ER17-MPG-01: NAT ER Project Final Meeting - WP1 Summary

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ITALIAN NATIONAL AGENCY FOR NEW TECHNOLOGIES, ENERGY AND SUSTAINABLE ECONOMIC DEVELOPMENT

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December 13th 2018

Outline

I. General summary of WP1 activities
- Milestones and deliverables
- Publications, Conferences and Outreach

II. WP1: theory summary
- Phase space zonal structures and transport equations
- Weak turbulence expansion and reduced models

III. WP1: summary of simulation and modeling
- Numerical simulations of the beam-plasma system, constructed by mapping from the AE case in ITER
- Recent progress and ongoing work

General summary of WP1 activities

- Milestones and deliverables:

  - *Derivation of nonlinear model equations for the self-consistent evolution of SAW/DAW and ZS/PSZS for the "fishbone paradigm"; generalization of resonance broadening theory (2017)*

    ⇒ The complete theoretical framework was presented at the NAT review meeting (Feb. 27th, 2018) and provided the basis for the "plenary invited talk" (F. Zonca et al) at the 2.nd AAPPS-DPP Conference in Kanazawa, Nov. 12-17, 2018.

    ⇒ Two journal papers are expected in 2019 from this research activity

    ⇒ Outreach and published cross-WP results with WPDTT1/DTT2 (2 papers), ER17-ENEA-10 (2 papers) and ER17-CIEMAT-03 as well as MST1 Campaigns on Medium-Sized Tokamaks-6 (3 papers) (also presented at the 27th International Toki Conference, Toki, Nov. 19-22, 2018).

- *Numerical solution of model equations in the uniform plasma limit and V&V against numerical solution of Hamiltonian formulation of the bump-on-tail paradigm. Solution of model equations in non-uniform plasmas (fishbone paradigm) and applications to ITER and DEMO (2018).*

  ⇒ Vlasov-Poisson diagonal reduced model successfully tested against fully non-linear N-body BoT simulations (2 papers).

  ⇒ ITER 15MA baseline scenario [M. Schneller et al. PPCF 58, 014019 (2016)] EP transport and spectral evolutions have been properly reproduced (2 papers), underling avalanches and convective spectral transfers.

  ⇒ BoT paradigm has been also applied for the prediction of the non-linear velocity redistribution related to the EGAM dynamics (2 papers).

□ Publication & Conference Summary: 4+9 papers published+submitted(to be) ⊕ 3 Conference papers ⊕ 5 Conference participations (1 invited; 2 orals)

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WP1: theory summary

- Construct the PSZS analysis for the beam-plasma system (Dyson equation):

$$\partial_t f_0 = -i\frac{e}{m}\sum_k k\left[\delta\phi_k \frac{\partial}{\partial v}\delta f_{-k} - \delta\phi_{-k}\frac{\partial}{\partial v}\delta f_k\right] \quad .$$

- Note similarities and differences with fishbone paradigm in tokamak (reduced description to 1 degree of freedom):

$$\partial_t \delta\bar{G}_z = \overline{\left[e^{iQ_z}\left(-\frac{e}{m}\frac{\partial \bar{F}_0}{\partial \mathcal{E}}\frac{\partial}{\partial t}\left\langle \delta L_g\right\rangle_z\right)\right]} + \frac{c}{d\psi/dr}\frac{\partial}{\partial r}\sum_{\boldsymbol{k}} in\left[\overline{e^{iQ_z}\left\langle \delta L_g\right\rangle_{-k} e^{-iQ_k}}\delta\bar{G}_k\right]$$

- Based on weak turbulence expansion, construct models of increasing simplification: (connection with ER19-ENEA-05 MET Project)

  - Weak turbulence expansion $\longrightarrow$ Quasi-linear description

- Phase space zonal structure description and transport theory:

$$\partial_t \delta \bar{G}_z = \overline{\left[e^{iQ_z} \left(-\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \langle \delta L_g \rangle_z \right) \right]} + \frac{c}{d\psi/dr} \frac{\partial}{\partial r} \sum_{\boldsymbol{k}} in \left[\overline{e^{iQ_z} \langle \delta L_g \rangle_{-k} e^{-iQ_k} \delta \bar{G}_k} \right]$$

- Issues arising:
  - This approach includes quasi-linear description but is not limited to it: general description
  - Progressively introduce approximations by similarity with the beam-plasma problem
  - Weak turbulence expansion $\longrightarrow$ Quasi-linear description
  - Accurate test for reduced models

Non-linear features of beam-plasma systems

[NC, G. Montani, F. Zonca, *45th EPS Conference on Plasma Physics* **42A**, P5.1067 (2018)]

Hamiltonian (N-body) formulation of the bump-on-tail paradigm: fast **electron beam** injected into a **1D cold plasma** supporting longitudinal electrostatic **Langmuir waves**.

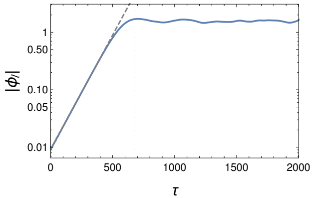
- **Single isolated resonance** -

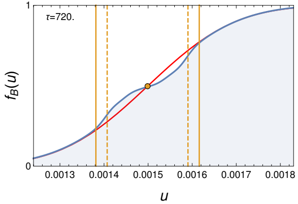
  Mode exponential growth $\rightarrow$ non-linear saturation ($|\phi|^S$) $\rightarrow$ trapped particles (phase-space rotating clumps) $\rightarrow$ flattening of velocity distribution function.

  - **Growth-rate**: $\gamma = \gamma_D - \gamma_d$ (linear disp. rel.).
  - **Quadratic scaling**: $|\phi|^S = \beta\gamma^2$.
  - **Trapping freq.**: $\omega_B = \sqrt{2\ell^2|\phi|^S} \simeq 3.31\gamma$.
  - **Clump width**: $\Delta u^c_{NL}/u_r \simeq 6.64\gamma$ (dashed).

- **Dynamic role of un-trapped particles** - Effective non-linear velocity spread evaluated introducing a **scale factor** to Δu^c_{NL}: proper characterization of the active resonance overlap

$$\Delta u_{NL} \simeq \alpha \Delta u^c_{NL} \simeq 8.5\gamma \quad \text{(solid)} \qquad \alpha \simeq 1.28$$





Vlasov-Poisson system: reduced model validation

[G. Montani, F. Cianfrani, NC, *submitted to Phys. Plasmas* (2018)]
[L. Chen, F.Zonca, *Rev. Mod. Phys.* **88**, 015008 (2016)]

Vlasov-Poisson coupled system for the BoT in Fourier space:

$$\partial_t f_k = -ikvf_k + \frac{e}{m}\sum_q E_{k-q}\partial_v f_q \qquad \partial_t E_k = -i\omega_p E_k + \frac{2\pi e\omega_p}{k}\int_{-\infty}^{+\infty} dv f_k$$

$f_{k=0}$: only component having non-zero initial condition (BoT initially homogeneous).

- **Diagonal assumption** - f_k receives mainly contribution by $q = k$:

$$\partial_t f_k = -ikv\, f_k + \frac{e}{m}E_k\partial_v f_0 \quad \Rightarrow \quad f_k(t,v) = \frac{e}{m}\int_0^t dt' E_k(t')e^{ikv(t'-t)}\partial_v f_0(t',v)$$

Substituting and using the complex-conjugate notation: **Dyson-like equation**

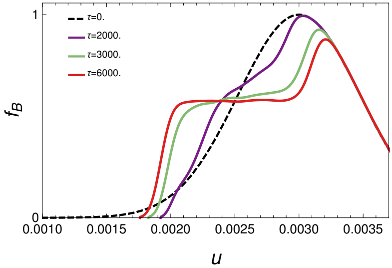
$$\partial_t f_0(t,v) - \frac{e^2}{m^2}\sum_k \Big[E_k^*\, \partial_v\Big(\int_0^t dy\, E_k(y)e^{ikv(t-y)}\partial_v f_0(y,v)\Big) + c.c.\Big] = 0\,.$$

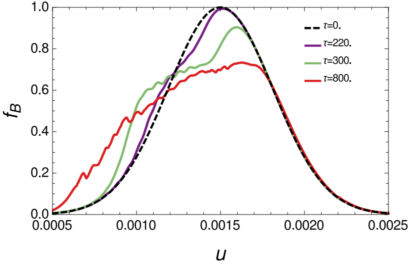
- **Dyson equation with external fields (only)** - Direct integration for given mode evolution extracted from sims ($f_0 \propto n_B f_B(u,\tau)$): (quasi) self-consistent flux eqs

$$\partial_\tau f_B = \partial_u \Gamma \qquad \Gamma = \sum_j \ell_j^2 (\phi_j \bar{G}_j^* + \phi_j^* \bar{G}_j)$$

$$\partial_\tau \bar{G}_j = -i\ell_j u \bar{G}_j + \bar{\phi}_j \partial_u f_B$$

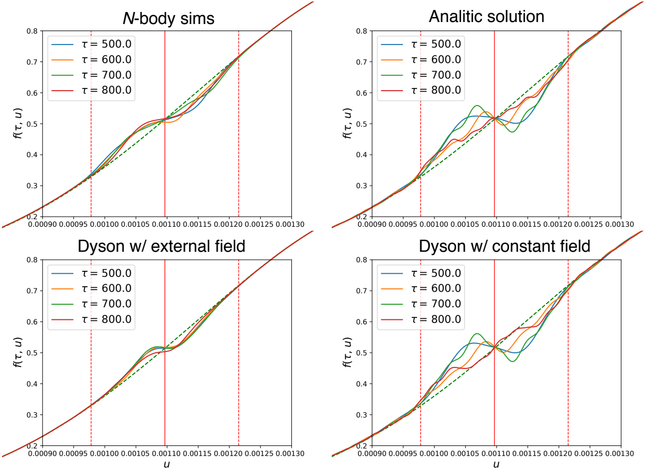
- Tabulated $\phi_j(\tau)$ from sims: Runge-Kutta (4*th*-order) evolves the system in time.
- Numerical results: **broad spectrum** (30 modes, Kubo no $\mathcal{K} \sim 0.03$).





- Morphology of the profile is well described but the system seems less efficacious: neglect mode-mode coupling (?)
- *Numerical problem for integration (u-mesh) to be solved.*

- **Analytic solution and self-consistency** - Dyson eq. analytically integreted [Al'tshul', Karpman (1966)] using Hermite eigenfunctions, with a **single constant mode** ($|\phi|_{Sat}$).



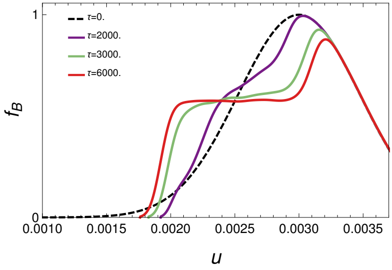
- Analytic solution well describe resonance position and non-linear velocity spread, but generates corrugation and gradient inversion.
- The shortcomings of the analytical sol is **NOT** the Hermite truncated expansion, but the constant mode assumption (self-consistency breaking).

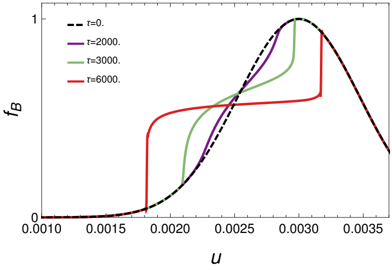
- **Quasi-linear model** - Broad spectrum assumption: $\sum_k (...) \to \int_{k_{min}}^{k_{\max}} dk\, \mathcal{N}(k)(...)$ (spectral density $\mathcal{N} = M/\Delta k$, $\Delta k = \mathrm{Max}[k_j] - \mathrm{Min}[k_j]$):

$$f_B = F_B + \bar{\mathcal{N}} M \eta^{-1}\, \partial_u(\mathcal{J} u^{-5})$$
$$\partial_\tau \mathcal{J} = \pi\eta M^{-1} u^2 \mathcal{J} \partial_u F_B + \pi \bar{\mathcal{N}} u^2 \mathcal{J} \partial_u^2 (\mathcal{J} u^{-5})$$

The spectrum is $\mathcal{J}(\tau, u) = |\phi|^2$ where $\bar{\phi}(\tau, s)$ is the continuous mode spectrum, derived from the discrete one, specified by res. conds.

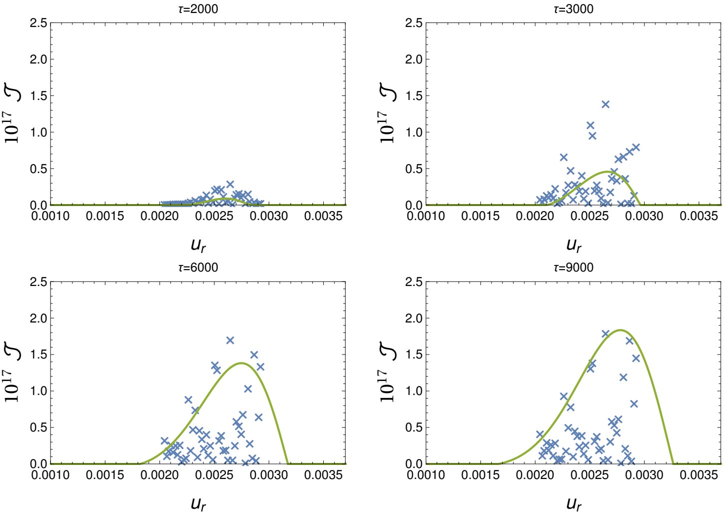
- Spectral PDE integrated with (linearized) Crank-Nicolson algorithm.
- Numerical results: **broad spectrum** (30 modes, Kubo no $\mathcal{K} \sim 0.03$).





- Time delay in the QL evolution: slower formation of the plateau. This confirms the assumed hypothesis of slow evolution of f_0 in the derivation.

- Spectral evolution: temporal delay



- QL model is not predictive in the early stages of evolution.
- Flattening width discrepancy: QL spectrum is larger (fixed number of simulated modes).

BoT reduced model for ITER relevant EP simulations

[NC, G. Montani, F. Zonca, P. Lauber, T. Hayward-Schneider, *to be submitted to Phys. Plasmas* (2018)]

Mapping procedure - The reduced **radial profile** of the burning plasma scenario corresponds to the **velocity space** of the BoT paradigm. From the resonance conditions:

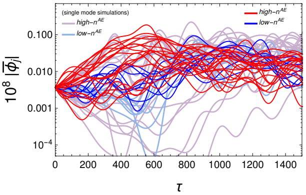
$$\boldsymbol{u = (1 - s)/\ell_1} \qquad \ell_{r(j)} = \ell_1/(1 - s_{r(j)})$$

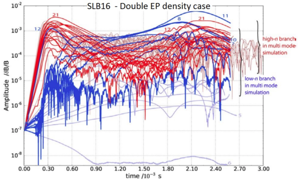
linear relation (local map), once the AE/EP system is **dimensionally reduced**.

- **Critical issue** - [1D uniform; 1 dof.] $\Rightarrow$ [3D non-uniform; 2 dof.]
  $\rightarrow$ EP response intrinsically different: drives are imposed as

$$\gamma_D/\omega_0 = \alpha\, \gamma_D^{AE}/\omega^{AE} \quad \text{with } \alpha \leqslant 1$$

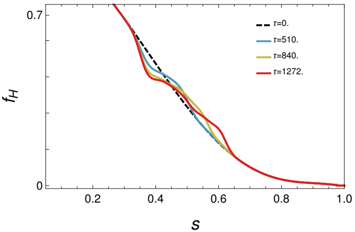
- **ITER case** - [M. Schneller, Ph. Lauber, S. Briguglio, *PPCF* **58**, 014019 (2016)]
  27 TAE: $n^{AE} \in [12, 30]$ main branch (red); $n^{AE} \in [5, 12]$ low branch (blue) ($\boldsymbol{\alpha = 0.4}$)

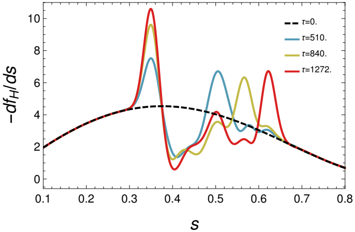




- Low-n^{AE} branch (blue) more efficiently and rapidly excited in multi-mode sims
- Mutual mode interaction $\Rightarrow$ **Avalanche transport** of particles in regions that can exite linearly stable modes.

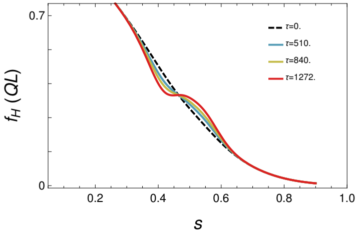
- **EP redistribution** - Spectral evolution reflects on the EP profile:

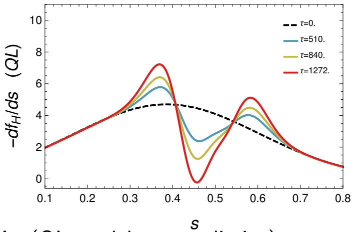




  - **Convective-like transport** toward the plasma edge (second peak for low branch).
  - SLB16: outer redistribution to $s \simeq 0.85$: importance of the **poloidal harmonics**.

- **Comparison with QL transport** - QL equations r-dimension using the mapping:





  - Absence of avalanche: two well **localized peaks** (QL model not predictive).

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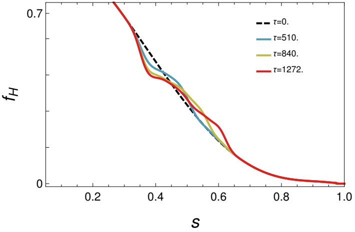

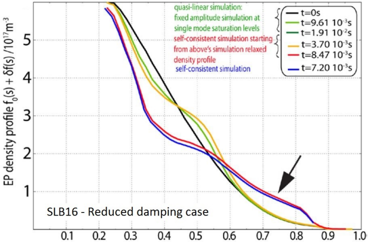

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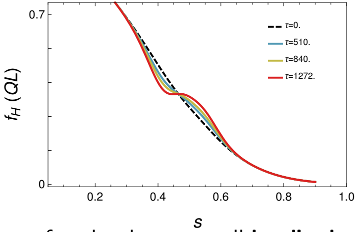

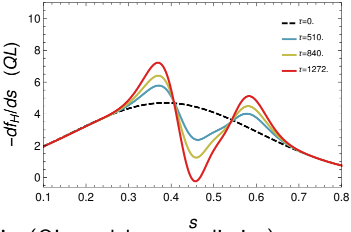

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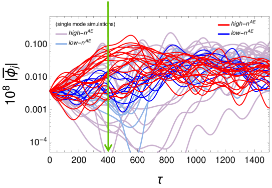

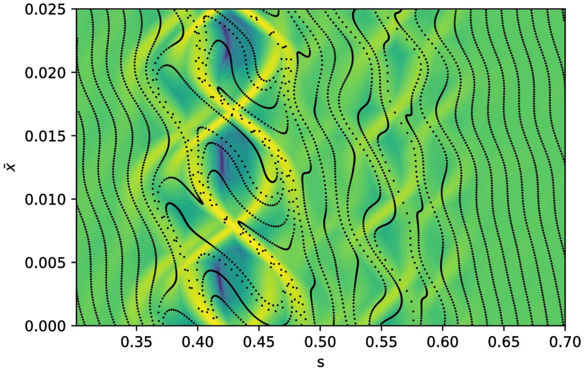

- Ridges in yellow: local (time dependent) transport barriers (clumps and trapped particles).
- Initial times: barriers more efficient for the high branch.
- Rich dynamics: overlapping and interchanging resonances.
- Progressive enhancement of low branch: representation of avalanche-like transport.

[Collaboration with M.V. Falessi]

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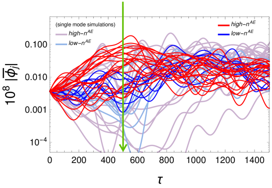

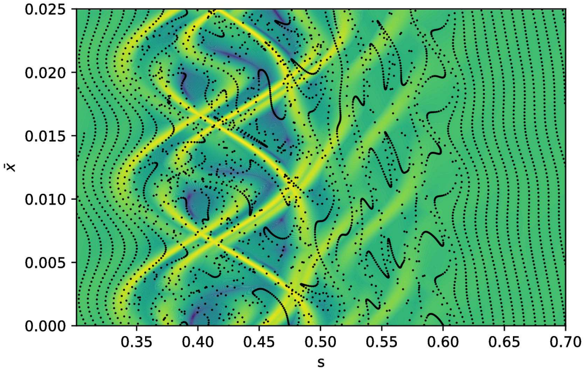

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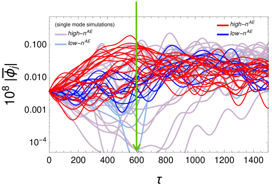

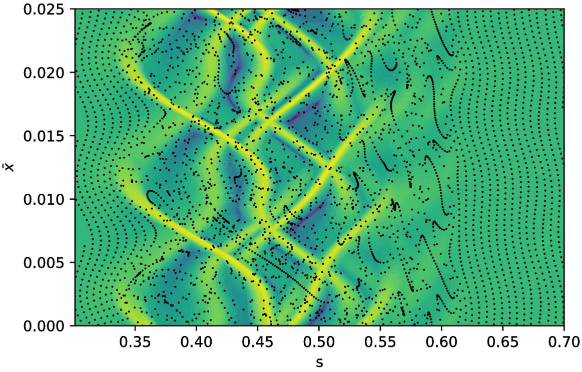

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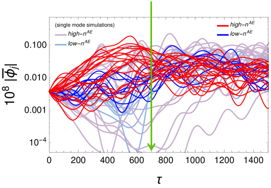

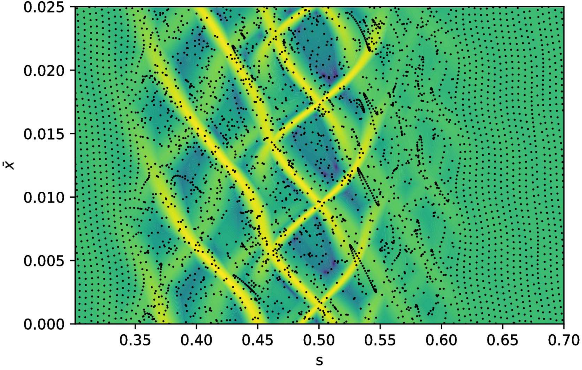

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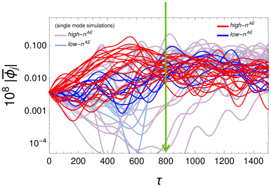

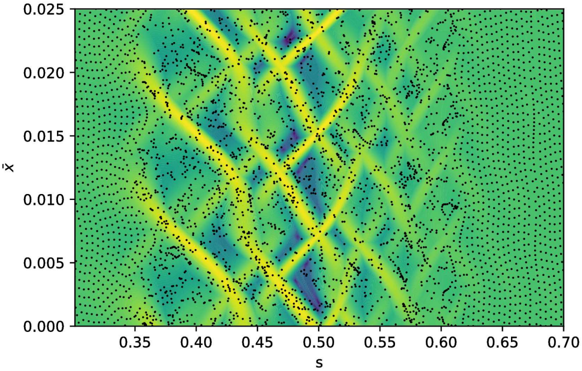

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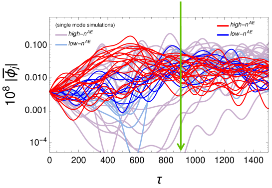

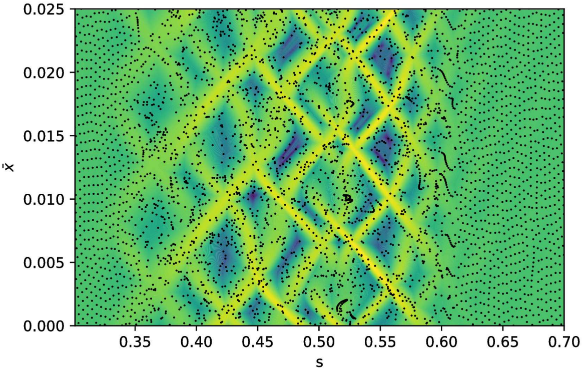

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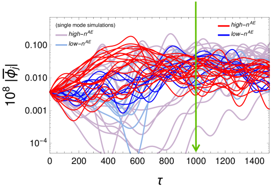

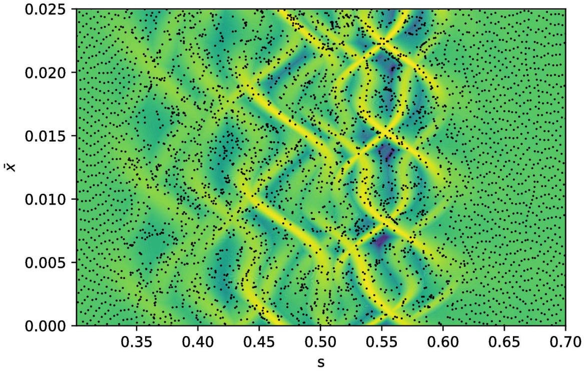

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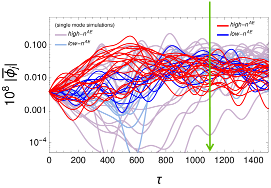

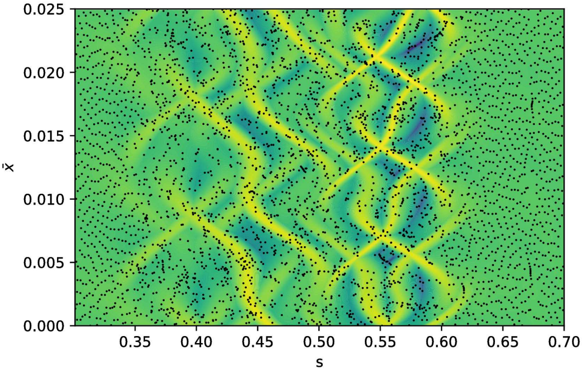

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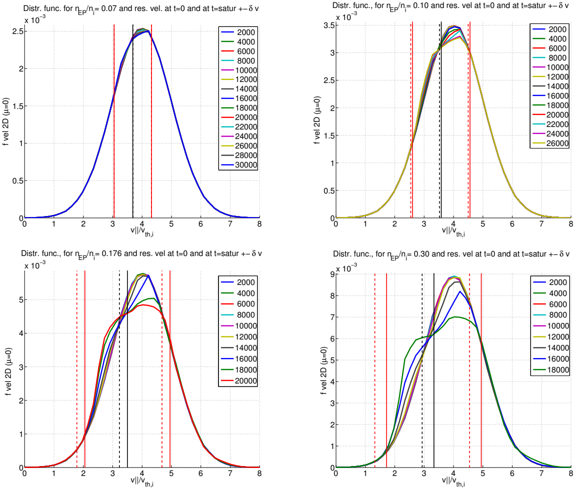
- Progressive enhancement of low branch: representation of avalanche-like transport.

[Collaboration with M.V. Falessi]

Cross-WP work: [A. Biancalani, NC, A. Bottino, G. Montani, Z. Qiu, *J. Plasma Phys.* **84**(6), 725840602 (2018).

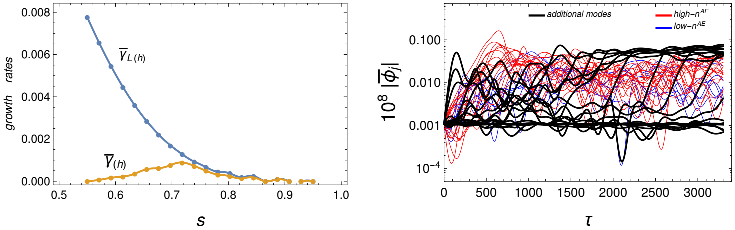
- Prediction, from BoT mapping, of the nonlinear parallel velocity spread.



- Low drive: a very good match between the estimated deviation and the nonlinear EP redistribution.
- Strong drive: relevant frequency chirping, the value remains very predictive but modifying resonance velocity accordingly.

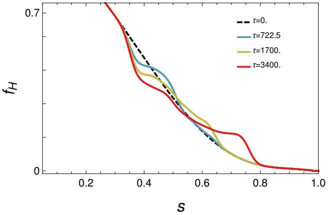
- **Bump-on-tail paradigm**: relevant scalings and effective non-linear velocity spread.
- **Validation of reduced models** for the BoT paradigm: Dyson equation and quasi-linear theory.
- **Mapping procedure** between velocity space of BoT model and radial configuration of fast ions interacting with TAE in a Tokamak device: successfully tested for an ITER relevant case.
- **EGAM**: non-linear velocity spread is predicted in terms of BoT dynamics.

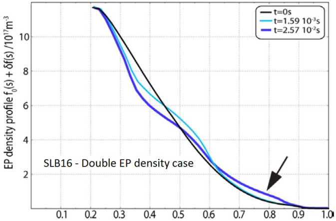
- Model with other 20 modes having wave numbers resonating with $0.55 \leqslant s \leqslant 0.95$. Given the linear setup (thus $\bar{\gamma}_{D(h)}$), specific damping rates $\bar{\gamma}_{d(h)}$ are introduced to get a **Gaussian distribution for growth-rates** $\bar{\gamma}_{(h)} = \bar{\gamma}_{D(h)} - \bar{\gamma}_{d(h)}$:



  - Mode evolution: **spectral transfer** clearly emerges.
    Additional modes progressively excited.

Increased **avalanche transport** toward large radial positions.
Outer redistribution is triggered around $s \simeq 0.75$:

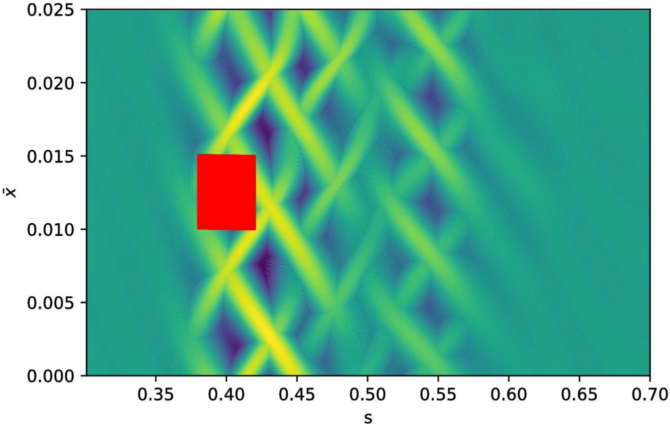




- Domino effects is **less efficient** in the BoT due to the fixed character of the spectrum which can not mimic the very details of the harmonic effects near the edge.
- Despite the differences, the **non-pure diffusive** character of the transport is enlightened.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

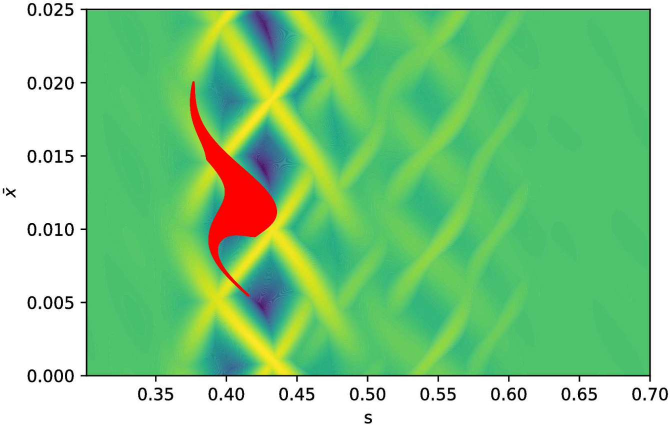
$\tau_0 = 200$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

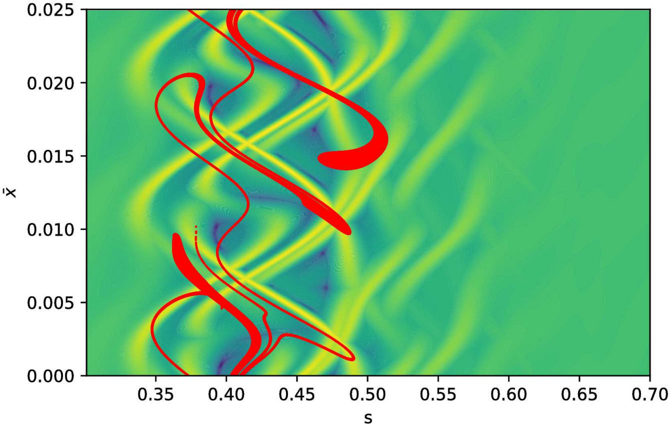
$\tau_0 = 300$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

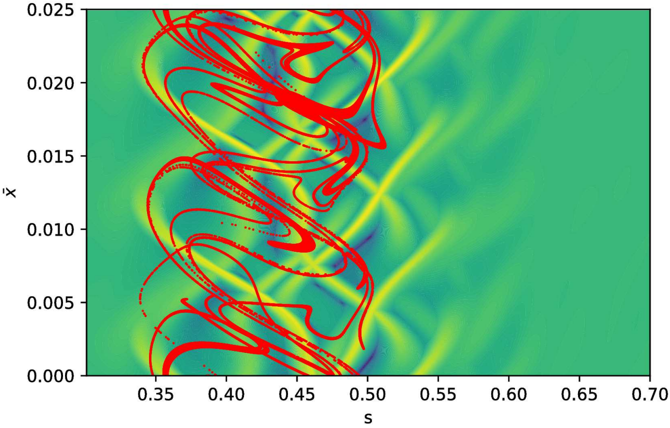
$\tau_0 = 500$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

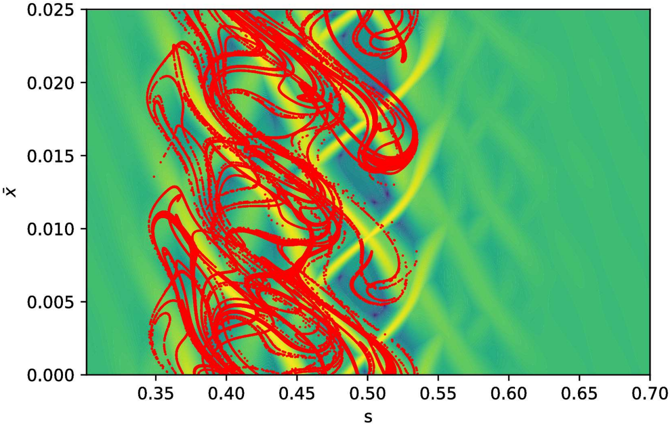
$\tau_0 = 600$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

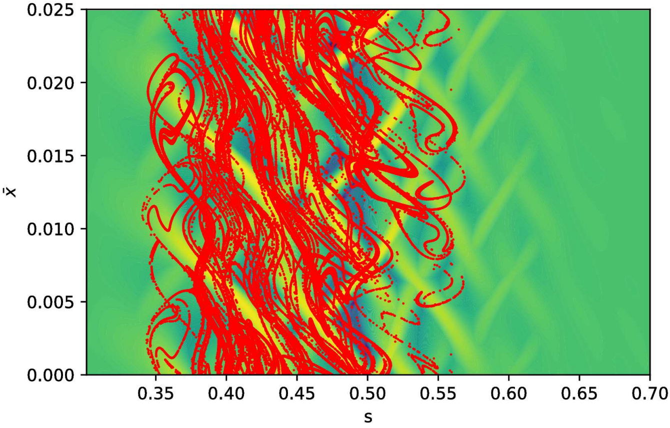
$\tau_0 = 700$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

- Failure of QL theory: non-diffusive dynamics expected (redistribution to the edge).

$\tau_0 = 800$



Tracers initialized at $s \simeq 0.4$. Clear emergence of avalanche-like transport.

Backup slides: Stable and unstable manifold