Pullback mitigation scheme in ORB5

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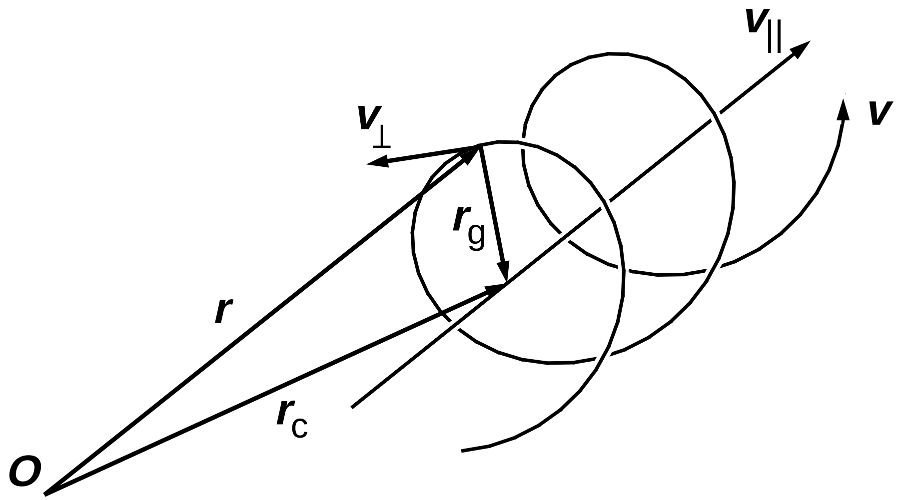
Motivation (pullback in ORB5)

- Kinetic effects on MHD instabilities
  - Finite ion gyroradius (e. g. comparable to boundary layer width – kink)
  - Finite electric field (electron inertia, electron pressure, collisionality)
  - Non-fluid "compressibility": trapped-particle effect (kink, ballooning)
- Kinetic destabilisation of MHD-stable modes
  - Fast-ion destabilisation of Alfvén eigenmodes: TAE, HAE, GAE, BAE . . .
  - Lower MHD destabilisation thresholds: KBM
  - Interaction between (marginal) MHD and fast ions: fishbones
- EM microturbulence: global profile evolution (drift-Alfvén, magnetic flutter)

1. Global approach is needed (intrinsic for MHD; needed for profiles)
2. Kinetic approach is needed (to address relevant physics)

Gyrokinetic theory



$$\epsilon_B = r_g / L_B \ll 1$$

$$\epsilon = \omega/\omega_c \sim k_\parallel / k_\perp \sim q\delta\phi/T \sim \delta B/B \ll 1$$

PERTURBATIVE ELIMINATION OF THE FAST GYROMOTION

Gyrokinetic SPC-IPP global PIC code suite

- 1995: GYGLES is developed at SPC(CRPP) with adiabatic electrons
- 1997: kinetic electrons implemented in GYGLES at IPP
- 1999: ORB5 is developed at SPC(CRPP)
- 1999: EUTERPE is developed at SPC(CRPP)
- 2004: EUTERPE implemented for W7-X at IPP
- 2004: GYGLES becomes electromagnetic at IPP
- 2008: ORB5 becomes electromagnetic, joint development IPP/SPC/UW
- 2009: EUTERPE becomes electromagnetic at IPP

All the codes share the equations solved, physics addressed and the discretisation principles applied. Deeper core routines are often very similar. Normalisation in EUTERPE and ORB5 is almost identical.

Cancellation problem

$$\underbrace{\frac{\beta_i}{\rho_i^2} A_\| + \frac{\beta_e}{\rho_e^2} A_\|}_{grid} - \nabla_\perp^2 A_\| = \mu_0 \underbrace{\left(\bar{j}_{\|i} + \bar{j}_{\|e}\right)}_{particles}$$

- The skin terms are "generated" by $p_\|$-formulation: Not physics!
- The electron skin term can be very large

$$\frac{\beta_e}{\rho_e^2} A_\| = \frac{\mu_0 n_0 e^2}{m_e} A_\|$$

- The adiabatic current is "generated" by $p_\|$-formulation: Not physics!

$$\bar{H}_1 = q_s \left(\langle\phi\rangle_s - v_\| \langle A_\| \rangle_s\right) \;, \quad F_e^{(\mathrm{ad})} = F_{0e}\, e^{-\bar{H}_1/T_e} \approx -\frac{q_e F_{0e}}{T_e}\left(\phi - v_\| A_\|\right)$$

- Adiabatic current (particles) coincides with the skin terms (grid): Must cancel each other!

$$\mu_0 \bar{j}_{\|s}^{(\mathrm{ad})} = \mu_0 q_s \int v_\| F_s^{(\mathrm{ad})} \mathrm{d}^3 v = \frac{\mu_0 n_0 e^2}{m_e} A_\| = \frac{\beta_e}{\rho_e^2} A_\|$$

Mixed-variable gyrokinetics

- Split the magnetic potential into the 'symplectic' and 'Hamiltonian' parts:

$$A_\| = A_\|^{(\mathrm{s})} + A_\|^{(\mathrm{h})}$$

- The perturbed guiding-center phase-space Lagrangian

$$\gamma = q\vec{A}^* \cdot \mathrm{d}\vec{R} + \frac{m}{q}\mu\,\mathrm{d}\theta + q\,A_\|^{(\mathrm{s})}\vec{b}\cdot\mathrm{d}\vec{x} + q\,A_\|^{(\mathrm{h})}\vec{b}\cdot\mathrm{d}\vec{x} - \left[\frac{mv_\|^2}{2} + \mu B + q\phi\right]\mathrm{d}t$$

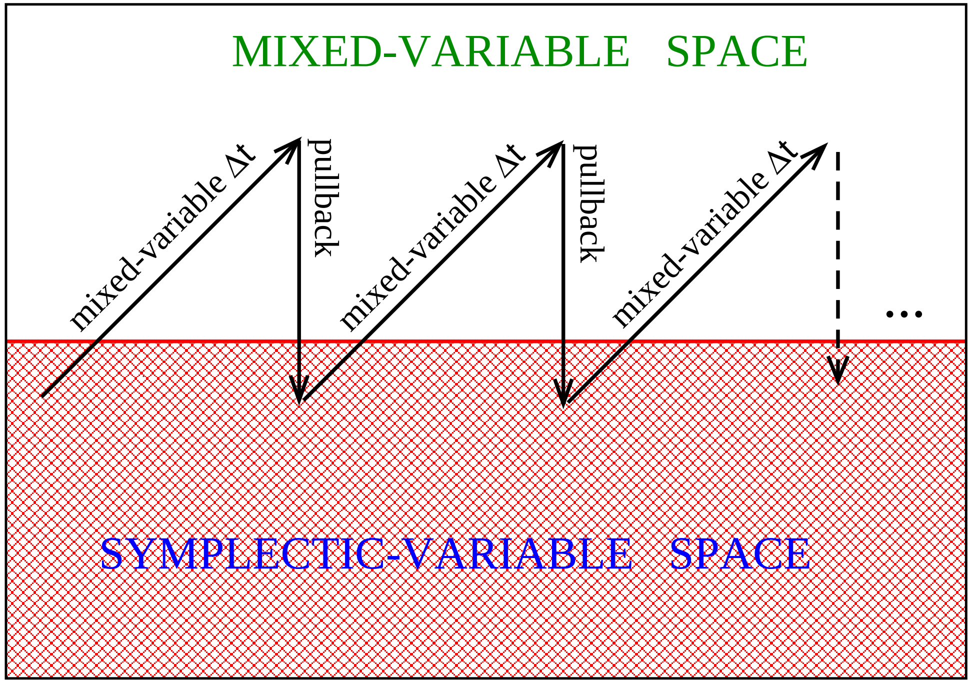
- "Mixed" Lie transform: $A_\|^{(\mathrm{h})} \longrightarrow$ Hamiltonian, $A_\|^{(\mathrm{s})} \longrightarrow$ symplectic structure

$$\Gamma = q\vec{A}^* \cdot \mathrm{d}\vec{R} + \frac{m}{q}\mu\,\mathrm{d}\theta + q\left\langle A_\|^{(\mathrm{s})}\right\rangle \cdot \mathrm{d}\vec{R} - \left[\frac{mv_\|^2}{2} + \mu B + q\left\langle \phi - v_\| A_\|^{(\mathrm{h})}\right\rangle\right]\mathrm{d}t$$

- An equation for $\partial A_\|^{(\mathrm{s})}/\partial t$ is needed

$$\frac{\partial}{\partial t}A_\|^{(\mathrm{s})} + \vec{b}\cdot\nabla\phi = 0$$

Mixed-variable formulation: pullback algorithm



Nonlinear pullback:

$$f_{1s}(Z_s, A_\parallel^{(s)}) = f_{1m}(Z_m, A_\parallel^{(s)}, A_\parallel^{(h)})$$

$$v_\parallel^{(s)} = v_\parallel^{(m)} - \frac{e}{m}\left\langle A_\parallel^{(h)}\right\rangle$$

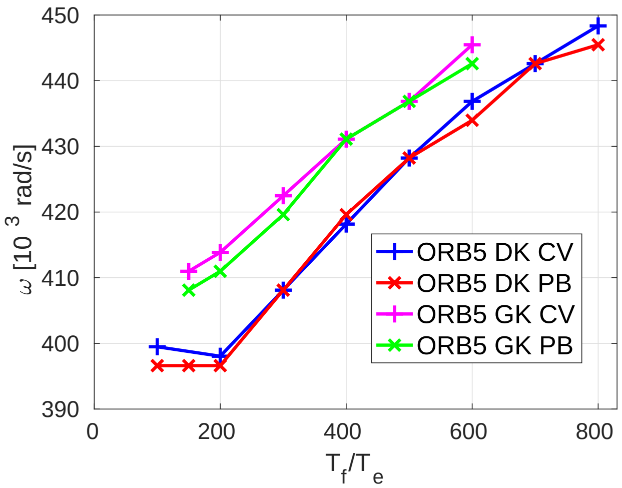
Additional nonlinear terms related to
$$\vec{B}^* = \vec{B} + \frac{mv_\parallel}{q}\nabla\times\vec{b} + \nabla\left\langle A_\parallel^{(s)}\right\rangle\times\vec{b}$$

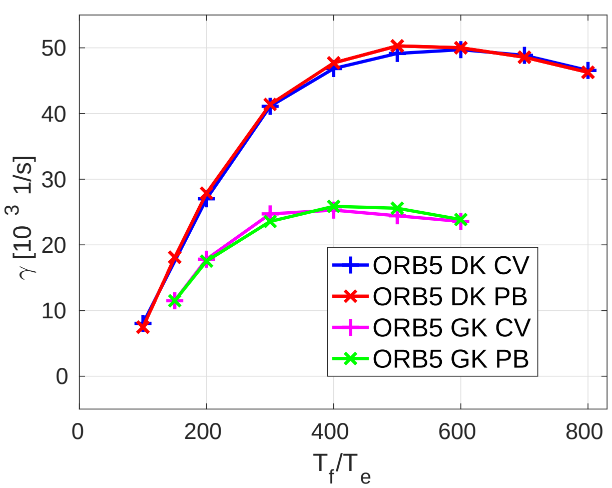
Linearised pullback:

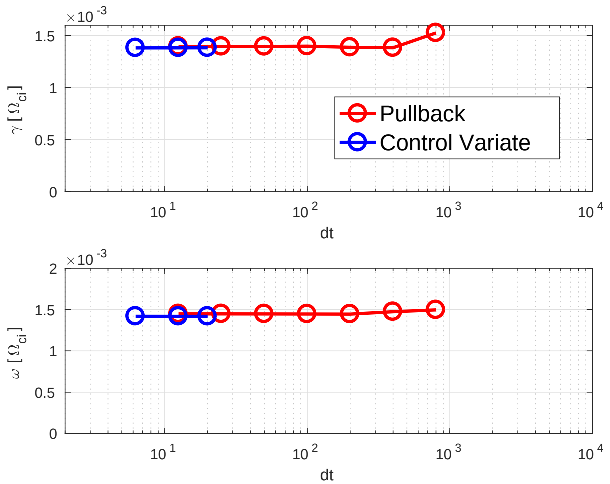
$$f_{1s(\mathrm{new})}^{(\mathrm{m})}(t_i) = f_{1s}^{(\mathrm{s})}(t_i) =$$
$$f_{1s(\mathrm{old})}^{(\mathrm{m})}(t_i) + \frac{q_s\left\langle A_{\parallel(\mathrm{old})}^{(\mathrm{h})}(t_i)\right\rangle}{m_s}\frac{\partial F_{0s}}{\partial v_\parallel}$$

1. Push coordinates and weights along the nonlinear mixed-variable trajectories
2. Transform from mixed variables into symplectic space (linear and nonlinear options)
3. Set $A_{\parallel(\mathrm{new})}^{(\mathrm{s})}(t_i) = A_\parallel(t_i) = A_{\parallel(\mathrm{old})}^{(\mathrm{s})}(t_i) + A_{\parallel(\mathrm{old})}^{(\mathrm{h})}(t_i)$ and $A_{\parallel(\mathrm{new})}^{(\mathrm{h})}(t_i) = 0$.

Pullback scheme implementation in ORB5

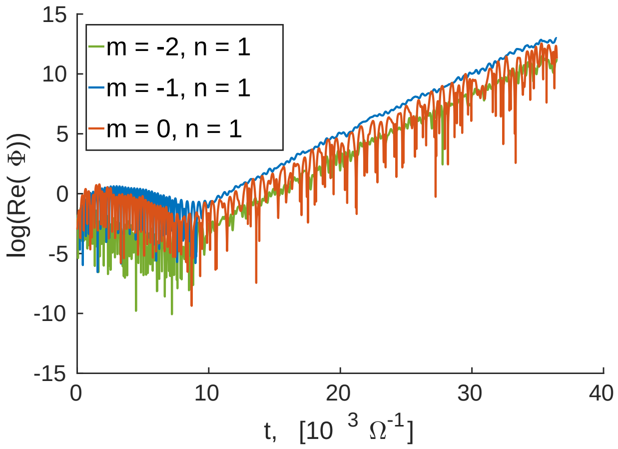


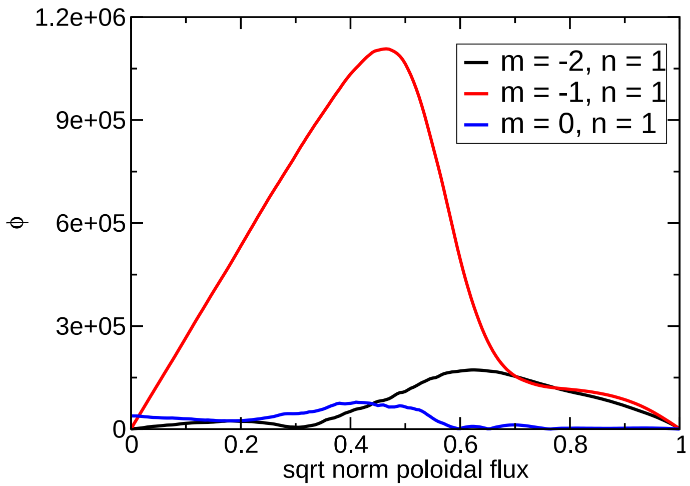


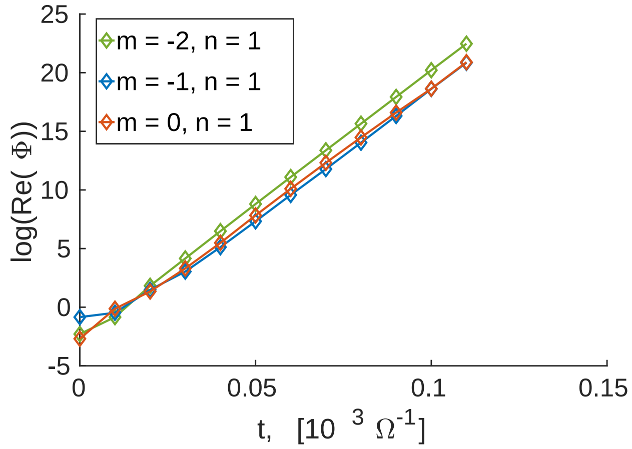


- Linear TAE instability (ITPA benchmark)
- Good agreement between pullback and control-variate simulations
- Pullback simulations much more efficient in terms of time step

Pullback scheme implementation in ORB5

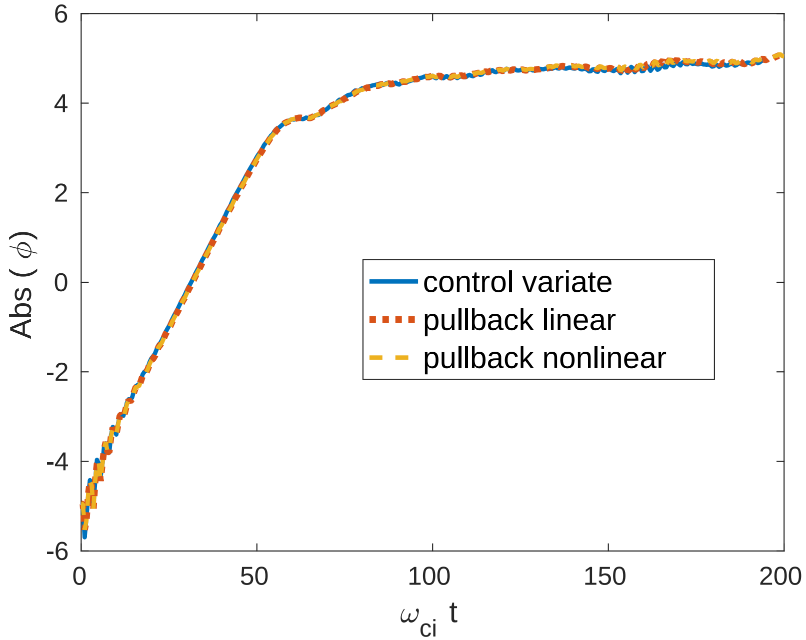






- Linear internal kink instability in ORB5 (tokamak)
- Simulation works fine for pullback mitigation
- Control variate is numerically unstable for the same parameters
- Control variate can be stabilised at smaller time step

Nonlinear pullback scheme in ORB5



- Control variate, linear pullback and nonlinear pullback compared
- Saturation level and linear evolution agree very well for all schemes
- Pullback mitigation is more efficient (time step can be larger)

Summary: going mainstream

- Cancellation problem has prohibited large-scale effort on GK PIC simulations (Reynders 1992, Cummings 1995)
- A lot of work has been done to mitigate the cancellation problem: control variate, mixed-variable pullback scheme
- The mitigation schemes can be used both in linear and nonlinear regime

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- Control-variate and pullback schemes have been implemented in ORB5
- Pullback scheme has been verified against control variate (ITPA-TAE benchmark) both in the linear and nonlinear regimes and for internal kink mode
- A considerable improvement of the code efficiency has been observed

In the outlook, ORB5 provides a unified framework for EM drift-wave turbulence, zonal flows / GAMs, fast particles, shear Alfvén waves (TAEs, BAEs, etc), MHD activity (internal kink instability) in axisymmetric tokamak geometry. This represents a vast field for future research.