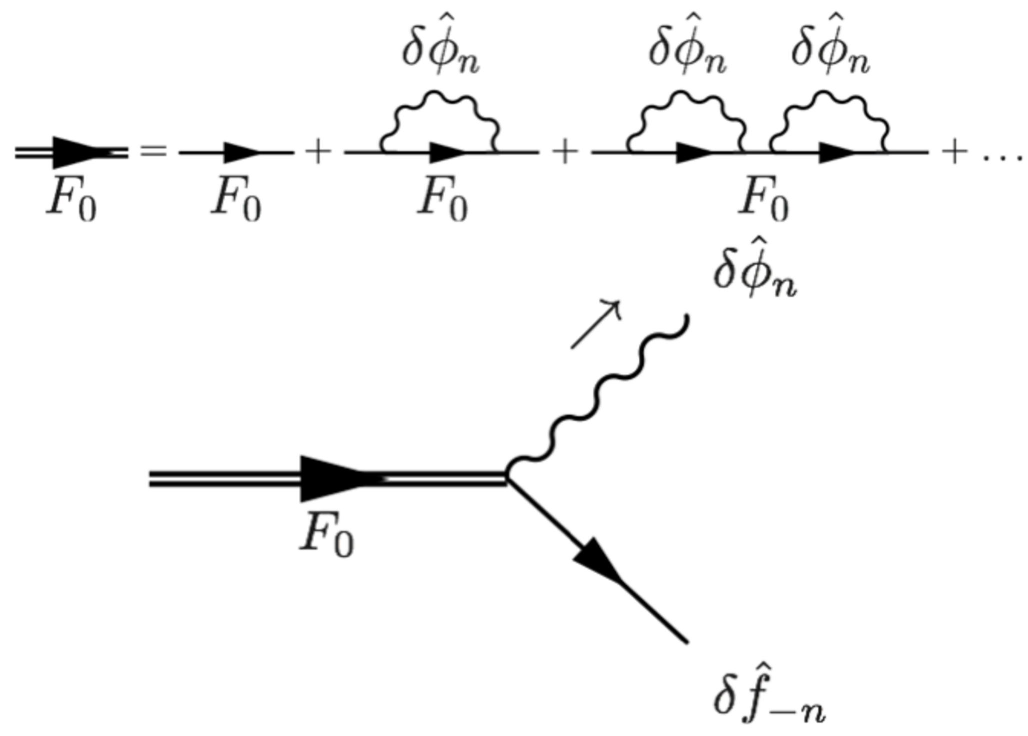
Nonlinear distortion of f_0 due to emission and absorption of the field $\delta\phi_k$.



The diagram of the process is defined in the top frame, while the solution of the "Dyson" equation corresponds to the summation of all terms in the Dyson series (bottom).

Dyson Equation: single-n coherent nonlinear interaction

- Dyson Equation describes fluctuation induced transport in the presence of a single-n quasi-particle $\Rightarrow$ Instability in strongly driven system.



- Non-perturbative interplay of SAW with Energetic Particles (EP).
- Mode structure evolution on same time scale of EP transport
- Self-consistent $\oplus$ non-adiabatic phase space dynamics
- Energetic Particle Modes (EPM).

[Chen RMP16]
[Zonca et al. NJP 2015]

- PSZS evolution equation contains both zonal flows and fields as well as the nonlinear effect of fluctuation-induced transport.

$$\partial_t \delta \bar{G}_z = \overline{\left[e^{iQ_z} \left(-\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \langle \delta L_g \rangle_{0,0} - \frac{c}{B_0} \boldsymbol{b} \times \boldsymbol{\nabla} \langle \delta L_g \rangle \cdot \boldsymbol{\nabla} \delta g \right) \right]}$$

- In turn, the feedback of phase space zonal structures onto δg_n $(n \neq 0)$ is

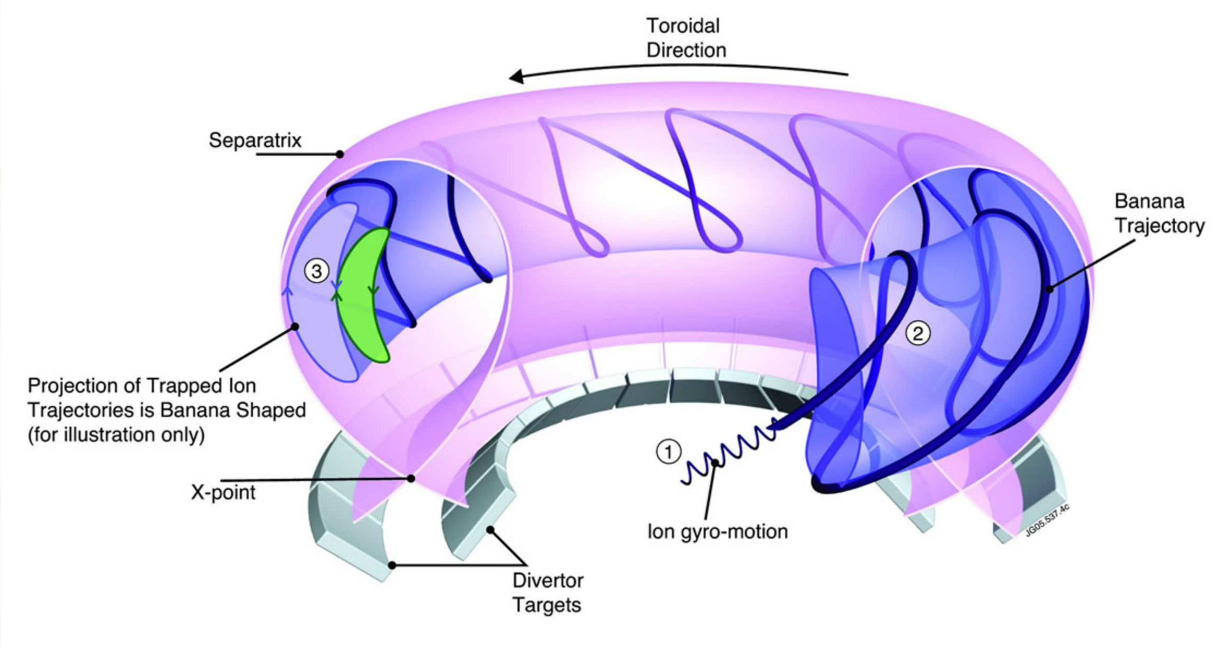
$$\left(\frac{\partial}{\partial t} - \frac{inc}{d\psi/dr} \langle \delta L_g \rangle_z \frac{\partial}{\partial r} + v_\parallel \nabla_\parallel + \boldsymbol{v}_d \cdot \boldsymbol{\nabla}_\perp \right) \delta g_n = i \frac{e}{m} \left(Q \bar{F}_0 - \frac{n B_0}{\Omega d\psi/dr} \overline{(e^{-iQ_z})} \frac{\partial \delta \bar{G}_z}{\partial r} \right) \langle \delta L_g \rangle_n \quad .$$

- Accounts for zonal flows/fields as well as corrugation of radial profiles.

$$Q \bar{F}_0 = i \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} - i \frac{\boldsymbol{b} \times \boldsymbol{\nabla} \bar{F}_0}{\Omega} \cdot \boldsymbol{\nabla} \quad .$$

- This forms a closed system of equations, once evolution equations for the zonal structures are given along with those of nonlinear $n \neq 0$ fluctuations [Z. Qiu et al., 2016-17].

Fishbone Paradigm for SAW-EP nonlinear interplay



- Consider $|\omega| \sim |\bar{\omega}_d| \ll |\omega_b| \Rightarrow$ 2 integrals of motion: μ and $J = \oint v_\parallel d\ell$.
- The system behaves as non-autonomous, **non-uniform** system with one degree of freedom. Reminiscence of 3D equilibrium system.
- Crucial difference with the beam plasma system: non-autonomous, **uniform** system with one degree of freedom.

[Zonca et al. NJP 2015]

- Introduce the standard Laplace transform notation; e.g. $\hat{F}_0(\omega) = (2\pi)^{-1}\int_0^\infty e^{i\omega t} F_0(t) dt$.

- The Dyson equation for $\hat{F}_0(\omega)$ and nearly periodic fluctuations, $\omega_{k0} = \omega_0(\tau) + i\gamma_0(\tau)$, becomes (introducing sources and collisions)

$$\hat{F}_0(\omega) = \frac{i}{\omega}\mathrm{St}\hat{F}_0(\omega) + \frac{i}{\omega}\hat{S}(\omega) + \frac{i}{2\pi\omega}\bar{F}_0(0) + \frac{e}{m}\frac{nc}{\omega(d\psi/dr)}\frac{\partial}{\partial r}\left\{\left[\frac{Q^*_{k_0,\omega_0(\tau)}}{\omega_0^*(\tau)} \times \frac{\hat{F}_0\left(\omega - 2i\gamma_0(\tau)\right)}{\omega - \omega_0(\tau) + n\bar{\omega}_{dk0}} + \frac{Q_{k_0,\omega_0(\tau)}}{\omega_0(\tau)}\frac{\hat{F}_0\left(\omega - 2i\gamma_0(\tau)\right)}{\omega + \omega_0^*(\tau) - n\bar{\omega}_{dk0}}\right]\hat{\omega}_{dk0}\left|\delta\bar{\phi}_{k0}(r,\tau)\right|^2\right\}$$

- This equation can be specialized to a variety of cases of practical interest, including EPM convective amplification via soliton formation [Zonca et al. NJP 2015] and the nonlinear fishbone cycle [Chen RMP16].

- It is instructive to move to the t-representation for nonlinear fishbone cycle. Assuming a rigid plasma displacement $\delta\boldsymbol{\xi}_{r0}$, the evolution equation for the PSZS can be cast as [Chen RMP16]

$$\frac{\partial}{\partial t}F_0(t) \simeq \mathrm{St}F_0(t) + S(t) + 2\left(\frac{\bar{\omega}_d}{\omega_0(\tau)}\right)\frac{\partial}{\partial r}\left[\int_{-\infty}^{+\infty} e^{-i\omega t}\left(\frac{\partial \hat{F}_0(\omega)}{\partial r} - \frac{\omega_0}{\bar{\omega}_d}\frac{\mathcal{E}}{R_0}\frac{\partial \hat{F}_0(\omega)}{\partial \mathcal{E}}\right)\right.$$
$$\left.\times\frac{(\gamma_0 - i\omega)}{(\bar{\omega}_d-\omega_0)^2+(\gamma_0-i\omega)^2}|\omega_0(\tau)|^2|\delta\boldsymbol{\xi}_{r0}|^2 d\omega\right] \quad .$$

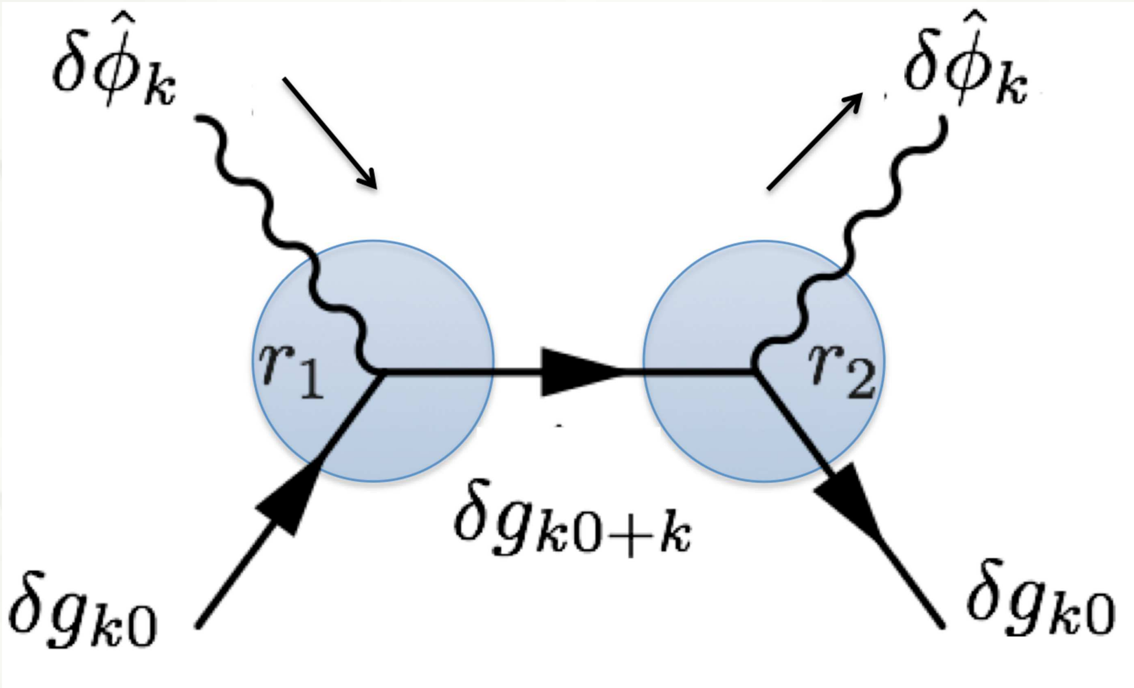
- Fishbone spatiotemporal structures affect EP transport and vice-versa This process is generally non-perturbative. ⇒ Phase locking [Chen RMP16].

- In the same way, one can write explicitly the expression of resonance broadening, due to fluctuation-induced wave-particle decorrelation. [Dupree 66] ⇒ Fluctuation induced diffusion in space rather than velocity space.

- Detailed expression depends on the assumed (or computed) fluctuation spectrum. Approach is fully consistent with a statistical analysis [Dupree 66].

- By extension of the PSZS evolution equation, and introducing the generator of coordinate transformation to banana centers, $\delta g_k = e^{-iQ_k}\delta\bar{G}_k$, with $v_\parallel \nabla_\parallel Q_k + \tilde{\boldsymbol{v}}_d \cdot \boldsymbol{\nabla}_\perp Q_k \equiv 0$,

$$(\bar{\omega}_d - \omega)_k \delta\bar{G}_k = i\,\overline{\left[e^{iQ_k}\left(Q\bar{F}_0\left\langle\delta L_g\right\rangle_k + \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta L_g\right\rangle\cdot\boldsymbol{\nabla}\delta g\right)\right]}$$



[Dupree 66; Laval & Pesme 84,99]

- Isolating the nonlinear harmonic generation (diagonal) from linear and other nonlinear response,

$$(\bar{\omega}_d - \omega)_{k0}\delta\bar{G}_{k0} = i\overline{\left[e^{iQ_{k0}}Q\bar{F}_0\left\langle\delta L_g\right\rangle_{k0}\right]} + [\text{OTHER NONLINEAR}]$$
$$- \sum_{\mathbf{k}} \frac{1}{V'}\frac{\partial}{\partial\psi}\overline{\left\{ce^{iQ_{k0}}\left(R^2\nabla\phi\cdot\nabla\left\langle\delta L_g\right\rangle_{-k}\right)\frac{e^{-iQ_{k+k0}}}{(\bar{\omega}_d-\omega)_{k+k0}}\right.}$$
$$\left.\times\overline{\frac{\partial}{\partial\psi}\left[cV'e^{iQ_{k+k0}}\left(R^2\nabla\phi\cdot\nabla\left\langle\delta L_g\right\rangle_k\right)e^{-iQ_{k0}}\delta\bar{G}_{k0}\right]}\right\}$$

- Formally, this equation can be written as (geometry effect through $\bar{\omega}_d$)

$$\left\{i\left[\bar{\omega}'_{d0}\rho - (\omega_0 - \bar{\omega}_{d0})\right] - D\frac{\partial^2}{\partial\rho^2}\right\}\delta\bar{G}_{k0} = \mathcal{L}_0 + [\text{NL OFF DIAGONAL}]$$

  - Resonant particle response (D real): resonance broadening
  - Non-resonant particle response (D imaginary): non-linear frequency shift

- Both effects are crucial for the nonlinear dynamics.

- Comparative study with the beam-plasma system [Carlevaro & Montani 2017]
  - importance of spectral density and intensity
  - crucial role of equilibrium geometry and non-uniformity
  - assess conditions for applicability of simplified/reduced models; e.g. weak turbulence theory
  - identify possible novel and/or “unexpected” behaviors in burning fusion plasmas

Summary and discussion

- Gyrokinetic theory & simulation provide a general framework for studying fluctuations and ensuing transport in strongly magnetized plasmas:
  - Complex behaviors due to many interacting degrees of freedom
  - Hierarchy of spatiotemporal scales and possibility of reduced NL dynamic descriptions depending on relevant time scales
  - Framework for bridging NL and transport time scales
- Applications: Fluctuation-induced (phase space) transport
  - Description in terms of particles interacting with quasi-particles
  - Phase space zonal structures bear fundamental information on the nonlinear evolution of plasma equilibria and related transport
  - Adding collisions, PSZS NL evolution suggest interplay of collisional and fluctuation induced transport on longer (than typical transport) time scales $\Rightarrow$ Important for burning plasma
  - Renormalized solution for the PSZS
    $\Rightarrow$ Crucial role of geometry, nonuniformity (advanced concepts)