(NAT WP3) Multiple-n simulations of Alfvénic modes

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Progress of Hybrid MHD-gyrokinetic simulations

EUROfusion enabling research projects:

- **NLED** Progress achieved in understanding nonlinear dynamics of single-n mode driven by energetic particles (saturation mechanism, non-adiabatic chirping etc.) due to nonlinear wave-particle interactions.

- **NAT** Progress achieved in understanding nonlinear dynamics of multiple-n modes simulations. In such cases, apart from nonlinear wave-particle interactions, wave-wave couplings are taken into account, which can modify qualitatively and/or quantitatively the nonlinear dynamics w.r.t. single-n simulations. Meanwhile, zonal flow physics is the first time investigated by (X)HMGC code, and is the first time carried out by a hybrid-code which includes kinetic thermal ion effects.

- **MET** Hybrid simulations (e.g. XHMGC and HYMAGYC) can help further investigation of multi-scale energetic particle transport in fusion devices.

(X)HMGC model equations

Hybrid reduced $O(\epsilon_0^3)$ MHD equations (HMGC) (Briguglio et al., Phys. Plasmas 2, 3711 (1995); Wang et al., Phys. Plasmas 18, 052504 (2011)).

$$\frac{\partial\psi}{\partial t} = \frac{R^2}{R_0}\nabla\psi\times\nabla\varphi\cdot\nabla U + \frac{B_0}{R_0}\frac{\partial U}{\partial\varphi} + \eta\frac{c^2}{4\pi}\Delta^*\psi + O(\epsilon^4 v_A B_\varphi),$$ (the evolution of the poloidal magnetic flux function)

$$\hat{\rho}\left(\frac{D}{Dt} + \frac{2}{R_0}\frac{\partial U}{\partial Z}\right)\nabla_\perp^2 U + \nabla\hat{\rho}\cdot\left(\frac{D}{Dt} + \frac{1}{R_0}\frac{\partial U}{\partial Z}\right)\nabla U = \frac{1}{4\pi}\mathbf{B}\cdot\nabla\Delta^*\psi + \frac{1}{R_0}\nabla\cdot\left[R^2(\nabla P + \nabla\cdot\mathbf{\Pi_s})\times\nabla\varphi\right] + O\left(\epsilon^4\rho\frac{v_A^4}{a^2}\right),$$ (the evolution of the scalar potential)

$$\Pi_s(t,x) = \frac{1}{m_s^2}\int d\bar{Z}D_{Z_c\to\bar{Z}}\bar{F}_s(t,\bar{\mathbf{R}},\bar{M},\bar{V})\times\left[\frac{\Omega_s\bar{M}}{m_s}\mathbf{I} + \mathbf{bb}\left(\bar{V}^2 - \frac{\Omega_s\bar{M}}{m_s}\right)\right]\delta(\mathbf{x}-\bar{\mathbf{R}})$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + v_\perp\cdot\nabla,\ U = -c\phi/B_0,\ Z^i = (\mathbf{R}, M, V)$ are the gyrocenter coordinates

ψ is the magnetic stream function

ϕ is the electrostatic potential

"s" stay for EP species, and/or thermal ions, …

Nonlinear simulations

HMGC code retains both fluid (wave-wave) and energetic particles nonlinearities. Nonlinear simulations are performed by the following steps:

- The magnetic flux function $\psi_{0,0}$ and $\psi_{0,1}$ are fixed for now, but can also evolved (stable simulation) by using radially dependent resistivity profile.
- The scalar potential $\phi_{0,0}$ are not evolved.

Step 1. Multiple-n simulations without wave-wave coupling: nonlinear coupling through EP drive

Step 2. Multiple-n simulations with wave-wave coupling: nonlinear coupling through both EP drive and wave-wave coupling (collaboration w.r.t WP6)

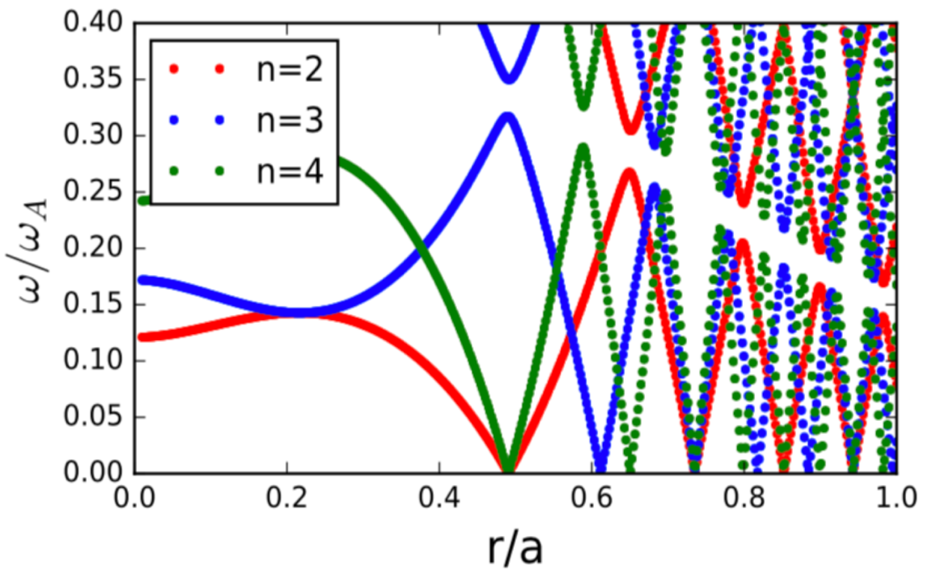
- The scalar potential $\phi_{0,0}$ are evolved.

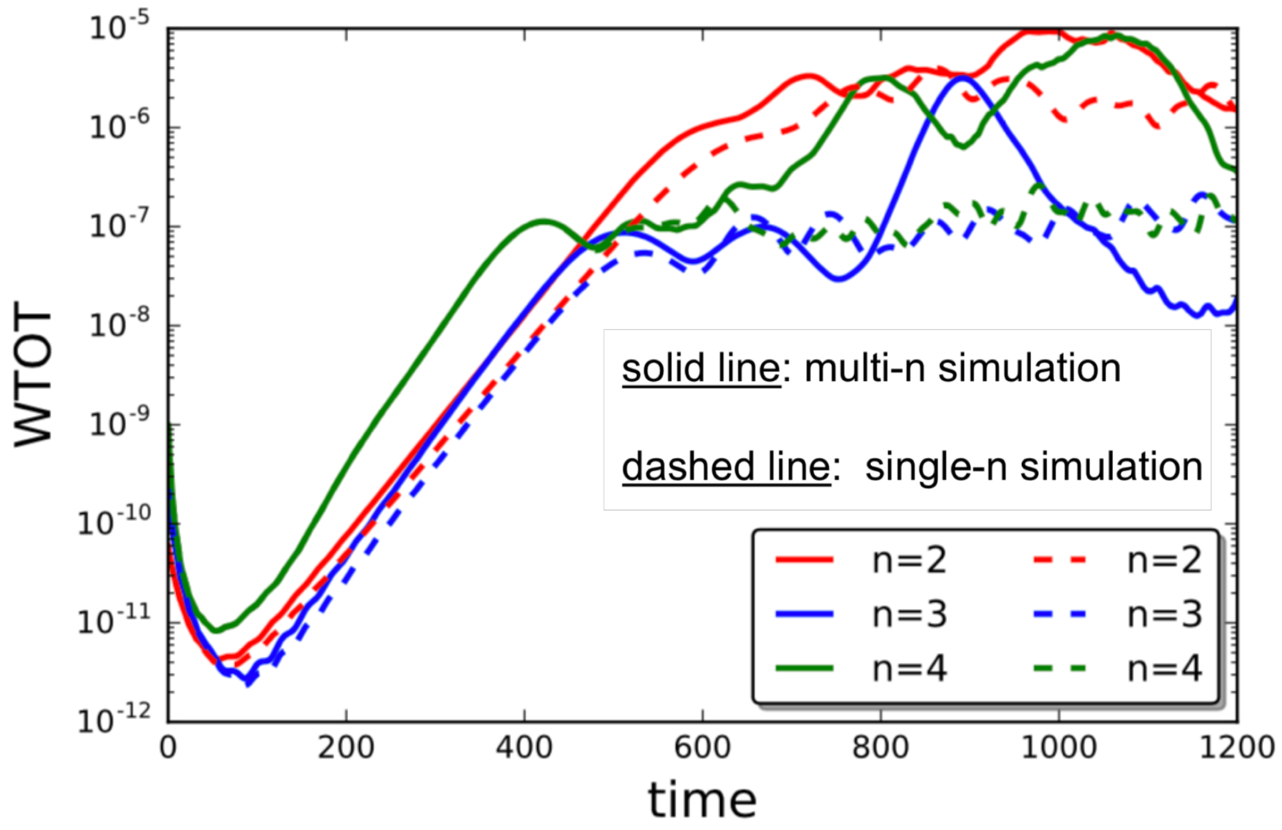
Step 1. Simplified electrostatic simulation: GAM and zonal flow residual level

Step 2. Nonlinear generation of zonal fields by an EPM mode

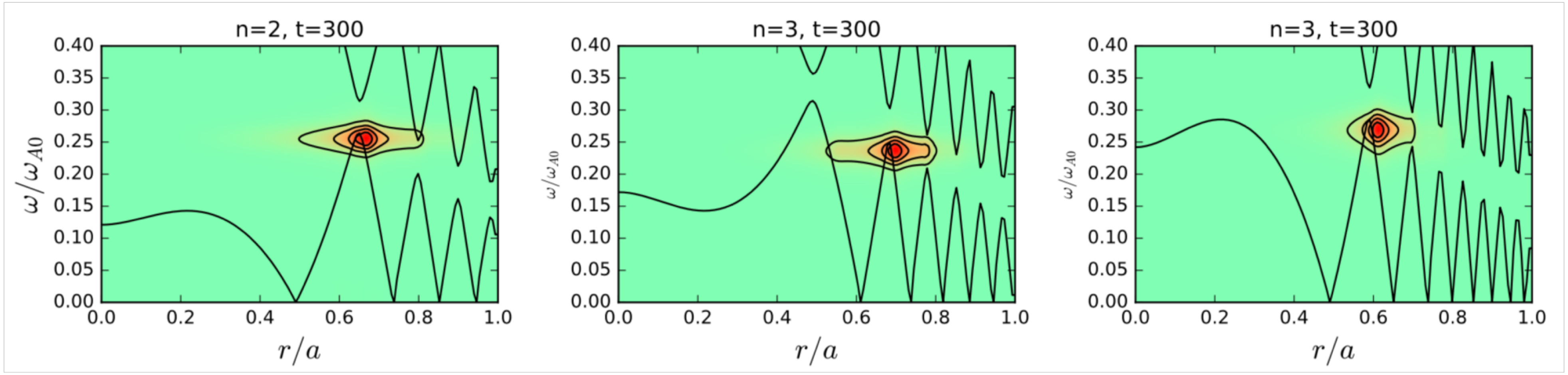
Nonlinear coupling through EP drive (i)

- continuum spectrum for n=2,3,4

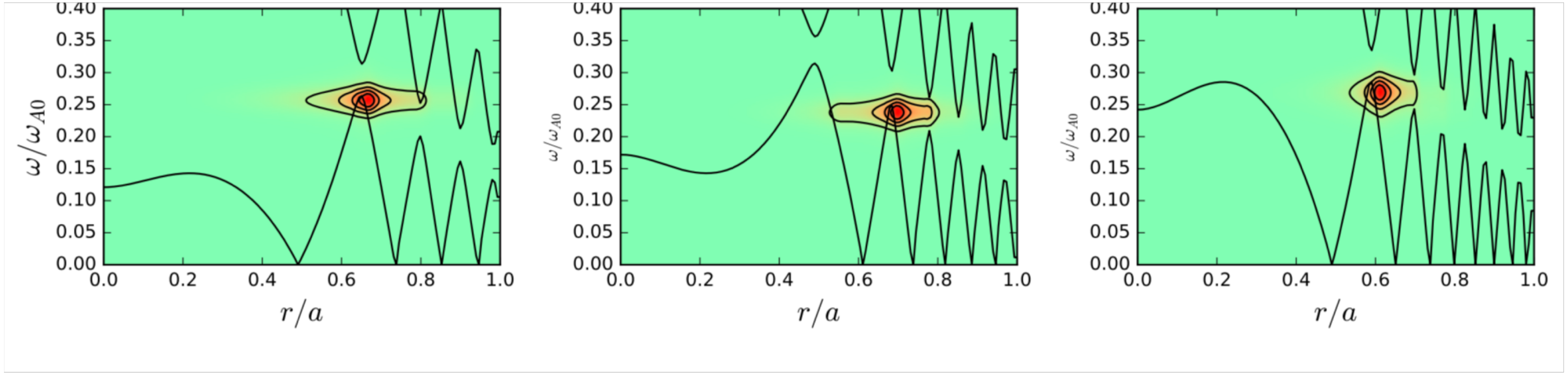




Nonlinear coupling through EP drive (ii)

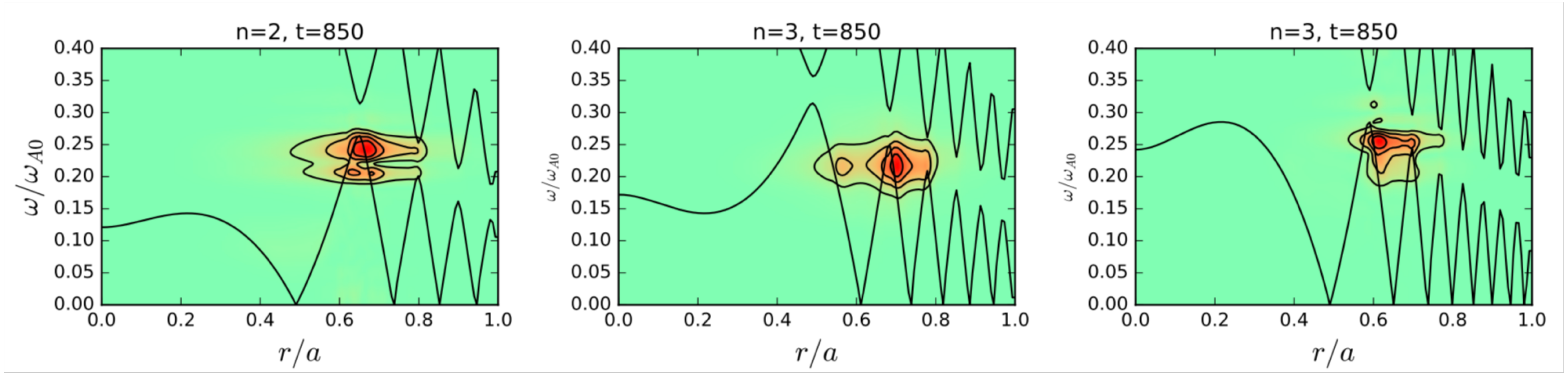


multi-n simulations

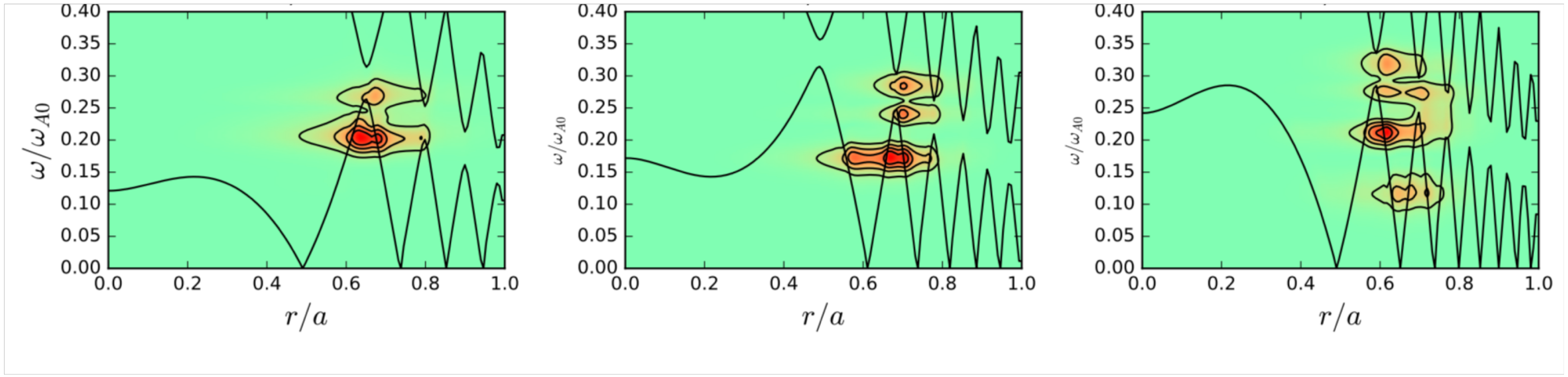


single-n simulations

Nonlinear coupling through EP drive (iii)



multi-n simulations

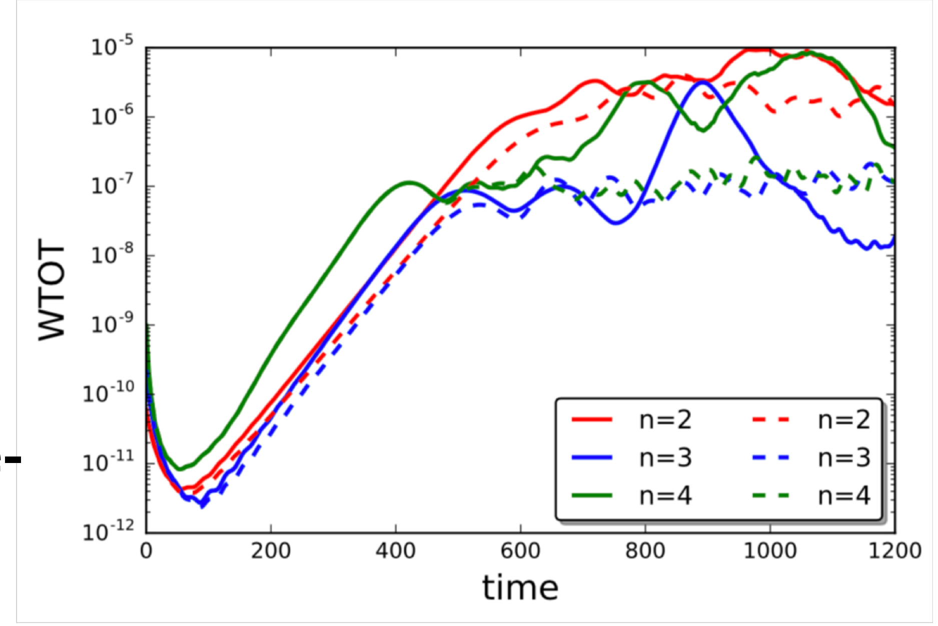


single-n simulations

Nonlinear coupling through EP drive (iv)

Multiple-n modes without wave-wave coupling

- Linear phase: most unstable mode not change, sub-dominant modes have slightly higher growth-rates.
- Once the sub-dominant mode (n=2) amplitude overtakes the most unstable one, both n=3 and n=4 modes amplitude are strongly modulated w.r.t. the single-n simulations.
- How the resonance structures overlap?
- How the phase space structure modified by the overlapping resonances?
- Although larger amplitudes observed, weaker frequency chirping happens.
- Enhancement of EP transport w.r.t. single-n simulations.



A paper will be prepared by answering those questions.

Nonlinear coupling through both EP drive and fluid NL (i)

Mode coupling (e.g. three wave scheme $n = n_1 + n_2$) through the EP term $\nabla \cdot \Pi_H$ [Vlad et al., 2018] can be recognized schematically by the following: $\Pi_H \propto \delta F_H$

$$\left(\frac{\partial}{\partial t} + \frac{dZ^i}{dt}\frac{\partial}{\partial Z^i}\right)\delta F_H = -\left(\frac{dZ^i}{dt}\right)_{pert}\frac{\partial}{\partial Z^i}F_{H;eq}$$

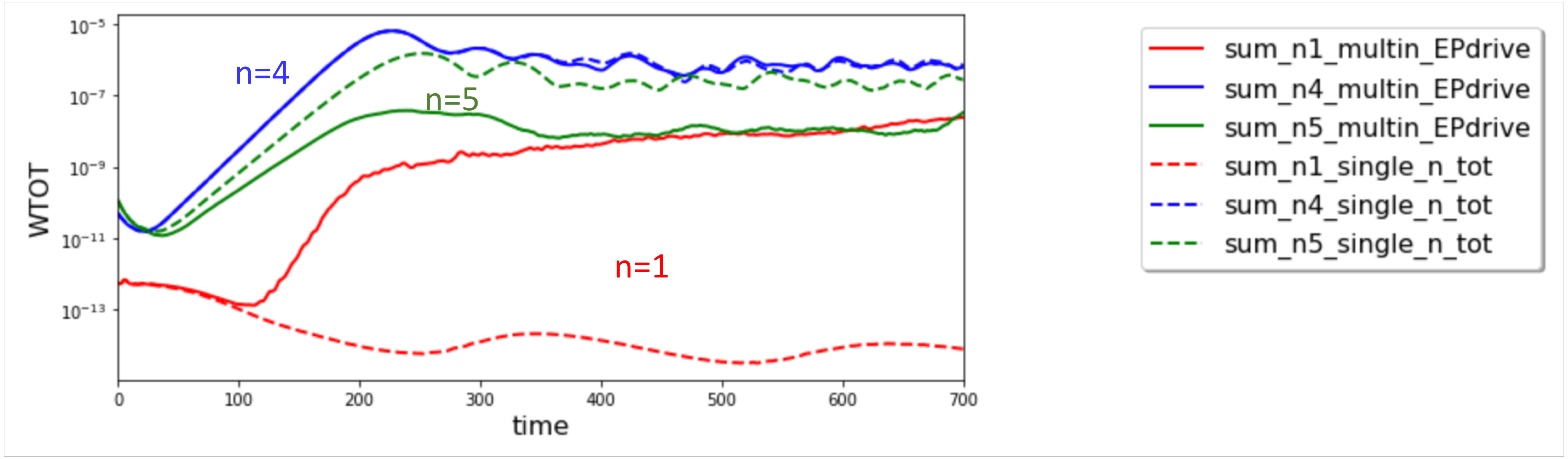
After formally splitting the generalized velocities in the l.h.s. in unperturbed (unpert) and perturbed (pert) ones:

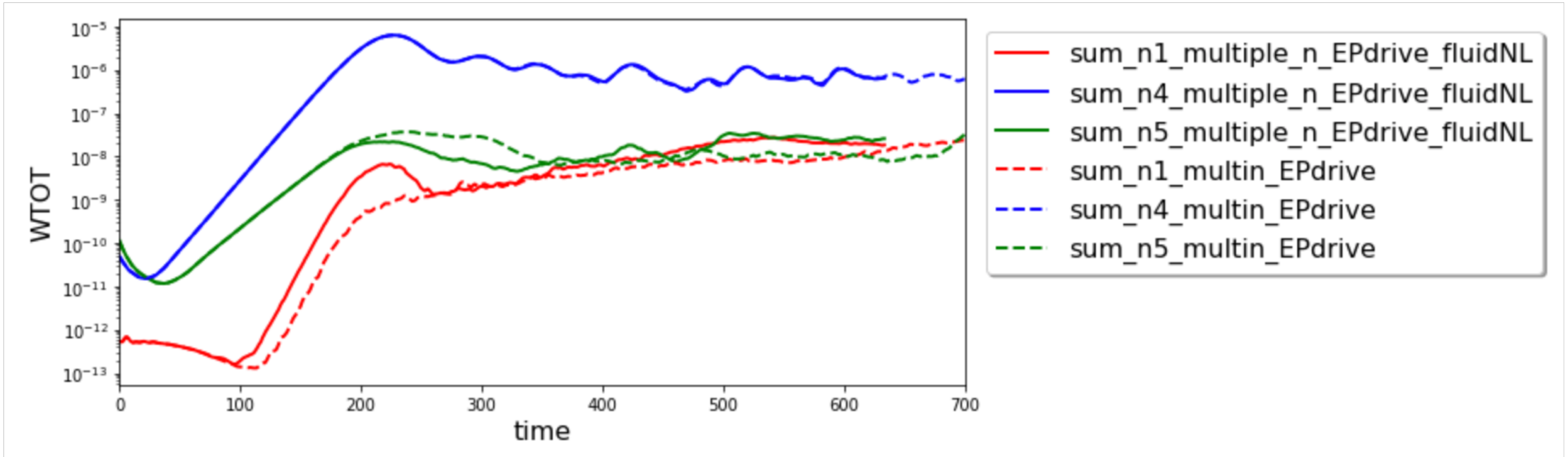
$$\left[\frac{\partial}{\partial t} + \left(\frac{dZ^i}{dt}\right)_{unpert}\frac{\partial}{\partial Z^i}\right]\delta F_H = -\left(\frac{dZ^i}{dt}\right)_{pert}\frac{\partial}{\partial Z^i}F_{H;eq} - \left(\frac{dZ^i}{dt}\right)_{pert}\frac{\partial}{\partial Z^i}\delta F_H$$

Passing to toroidal Fourier components:

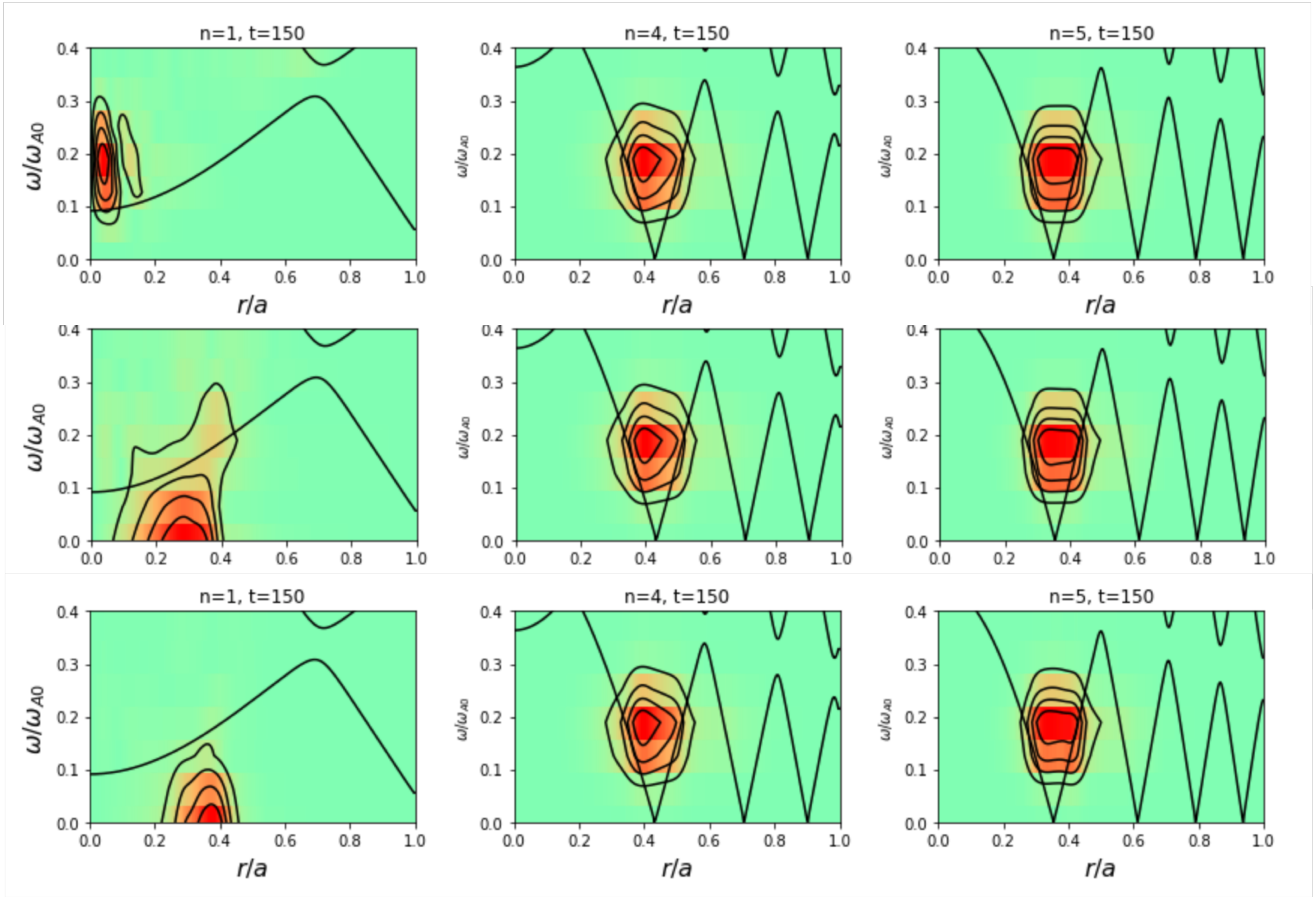
$$\left[\frac{\partial}{\partial t} + \left(\frac{dZ^i}{dt}\right)_0\frac{\partial}{\partial Z^i}\right]\delta F_{H;n} = -\left(\frac{dZ^i}{dt}\right)_n\frac{\partial}{\partial Z^i}F_{H0} - \Sigma_{\tilde{n}}\left(\frac{dZ^i}{dt}\right)_{n-\tilde{n}}\frac{\partial}{\partial Z^i}\delta F_{H;\tilde{n}}$$

Nonlinear coupling through both EP drive and fluid NL (ii)





Nonlinear coupling through both EP drive and fluid NL (iii)

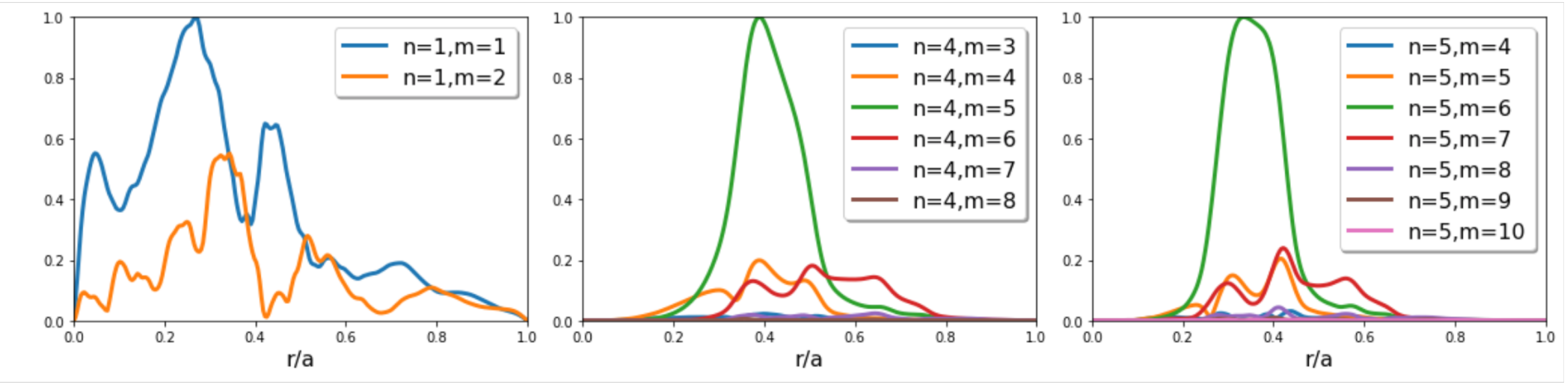


single-n simulations

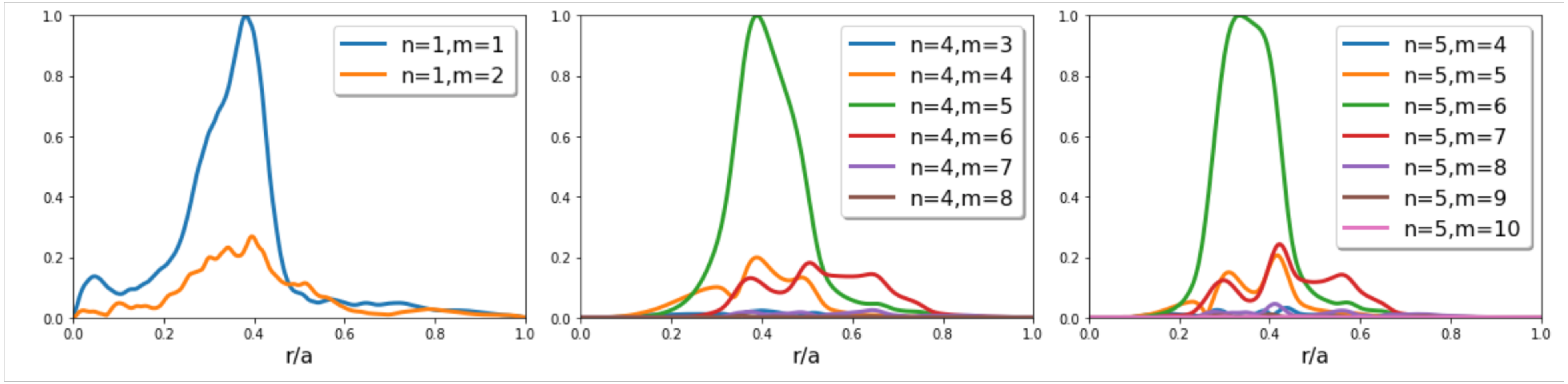
multi-n EP drive

multi-n EP drive and fluid NL

Nonlinear coupling through both EP drive and fluid NL (iv)



multi-n EP drive



multi-n EP drive and fluid NL

GAM and zonal flow (i)

Simplified vorticity equation

$$\hat{\rho}\left(\frac{D}{Dt}+\frac{2}{R_0}\frac{\partial U}{\partial Z}\right)\nabla_\perp^2 U+\nabla\hat{\rho}\cdot\left(\frac{D}{Dt}+\frac{1}{R_0}\frac{\partial U}{\partial Z}\right)\nabla U=\frac{1}{4\pi}\mathbf{B}\cdot\nabla\Delta^*\psi+\frac{1}{R_0}\nabla\cdot[R^2(\nabla P+\nabla\cdot\mathbf{\Pi_s})\times\nabla\varphi]+O\left(\epsilon^4\rho\frac{v_A^4}{a^2}\right)$$

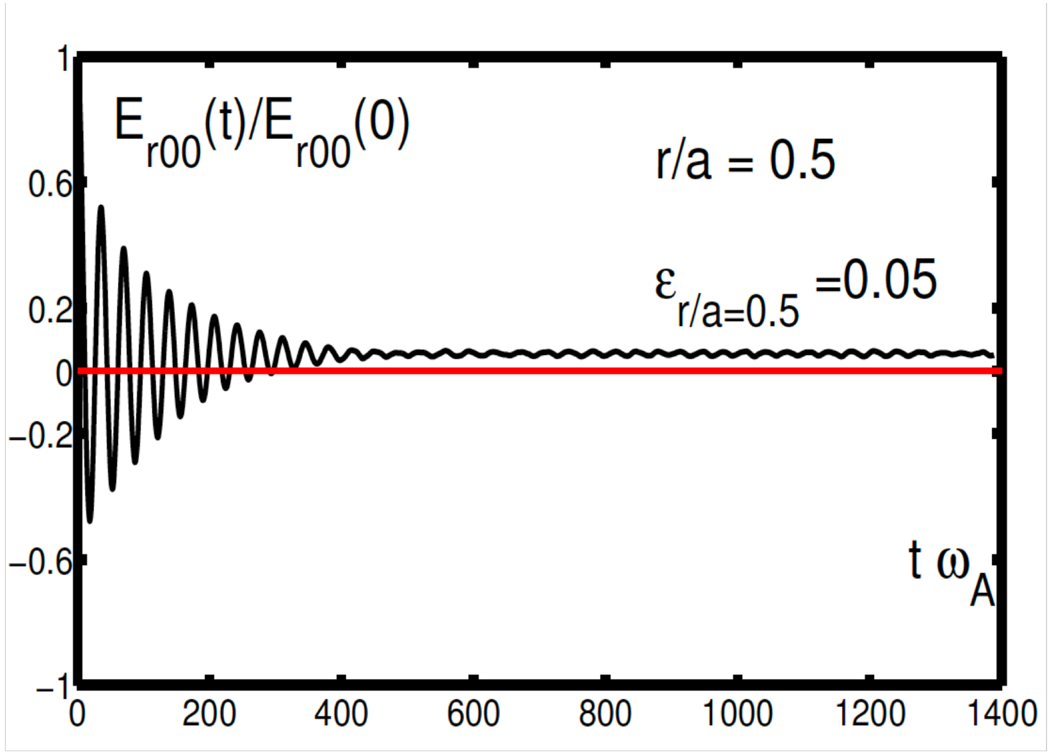
Simulation results:

1. Flat q profiles
2. $\epsilon = 0.1$
3. Scanning parameters are:

$q = [1.2, 1.4, 1.6, 2.0, 2.5]$,

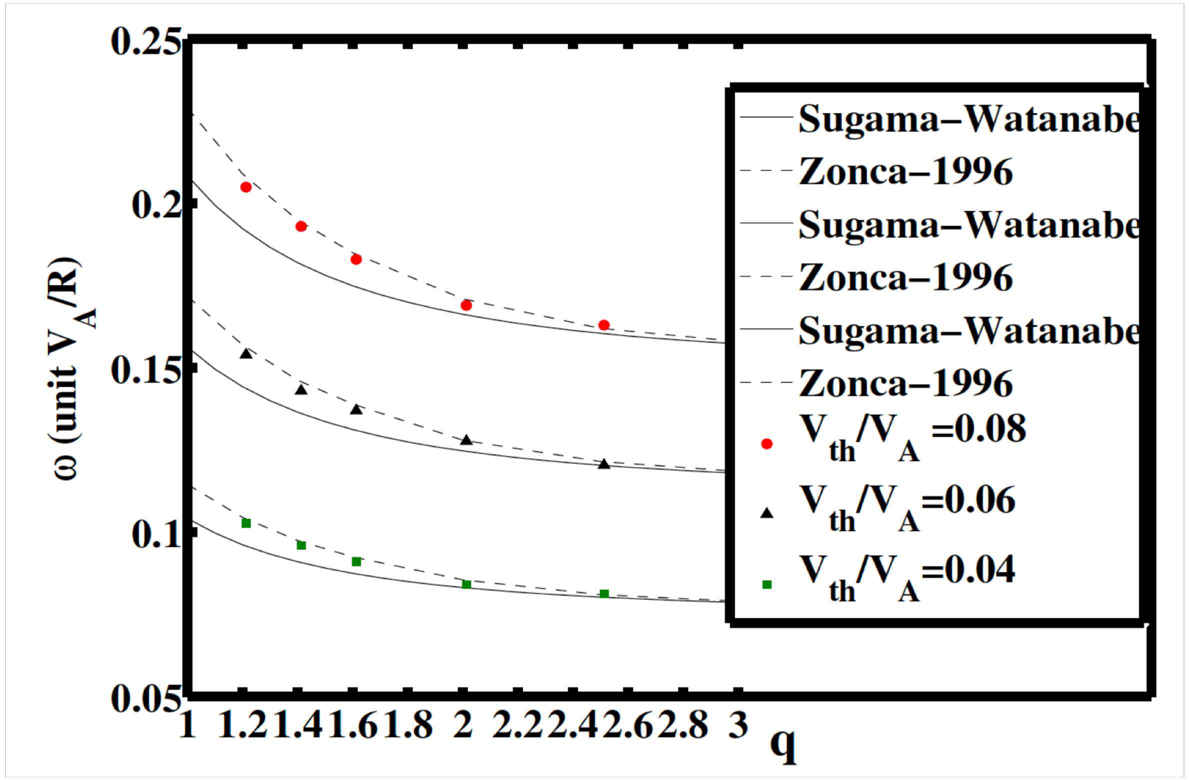
and $\frac{V_i}{V_A} = [0.04, 0.06, 0.08]$

and $\frac{\rho}{a} = [0.003125, 0.0046875, 0.00625]$

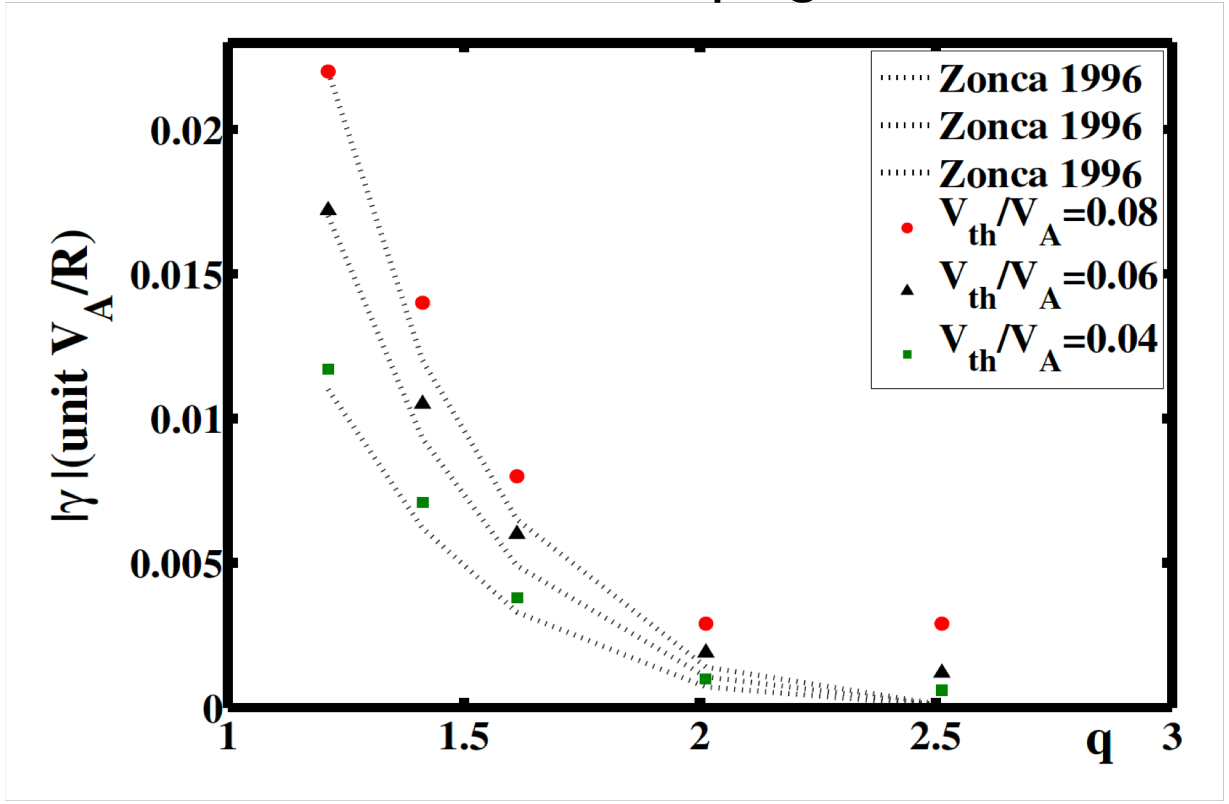


GAM and zonal flow (ii)

Real frequency

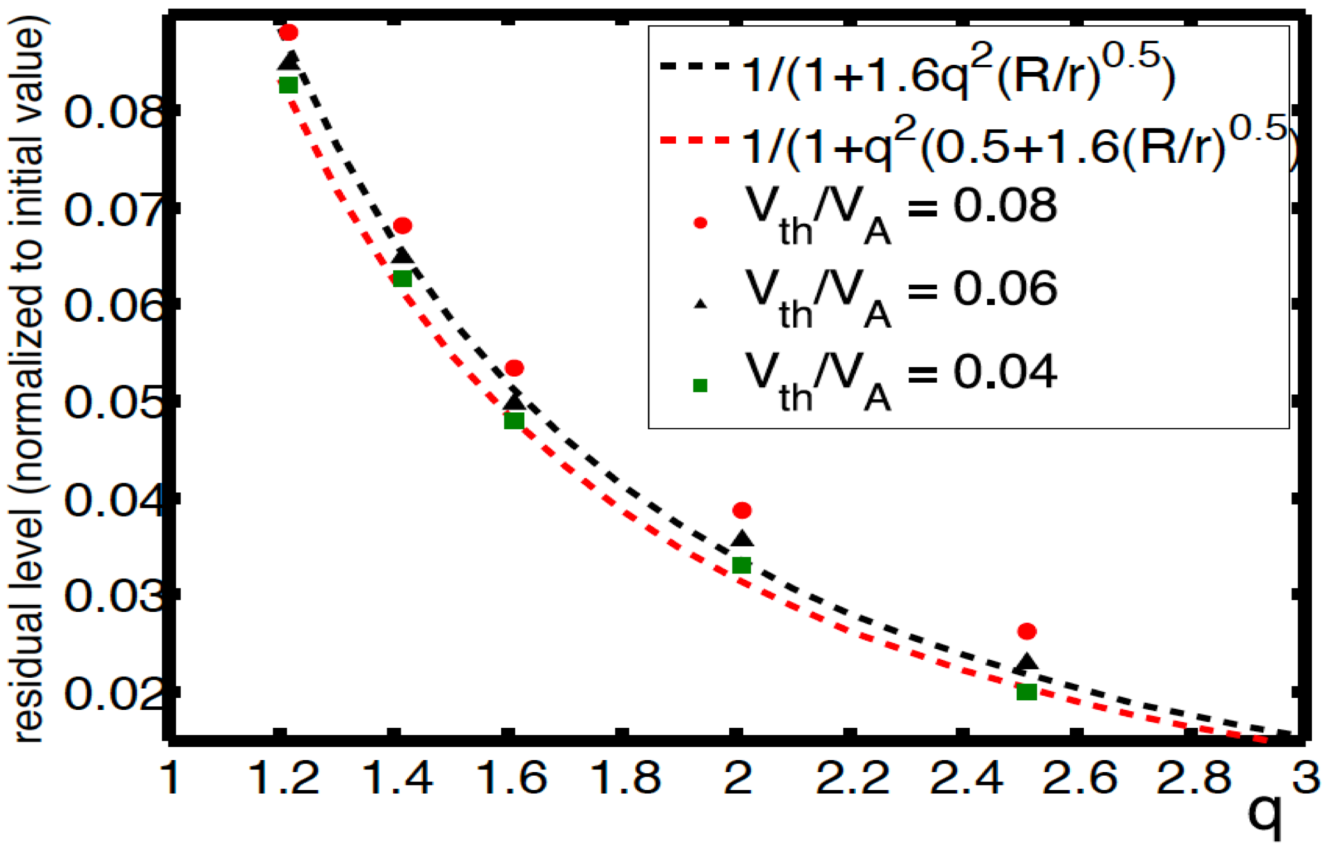


Damping rate

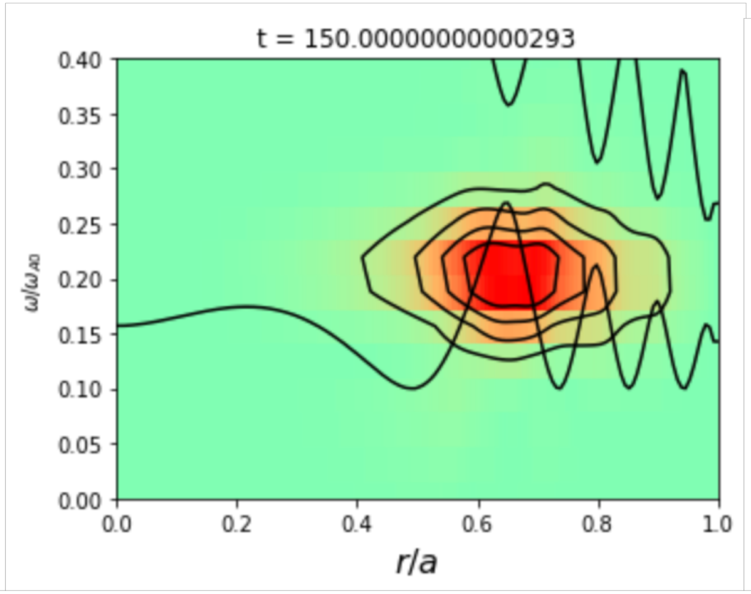


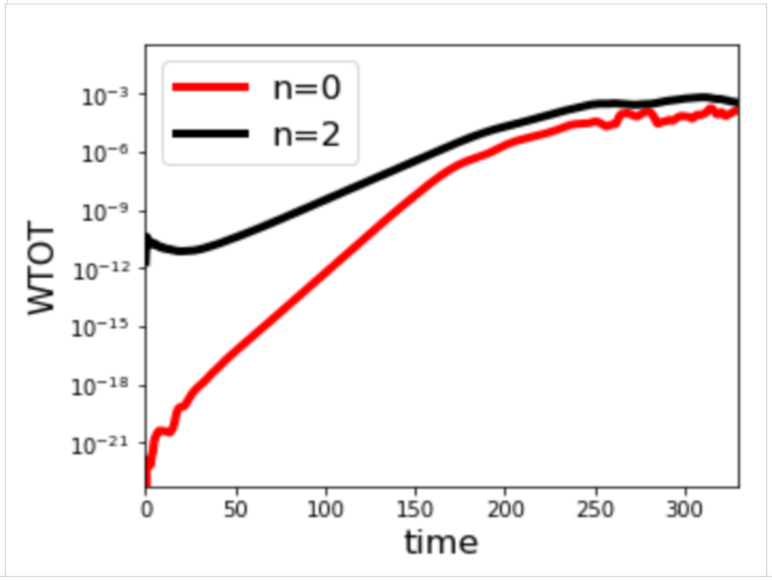
GAM and zonal flow (iii)

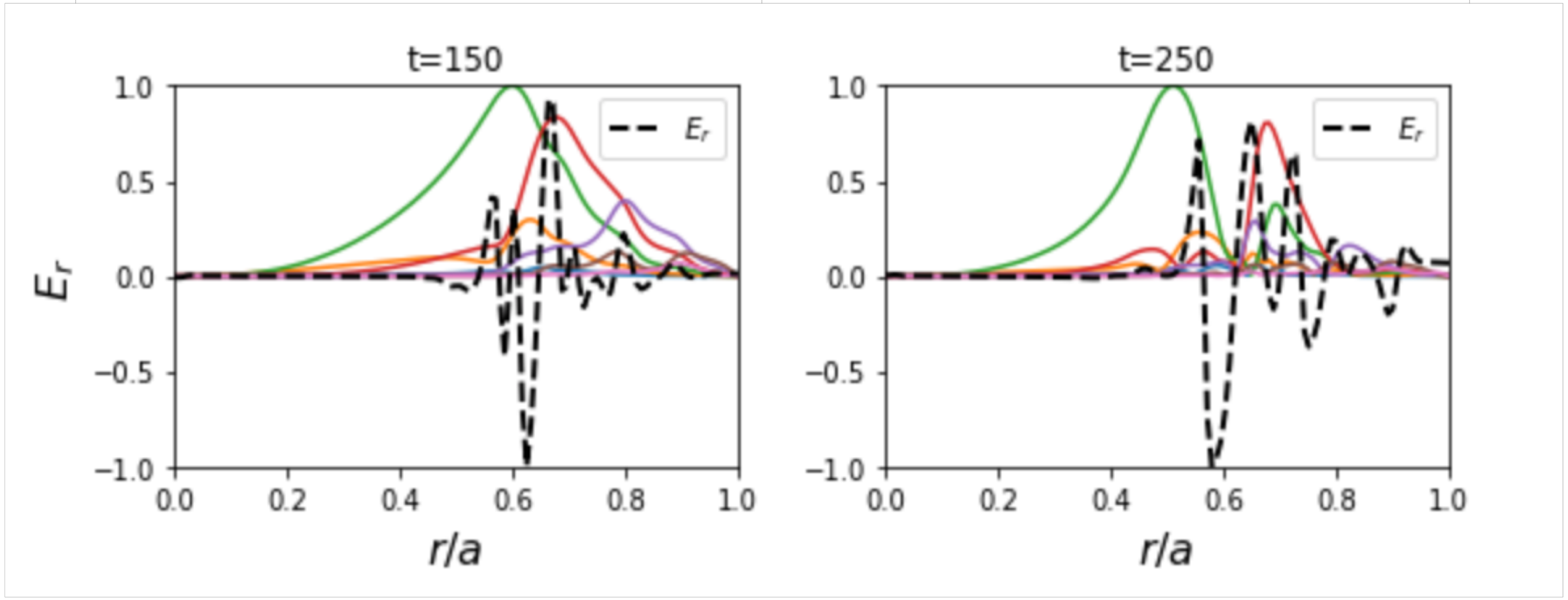
Residual level



Nonlinear generation of zonal fields by an EPM mode







Conclusions.

- Comparison between single-n and multiple-n simulations of Alfvénic modes has been performed.
- Without wave-wave coupling, the multiple-n simulation shows different amplitude, frequency and mode structure evolution of each mode w.r.t. the single-n simulations.
- With wave-wave coupling, NL coupling is shown from MHD terms and mediated by EP term.
- (X)HMGC is the first time used to simulate GAM and zonal flow. The real frequency, damping rate and the zonal flow residual level agree well with the analytical prediction.
- Nonlinear generation of zonal flow by a growth EPM has been simulated. The growth rate of zonal flow and the radial mode structure are investigated.

Report WP3 (X. Wang)
Milestones and Deliverables 2017/2018, Work package 3
Main contributors: X. Wang, Z. Lu

- develop and implement scheme to extract zonal current dynamics from resistivity effects (2017)
- benchmark with other models within the project, study importance of kinetic thermal ion response in comparison to MHD for the zonal dynamics (2018)

  - Nonlinear zonal flow generation by EP driven Alfvénic mode complete
  - no benchmarks with other code
  - Radially dependent resistivity can stable the simulation with zonal current evolved.
  - Instead: efforts contributed to multiple-n simulations