Report WP 2 (Ph. Lauber, T. Hayward-Schneider)

Milestones and Deliverables 2017/2018, Workpackage 2
main contributors: Ph. Lauber, T. Hayward

- Formulation of the extension of HAGIS model for three-wave interaction; implementation into the existing code (2017)
- Test implementation, benchmark in simple situations to other codes and apply to multi-mode TAE problem (2018)

3 wave interaction model for HAGIS

$$\mathcal{L}_w = \underbrace{\sum_j \left\{\frac{1}{2} m v_j^2 + e\left(\mathbf{A}_j \cdot \mathbf{v}_j - \Phi_j\right)\right\}}_{\mathcal{L}_{bulk}} + \underbrace{\frac{1}{2\mu_0}\int_V \left\{\frac{1}{c^2}E^2 - B^2\right\} d^3x}_{\mathcal{L}_{em}}$$

represent bulk plasma velocity by MHD fluid v and expand:

$$\mathbf{B} = \mathbf{B}_0 + \sum_{k=1}^{n_w} \tilde{\mathbf{B}}_k \qquad \mathbf{v} = \frac{\mathbf{E} \wedge \mathbf{B}}{B^2}$$

$$\begin{aligned}
\mathcal{L}_w \;=\; & \sum_{\substack{\text{bulk}\\ \text{plasma}}} \left[\frac{1}{2} m \left\{ \mathbf{v}_0 + \sum_k \frac{\tilde{\mathbf{E}}_k \wedge \mathbf{B}_0}{B_0^2} + \sum_{k,k'} \frac{\tilde{\mathbf{E}}_k \wedge \tilde{\mathbf{B}}_{k'}}{B_0^2} \right\}^2 \right. \\
& \left. + e \left\{ \left(\mathbf{A}_0 + \sum_k \tilde{\mathbf{A}}_k \right) \cdot \left\{ \mathbf{v}_0 + \sum_k \frac{\tilde{\mathbf{E}}_k \wedge \mathbf{B}_0}{B_0^2} + \sum_{k,k'} \frac{\tilde{\mathbf{E}}_k \wedge \tilde{\mathbf{B}}_{k'}}{B_0^2} \right\} - \sum_k \tilde{\Phi}_k \right\} \right] \\
& + \frac{1}{2\mu_0} \int_V \left\{ \frac{1}{c^2} \sum_k \tilde{E}_k^2 - B_0^2 - \sum_k (2\mathbf{B}_0 \cdot \tilde{\mathbf{B}}_k + \tilde{B}_k^2) \right\} d^3x,
\end{aligned}$$

[S.D. Pinches, PhD Thesis]

3 wave interaction model for HAGIS

[S.D. Pinches, PhD Thesis]

$$\mathcal{L}_0 = \int_V \left\{ \frac{1}{2} n_i m v_0^2 + n_i e \mathbf{A}_0 \cdot \mathbf{v}_0 - \frac{1}{2\mu_0} B_0^2 \right\} d^3x,$$

$$(\text{Equilibrium force balance} \Rightarrow \mathcal{L}_0 \equiv 0)$$

$$\mathcal{L}_1 = \sum_k \int_V \left\{ n_i m \mathbf{v}_0 \cdot \frac{\mathbf{E}_k \wedge \mathbf{B}_0}{B_0^2} + n_i e \left(\mathbf{A}_0 \cdot \frac{\mathbf{E}_k \wedge \mathbf{B}_0}{B_0^2} - \Phi_k \right) - \frac{1}{\mu_0} \mathbf{B}_0 \cdot \tilde{\mathbf{B}}_k \right\} d^3x,$$

$$(\text{1st order force balance} \Rightarrow \mathcal{L}_1 \equiv 0)$$

$$\mathcal{L}_2 = \int_V \left\{ \frac{1}{2} n_i m \left(\sum_k \frac{(\mathbf{E}_k \wedge \mathbf{B}_0)^2}{B_0^4} + 2 \sum_{k,k'} \mathbf{v}_0 \cdot \frac{\mathbf{E}_k \wedge \mathbf{B}_{k'}}{B_0^2} \right) \right.$$
$$\left. + n_i e \sum_{k,k'} \left(\mathbf{A}_0 \cdot \frac{\mathbf{E}_k \wedge \mathbf{B}_{k'}}{B_0^2} + \mathbf{A}_k \cdot \frac{\mathbf{E}_{k'} \wedge \mathbf{B}_0}{B_0^2} \right) - \frac{1}{2\mu_0} \sum_k \left(\frac{1}{c^2} E_k^2 - B_k^2 \right) \right\} d^3x,$$

second order mode-mode interaction terms disappear when orthogonality of MHD modes and $E_\parallel=0$ condition are used

$E_\parallel \neq 0$ included by M. Schneller [PhD thesis]; non-orthogonality effects e.g. mode envelope modification etc neglected here; their significance to be determined by benchmarks with other codes, in particular XHMGC - see below!

3 wave interaction model for HAGIS

use v= v_0+v_Z → 3rd order terms, Lagrangian → vary

$$L_3 = -m_i n_i \sum_{k,k'k''} = \frac{(\mathbf{E}_k \times \mathbf{B}_0)\cdot(\mathbf{E}'_k \times \mathbf{B}''_k)}{B_0^4} = \frac{(\mathbf{B}_0 \cdot \mathbf{E}'_k)(\mathbf{E}_k \cdot \mathbf{B}''_k)}{B_0^4}$$

$$L_{int} = \sum_{j=1}^{N_p}\sum_{k=1}^{N_w}\frac{1}{\omega_k}\sum_m (k_\parallel v_{\parallel j} - \omega_k)\cdot\left[\Xi_k C_{jkm} + Y_k S_{jkm}\right] + \boxed{\sum_{k=1}^{N_w}\sum_{k'=1}^{N_w} i\varrho_z B_{k'}\omega_k C_{k,z,k'}\epsilon_{k,z,k'}}$$

lead to Manley-Rowe relations: energy and wave action conservation

$$i\omega_1 \frac{dA_1}{dt} = C_{jkm} A_2^*(t) A_3^*(t)$$

$$i\omega_2 \frac{dA_2}{dt} = C_{jkm} A_1^*(t) A_3^*(t)$$

$$i\omega_3 \frac{dA_3}{dt} = C_{jkm} A_1^*(t) A_3^*(t)$$

$$\epsilon_{k,z,k'} = \exp[i(\omega_1 + \omega_1 + \omega_3)]$$

and $k_{1\parallel}+k_{2\parallel}+k_{3\parallel}=0$

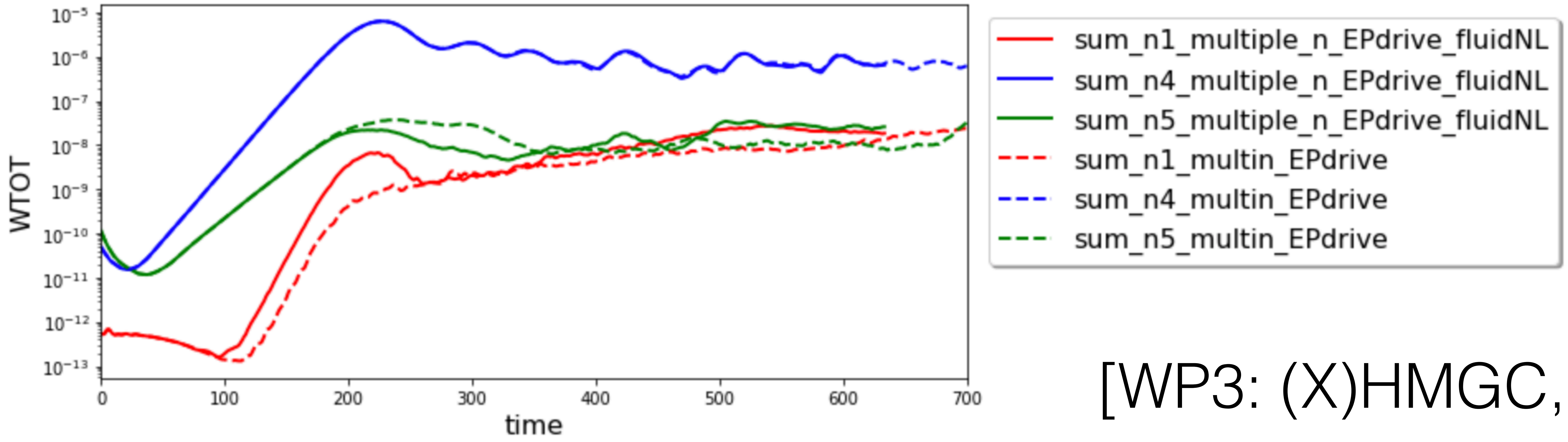
MHD model for coupling coefficient:

$$C_{jkm} = \frac{\alpha}{2 v_s \varrho^{1/2}}\sqrt{\omega_1\omega_2\omega_3}$$

$$\alpha = \sqrt{\frac{c_+^2 - v_s^2}{c_+^2 - c_-^2}}$$

[Webb, 1999]

coupling coefficient (HAGIS model) vanishes for 3 orthogonal Alfvén waves!



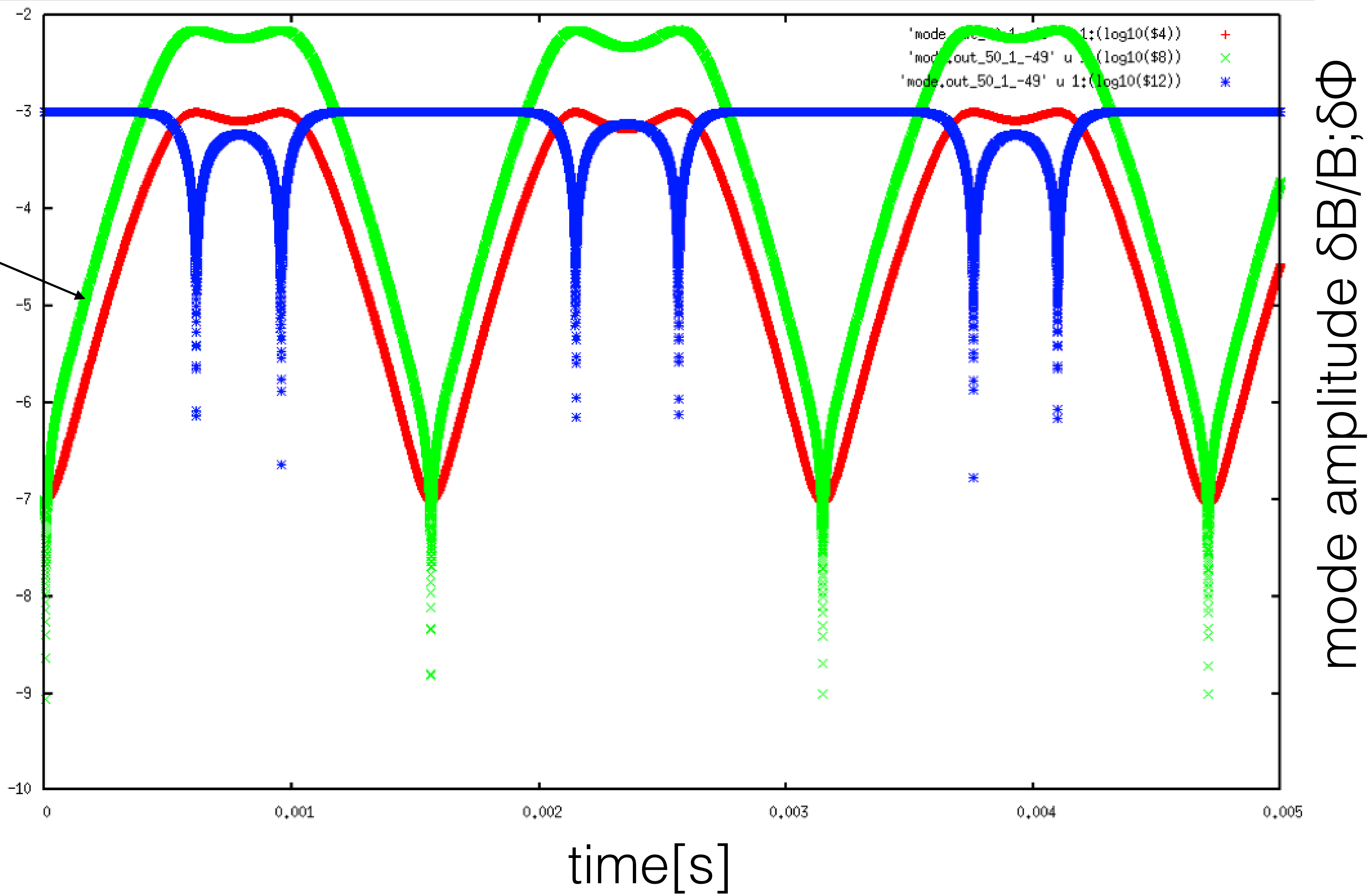
[WP3: (X)HMGC, X Wang]

XHMGC finds - as expected - only small effect: probably mode envelope modifications?

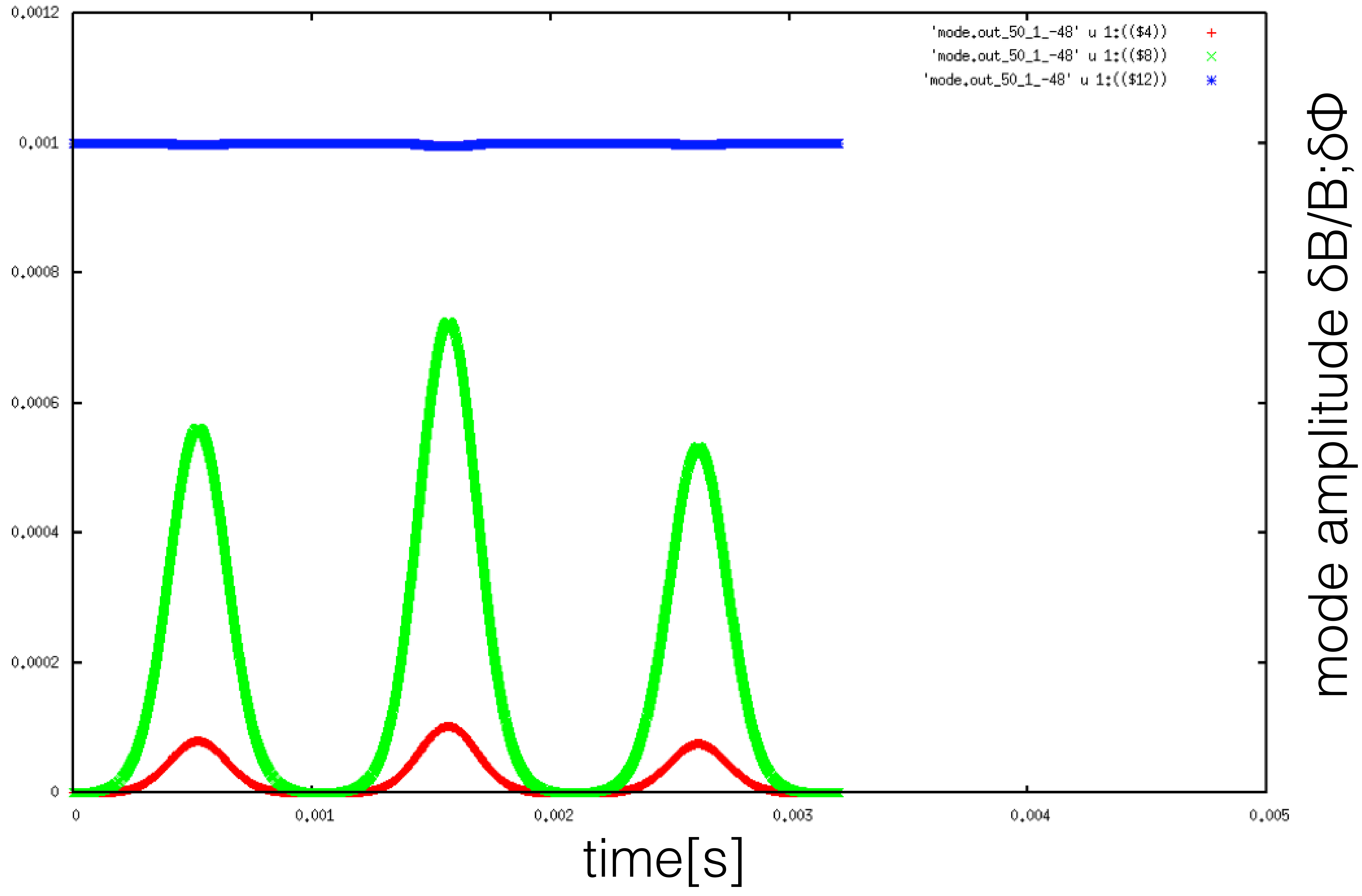
mode ‘beating’ for $f_1+f_2+f_3=(50-1-49)$ kHz=0

initially: pump wave conditions

secular growth



pump wave test: frequency mismatch: $f_1+f_2+f_3=(50-1-48)$ kHz



ZF model for HAGIS: two Alfven waves and 'sound-like' wave

forced oscillation: n=0 'eigenmode' structure: (kr ρs →0)

$$\frac{\phi(t)}{\phi_0} = A + \left(1 - A\right)\cos(\omega_{GAM}t)e^{-\gamma t} \qquad A = \frac{1}{1 + 1.6q_0^2/\sqrt{\epsilon}} \qquad \boxed{\gamma = \omega_{GAM}e^{-q_0^2}}$$

or more detailed expression [Xiao,Catto]

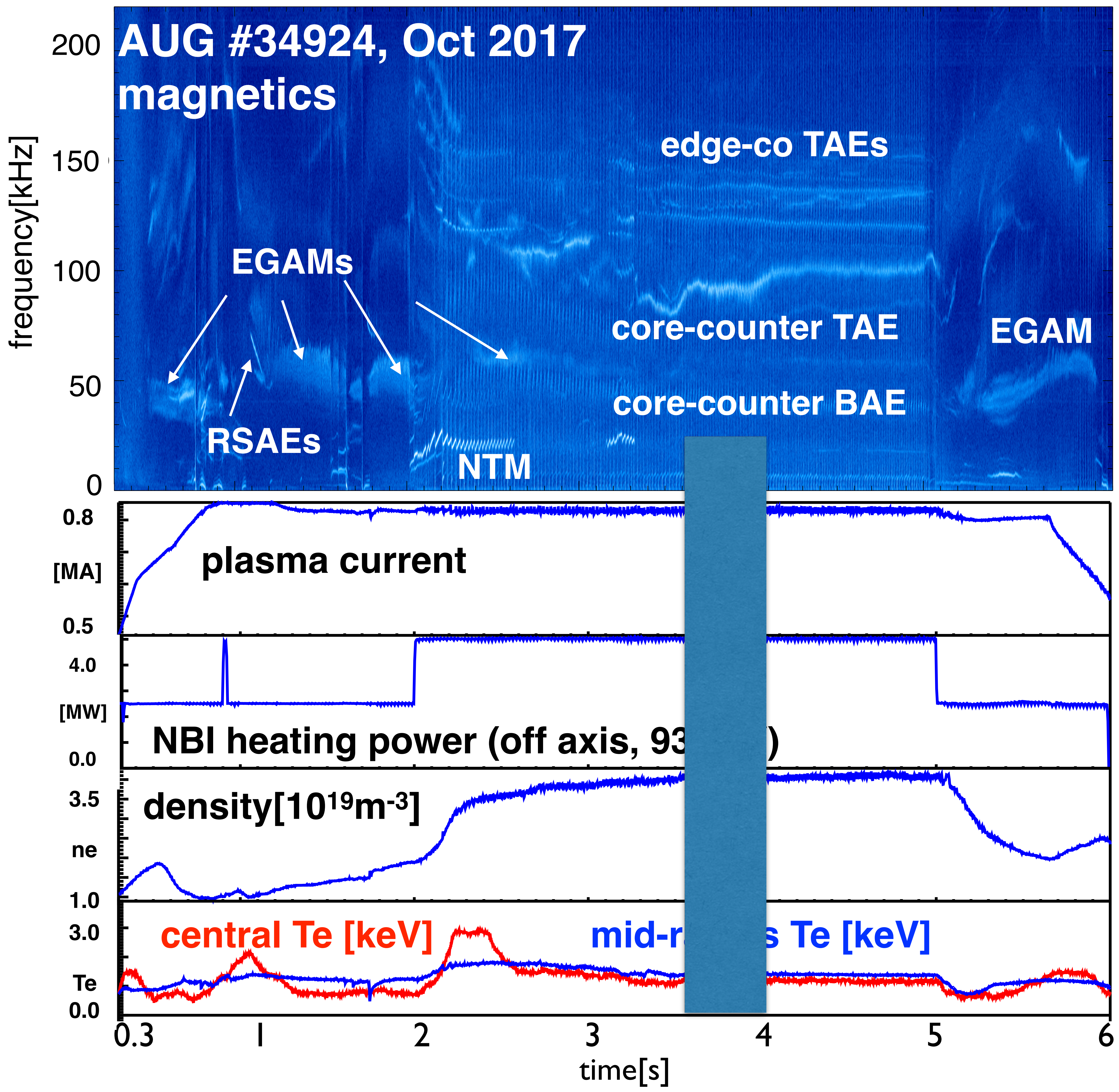
assume (for now) fixed spatial of Φ_0: radial envelope of pump AEs

spatial structure of v = E x B:

$$v_Z = \frac{-\nabla\phi(r,t)}{B}\left(0, 1, -2q\cos(\theta)\right)$$

unexpected opportunity: use actual experimental case - probably first ever AUG data on nonlinear Alfvénic wave-wave interaction physics!

new scenario with strong mode activity induced by energetic particles (EPs) was established at ASDEX Upgrade



[Lauber, IAEA FEC 2018]

I=800kA
B=-2.5T

q≥2
slightly
reversed

stationary flat top conditions!
new, complementary data to NLED AUG base case (bursting TAEs and EGAM)

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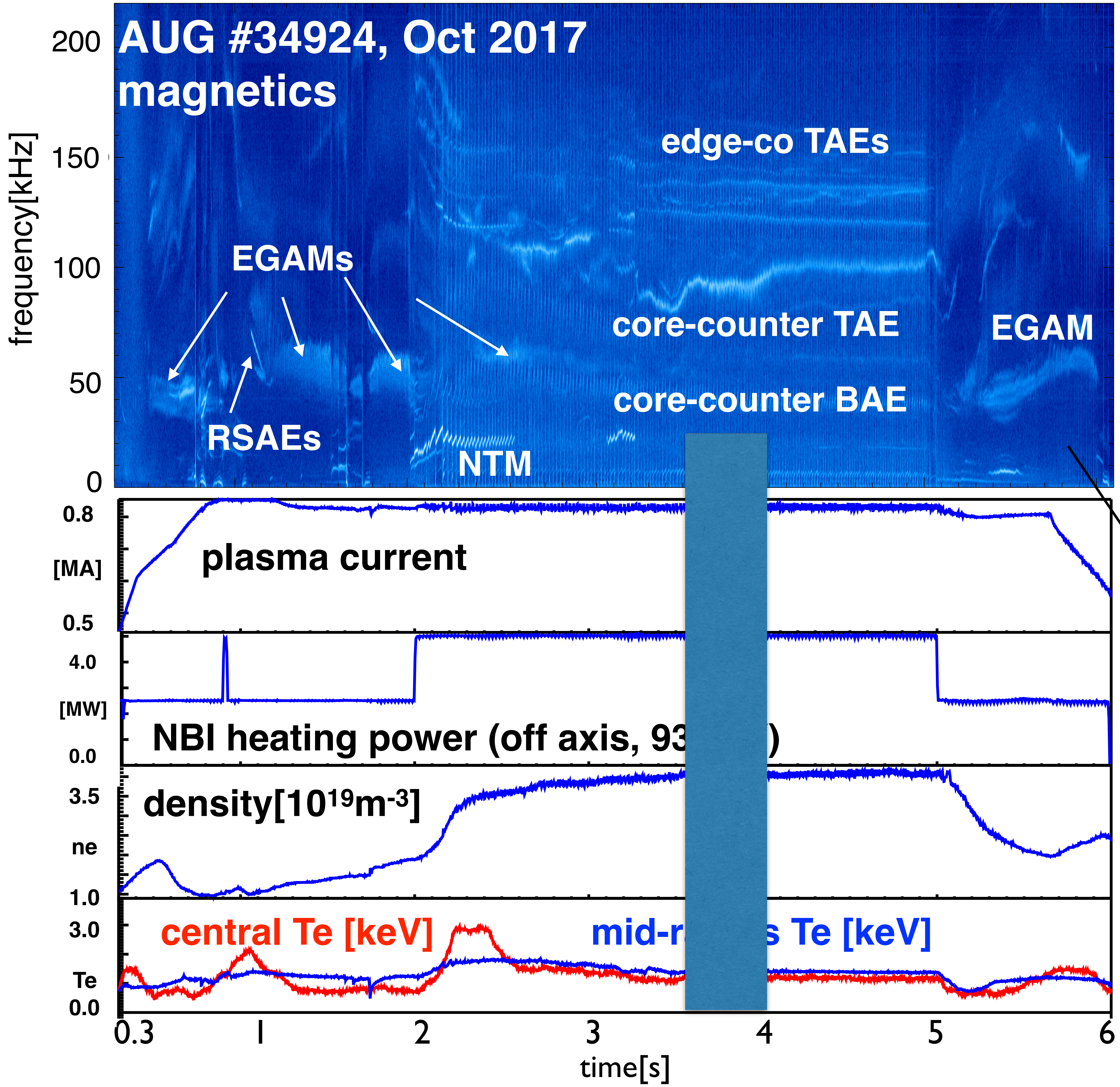

[Lauber, IAEA FEC 2018]

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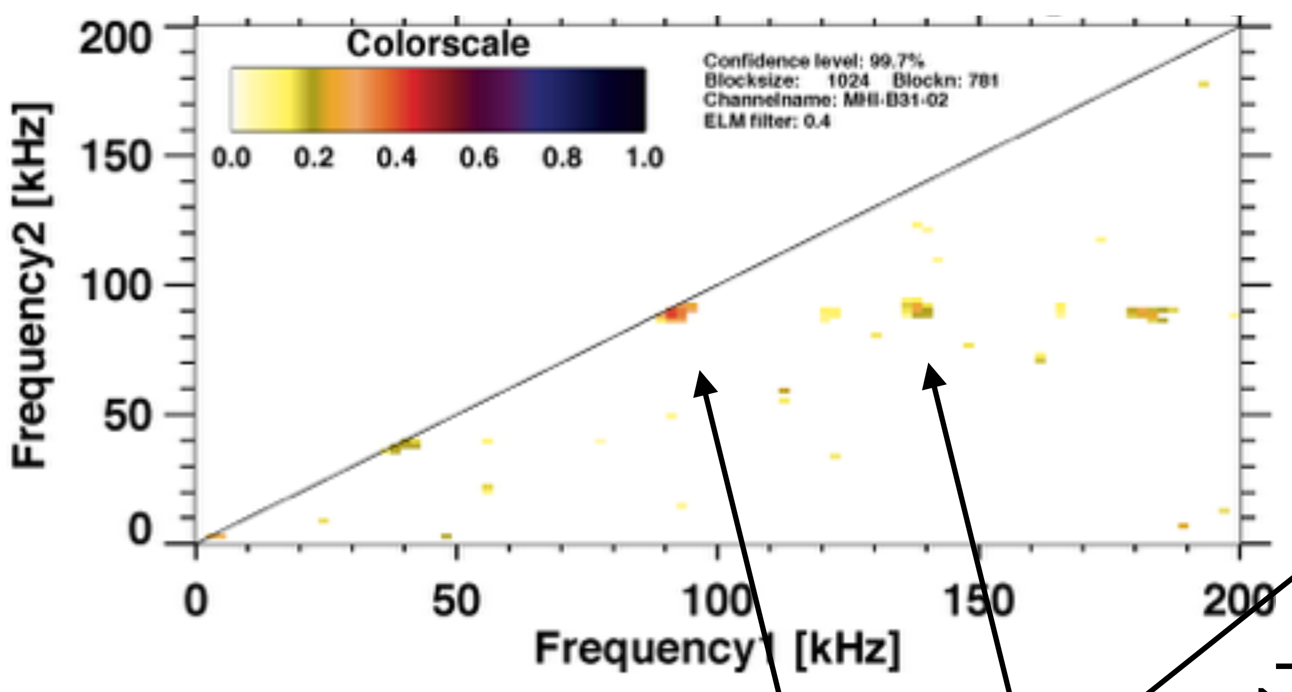
you can also listen to this shot now ;)

http://www2.ipp.mpg.de/~pwl/NAT/ENR_NAT.html

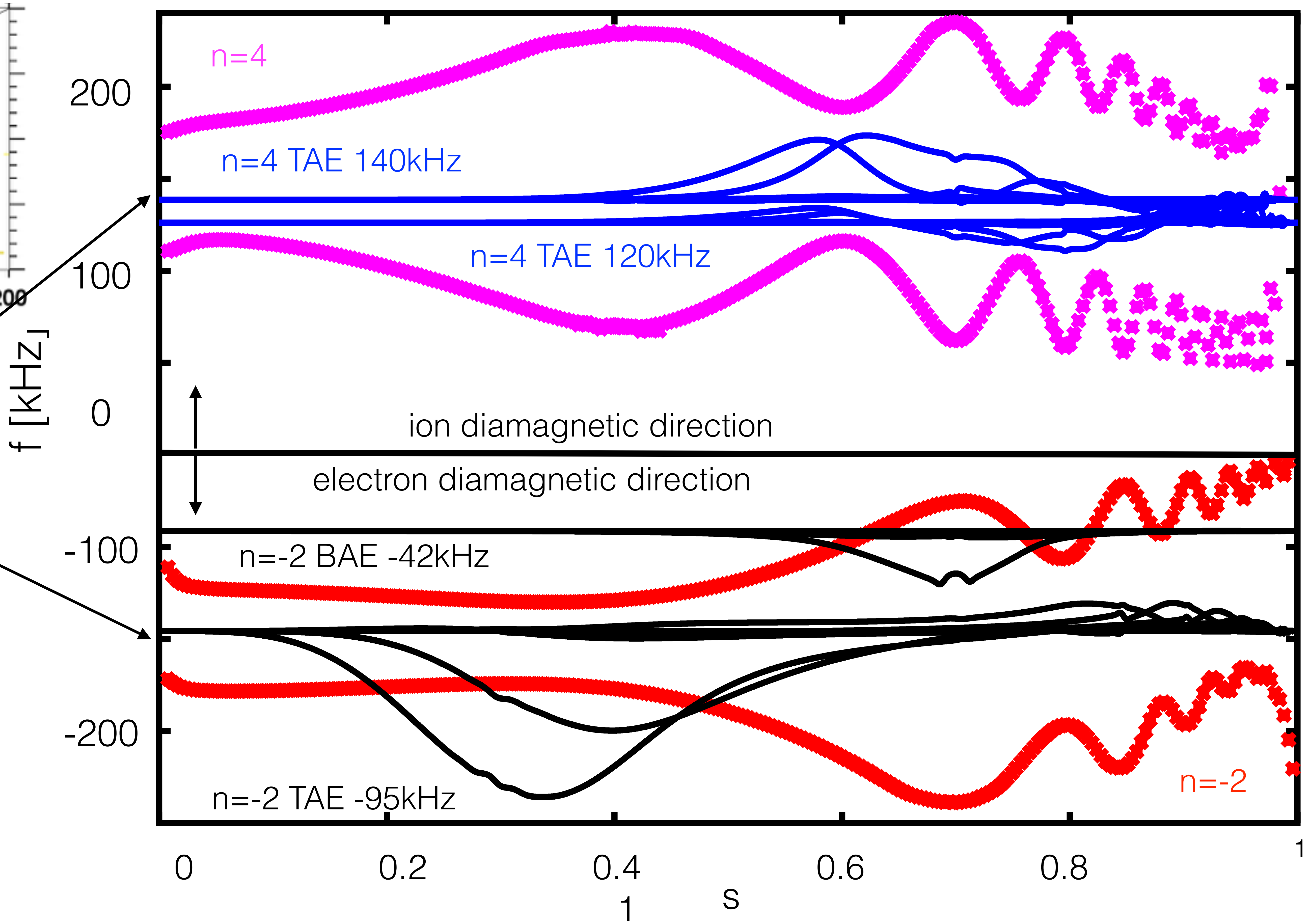
bottom of page

LIGKA: unstable modes, 3 wave coupling analysis



see also WP6

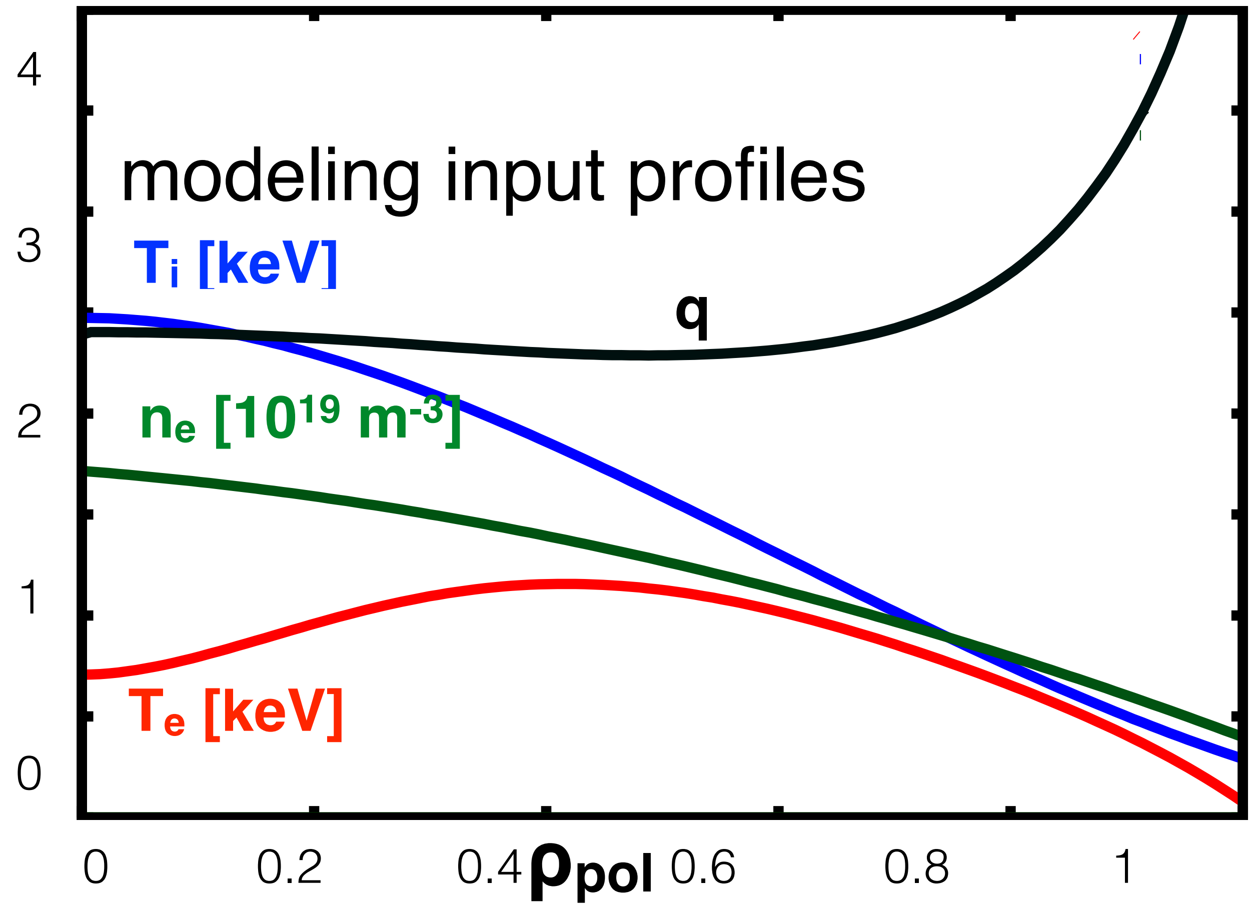
2nd type:



- after subtracting/adding rotation (7kHz): $\omega_{TAE-2}-\omega_{TAE+4}=0$
- also: $k_{\parallel TAE-2}+k_{\parallel TAE+4}= 1/(2\ q_{TAE-2}\ R)-1/(2\ q_{TAE+4}\ R)=0.222-0.211\approx 0$
- fulfil matching conditions with zero frequency zonal structure: modified parametric decay constellation *[Biancalani FEC 2016,TH/P2-9 2018]*

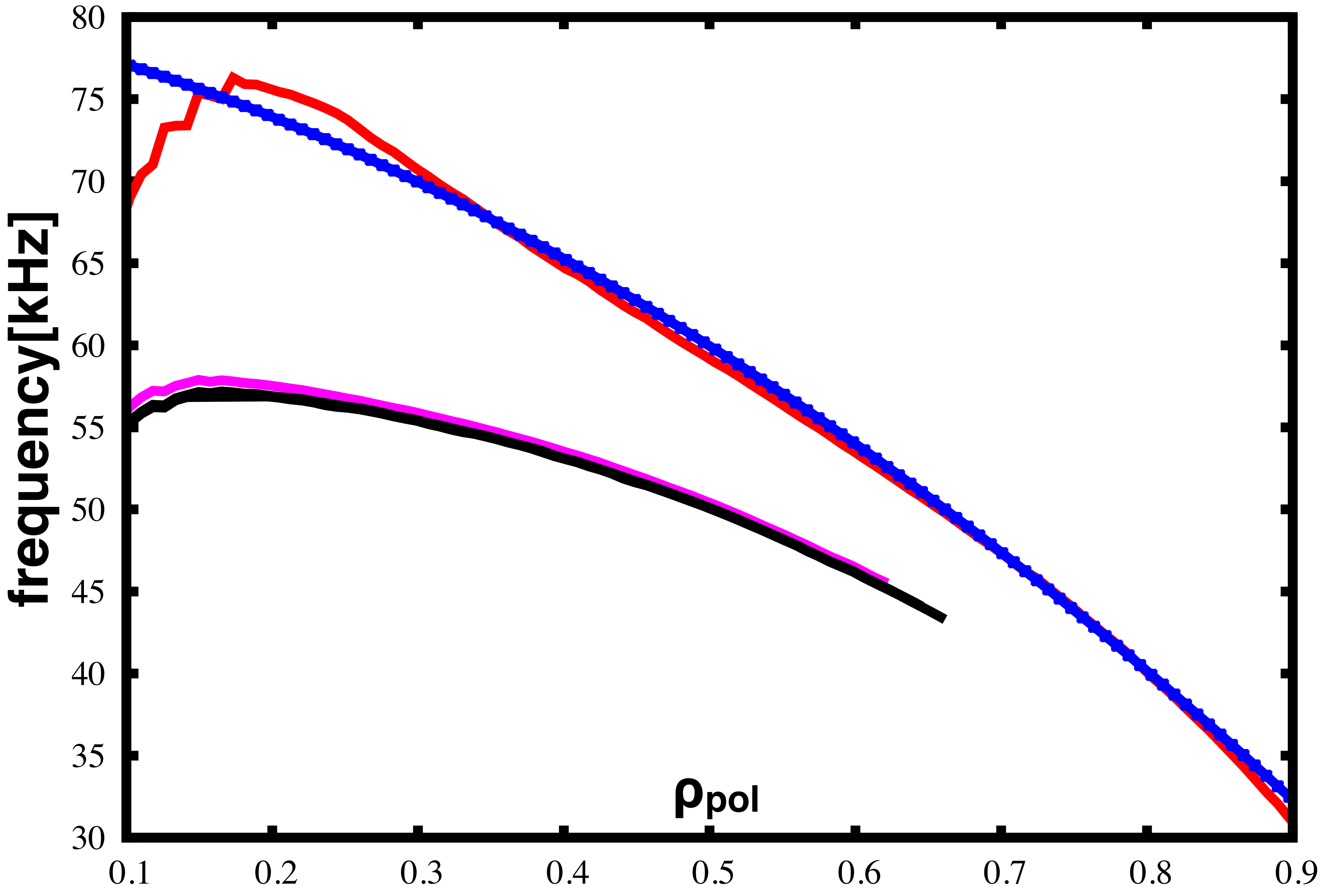
which mode structures? damping?

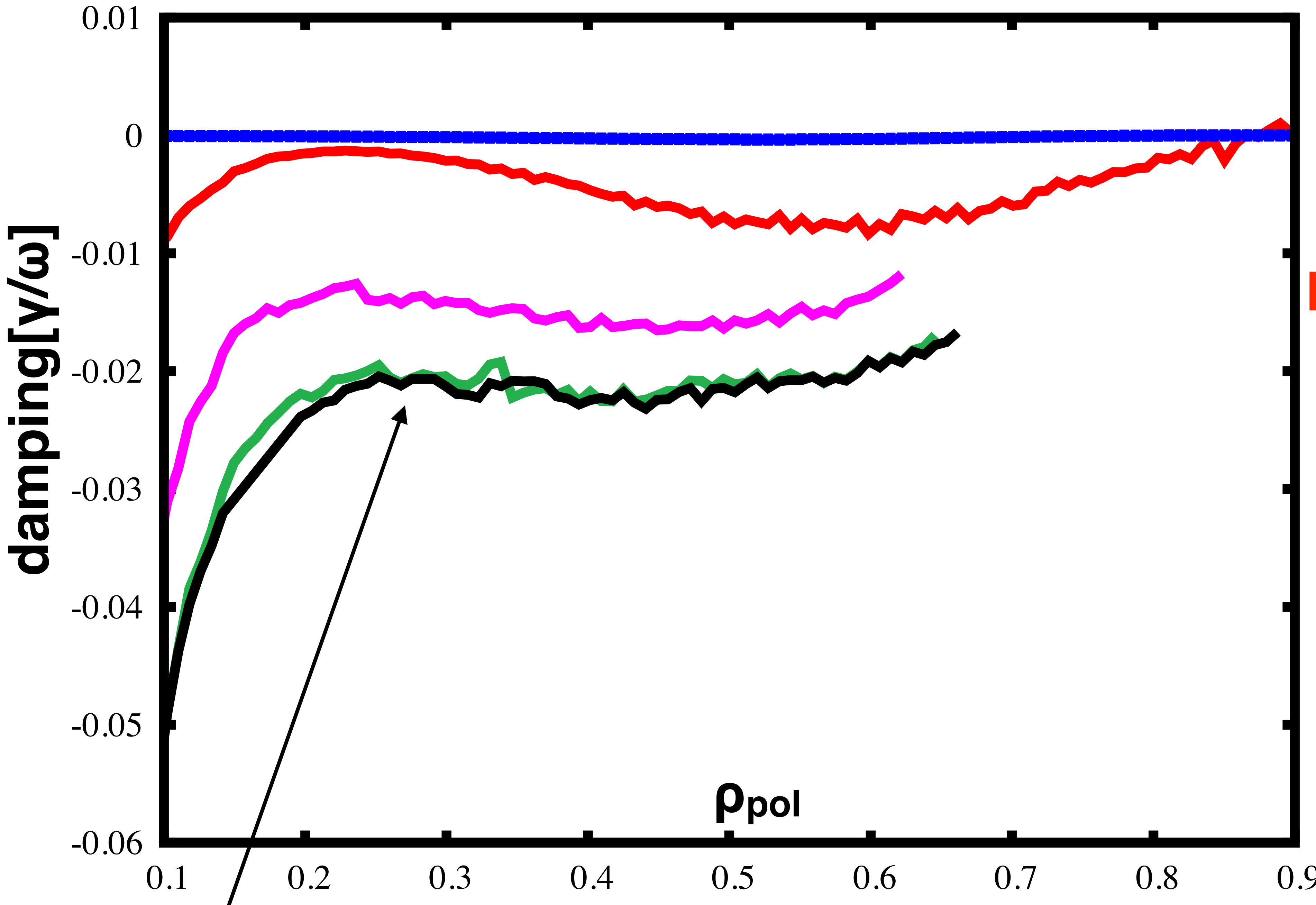
GAM continuum: local calculations [LIGKA]



at each radial position, solve linear dispersion relation:

- analytical, circular equilibrium
- numerical, circular eq., all ion resonances
- numerical, shaped equilibrium κ~1.6 ; $\omega \sim \sqrt{2/(1+\kappa^2)}$
- numerical, add trapped electrons
- numerical, trapped + circulating electrons

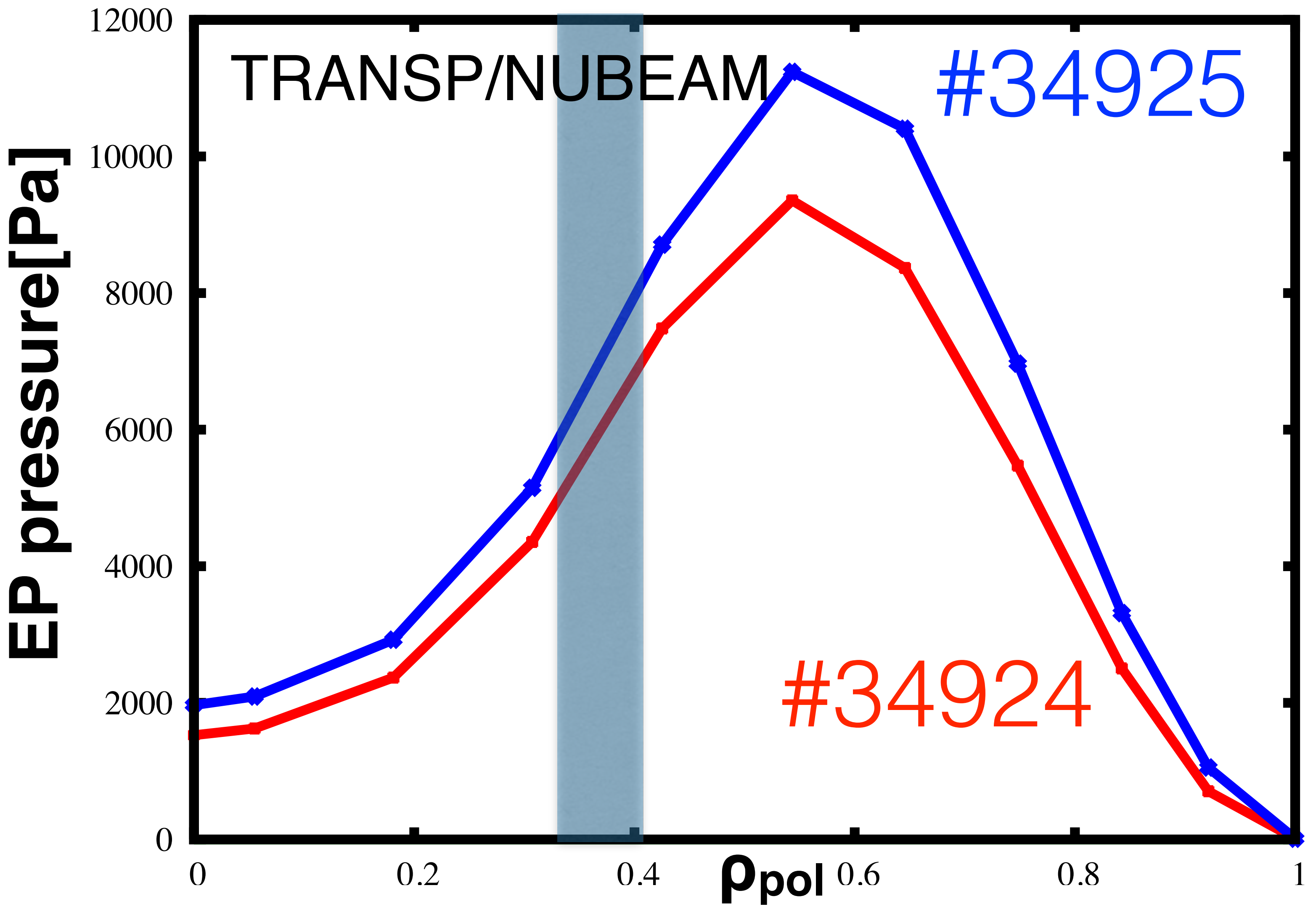




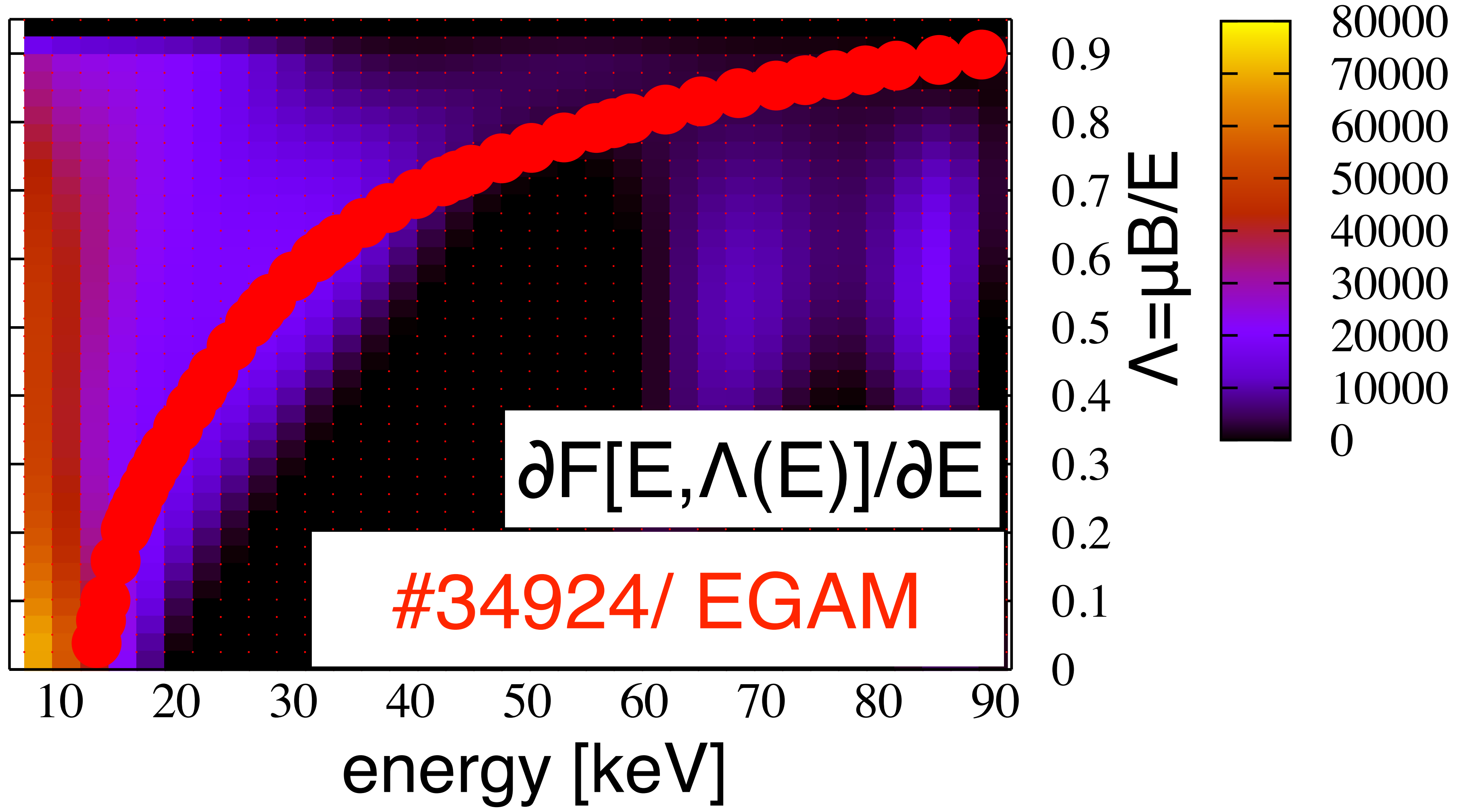
similar results for the role electron damping by ORB5 see WP4 [I Novikau]

profiles (q,Ti,Te) create (flat) minimum in GAM damping rate

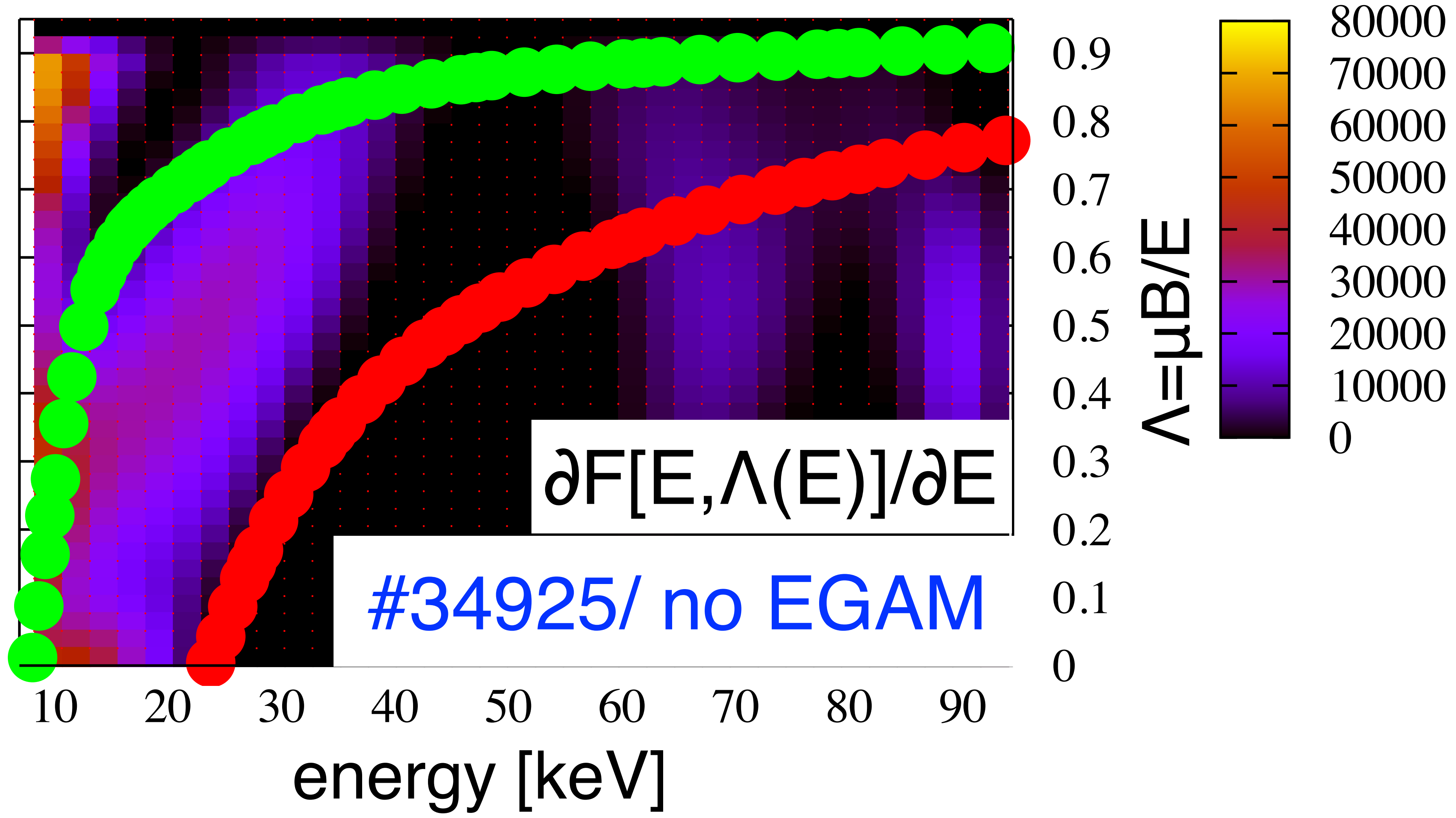
EP phase space analysis: ∂F/∂E



resonance with local f_{GAM}

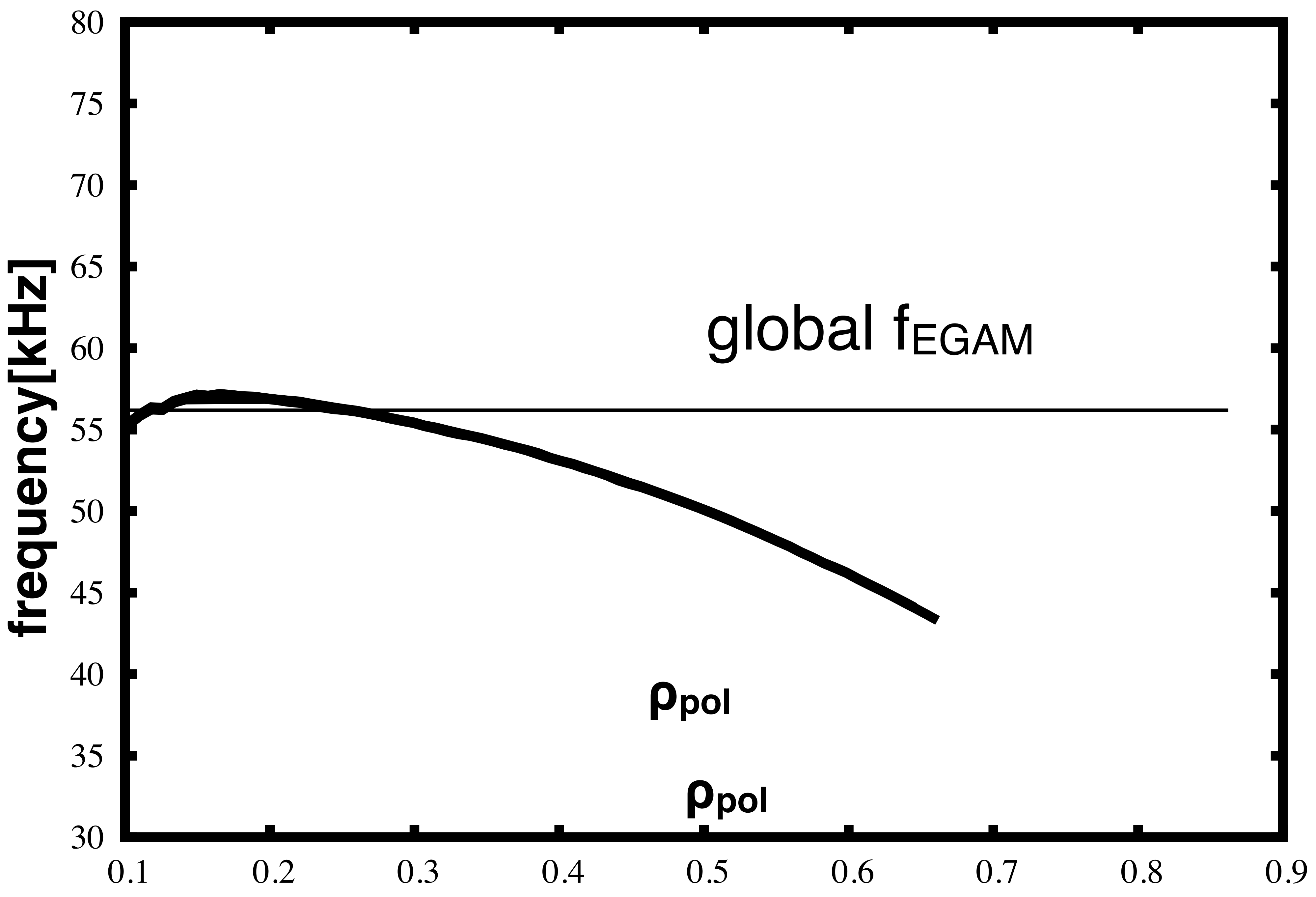


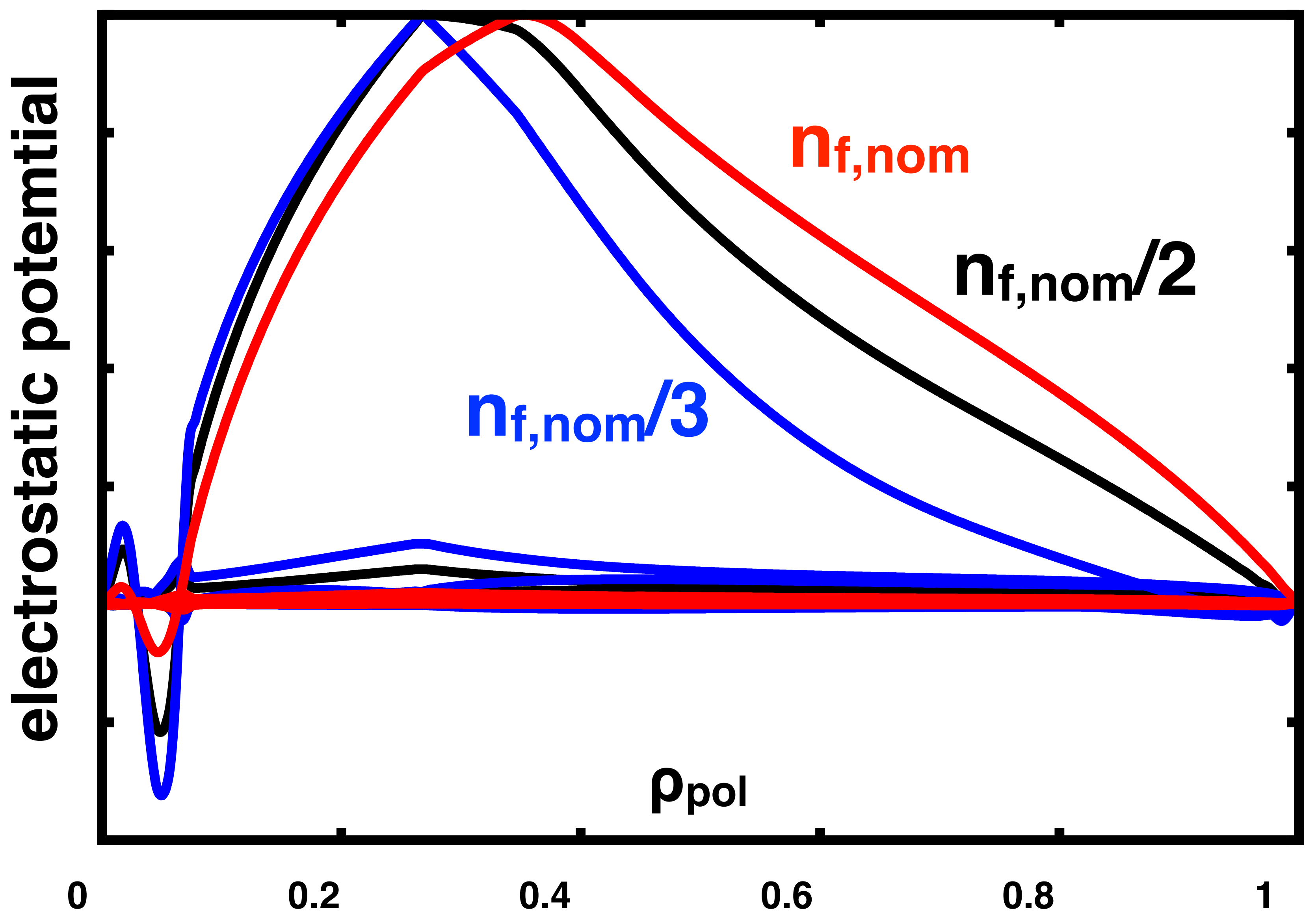
resonances with local f_{GAM}

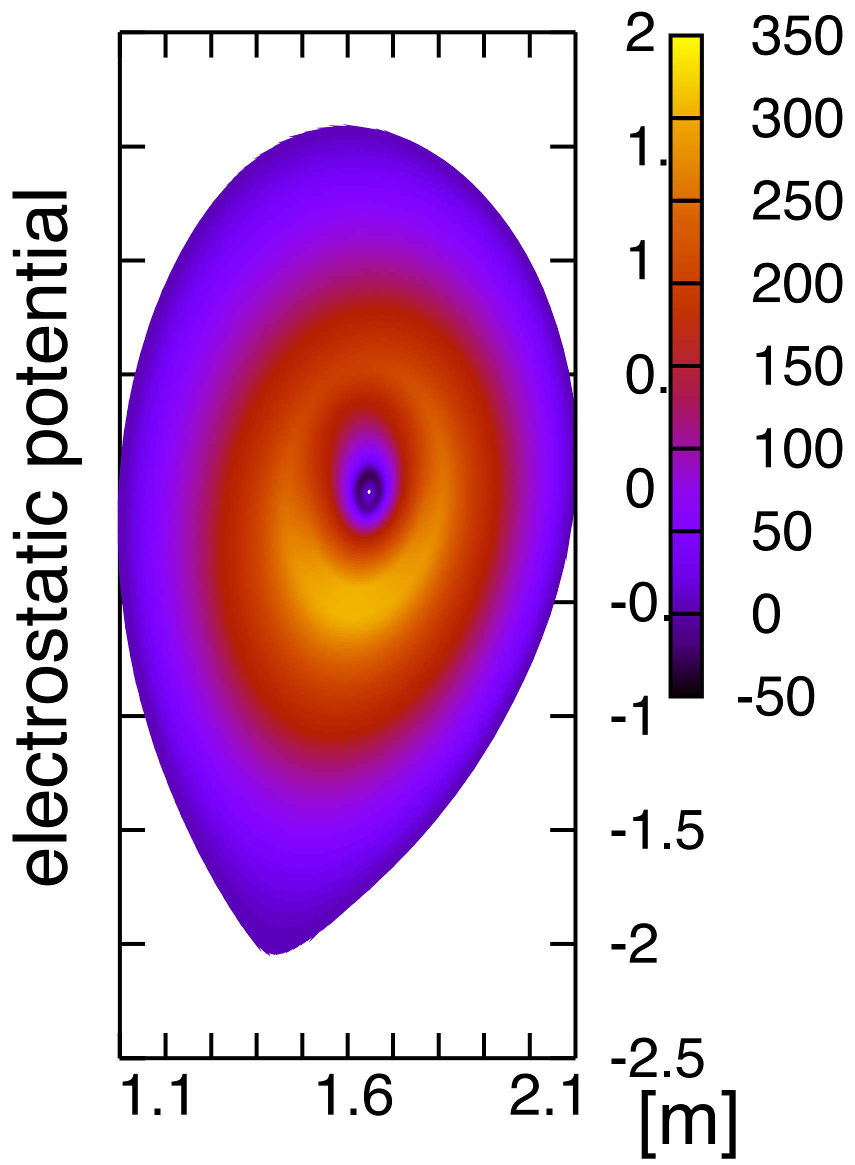


- EGAM drive is determined by integral along resonance line $\omega-\omega_t=0$
- no drive due to mismatch of drive region and local GAM frequency
- 2nd resonance $\omega - 2\omega_t=0$ suffers from damping of thermal background - ‘anomalous ion heating’ *[LHD, Ido 2014, H. Wang 2018]*

global EGAM structure [LIGKA]



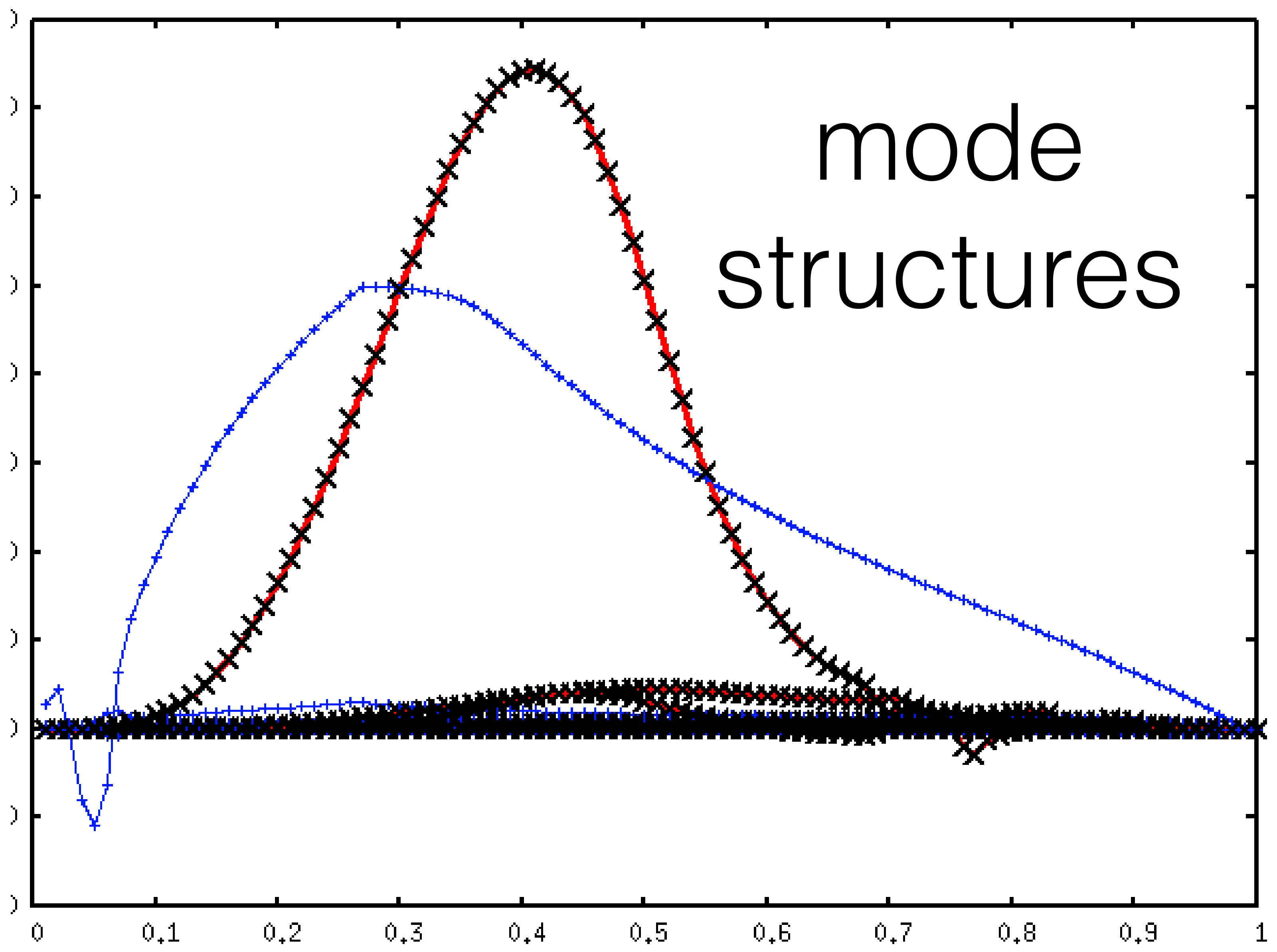




- global EGAM frequency stays roughly constant with increasing n_{EP}, and close to flat part of the GAM continuum
- change in mode structure is observed with increasing n_{EP}

challenging: global, electromagnetic non-perturbative calculations with kinetic electrons and anisotropic F_{EP} (still linear, though)

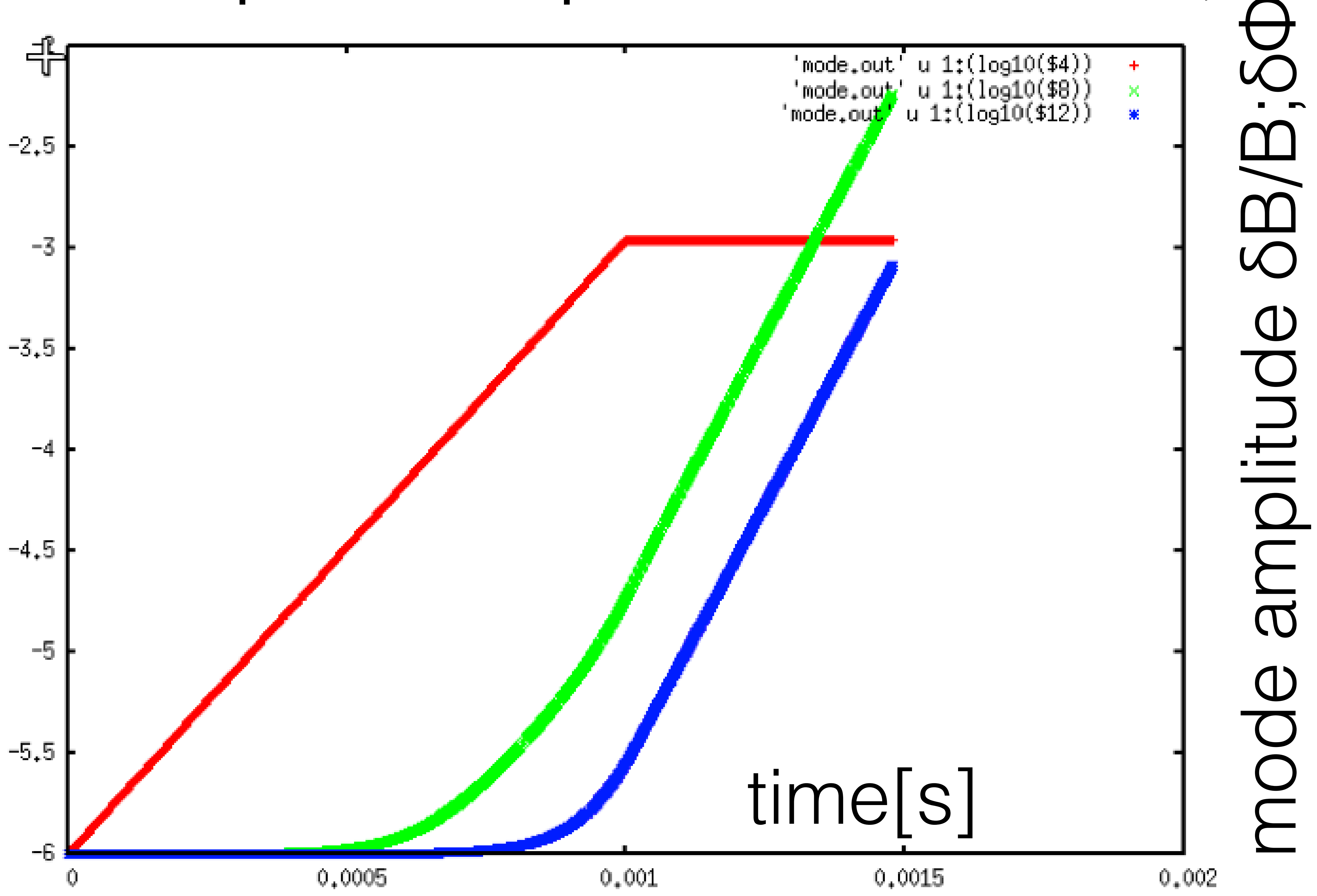
use NLED-AUG case based mode structures for 3-wave studies



n=2 RSAE/TAE propagating in ion diamagnetic direction
n=-2 RSAE/TAE propagating in electron diamagnetic direction
n=0 GAM/EGAM

pump-wave setup, like in experiment:
n=2 is driven by EPs: here prescribed amplitude evolution,
no explicit wave-particle non-linearities; no modification

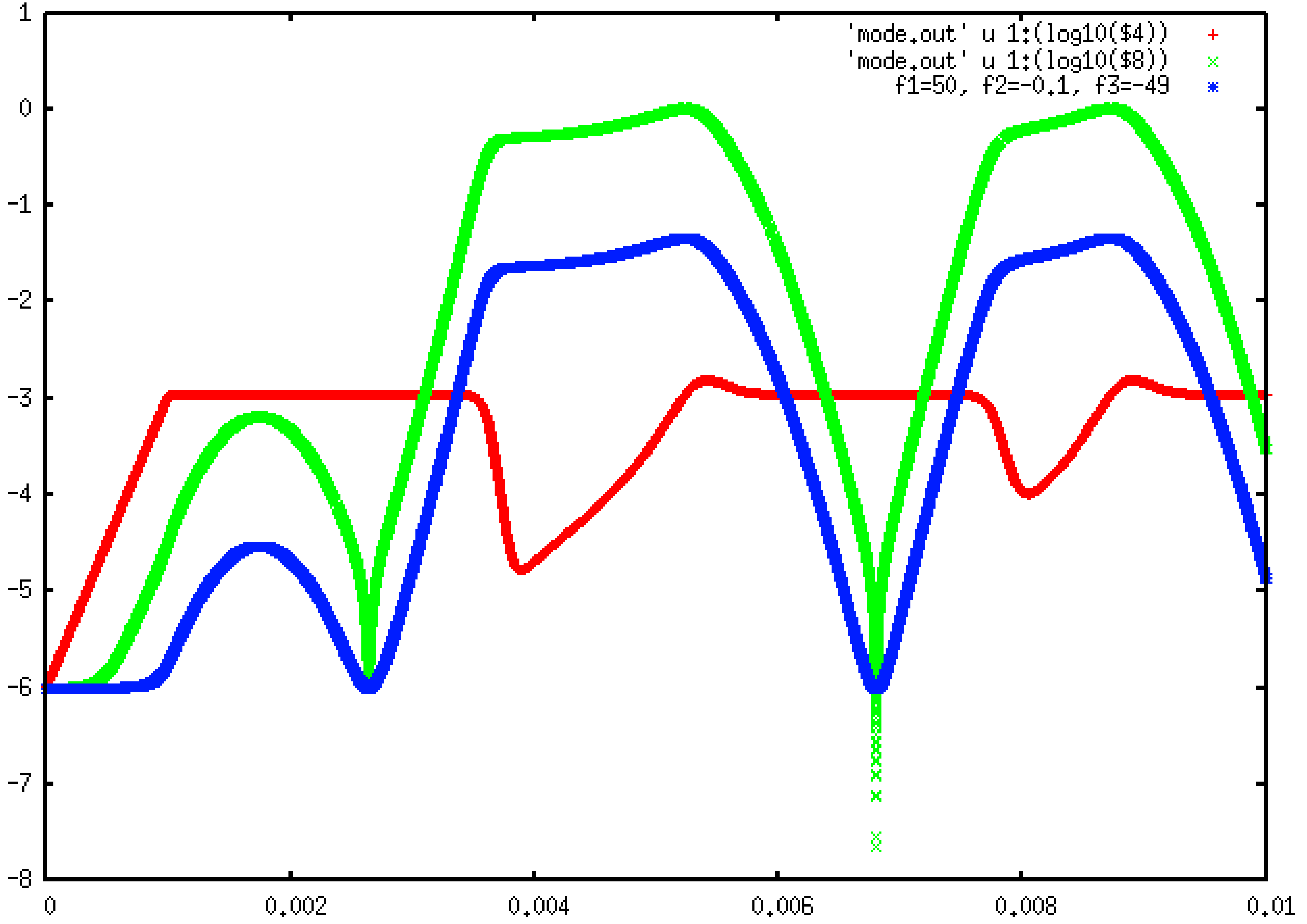
no modification of pump wave (red) allowed:
system explodes for exact frequency match



use NLED-AUG case based mode structures for 3-wave studies

modification of pump wave (red) allowed:
pulsating on time scales given by (small) frequency mismatch (50/49/0.1 kHz)

presently working on taking account damping - benchmark with XHMGC possible (WP 3)



Milestones and Deliverables 2017/2018, Workpackage 2

main contributors: Ph. Lauber, T. Hayward

- Formulation of the extension of HAGIS model for three-wave interaction; implementation into the existing code (2017) fully achieved
- Test implementation, benchmark in simple situations to other codes and apply to multi-mode TAE problem (2018)

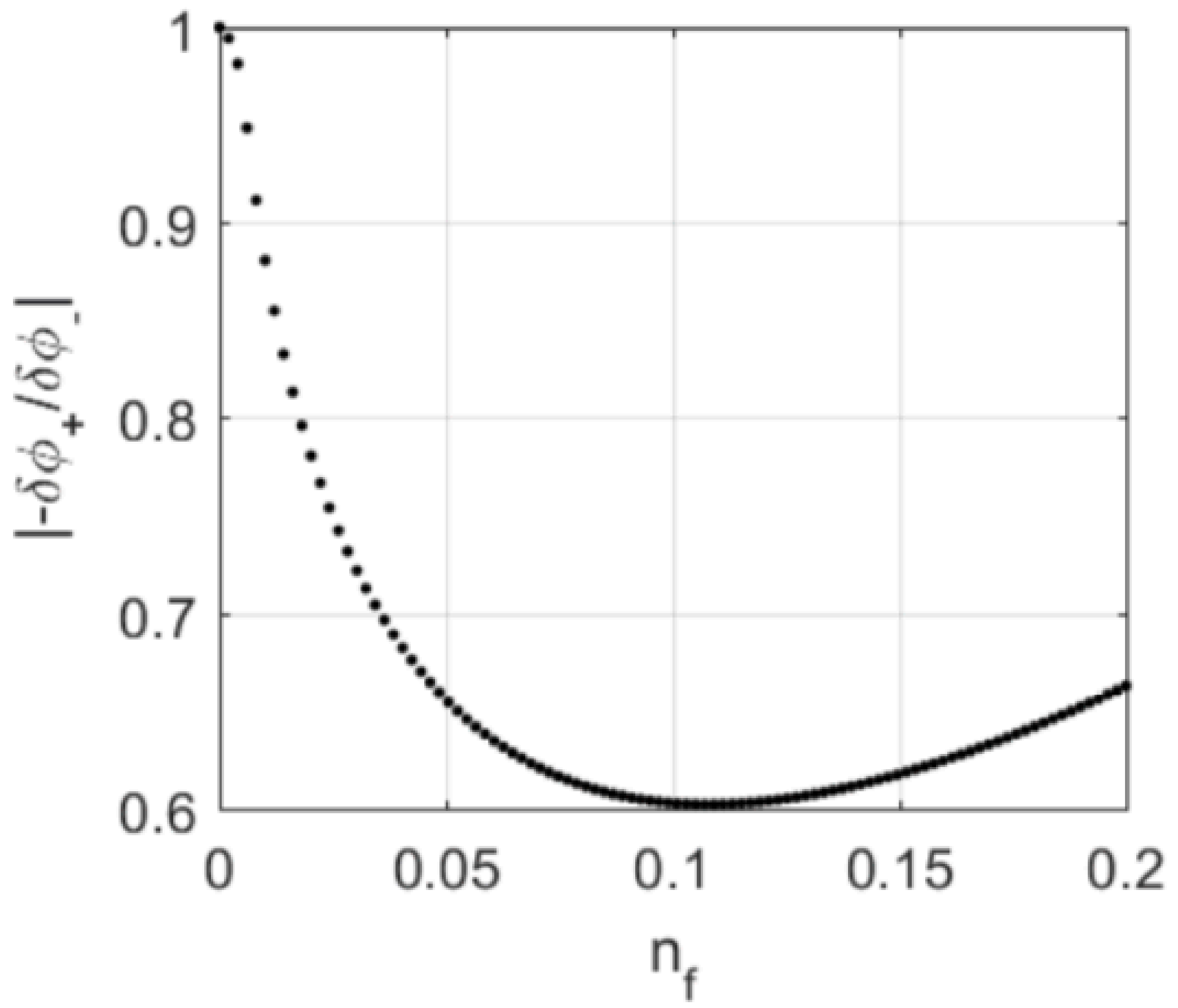
fully/partly achieved:

- implementation tests complete
- apply to multi-mode TAE problem based on AUG data: case setup and first tests complete
- benchmarks with other codes started - not complete
- instead: considerable effort to model linear EGAM onset with LIGKA with experimentally relevant parameters (global, kinetic electrons, anisotropic EP distribution function); analyse new, better AUG data for wave-wave studies

- Ph. Lauber and Z. Lu: Analytical finite-Lamor-radius and finite-orbit-width model for the LIGKA code and its application to KGAM and shear Alfvén physics; Journal of Physics: Conference Series; 1124 (1) 012015 (2018)
- Ph. Lauber et al "Strongly non-linear energetic particle dynamics in ASDEX Upgrade scenarios with core impurity accumulation", Oral at the 27th IAEA Fusion Energy Conference, Ahmedabad, India, 22-27 October 2018, EX1/1; Preprint:https://nucleus.iaea.org/sites/fusionportal/Shared%20Documents/FEC%202018/fec2018-preprints/preprint0319.pdf

additional work directly related to NAT (not in original proposal)

mode symmetry breaking studies on ITG and BAEs have been extended to ZF and EGAM physics

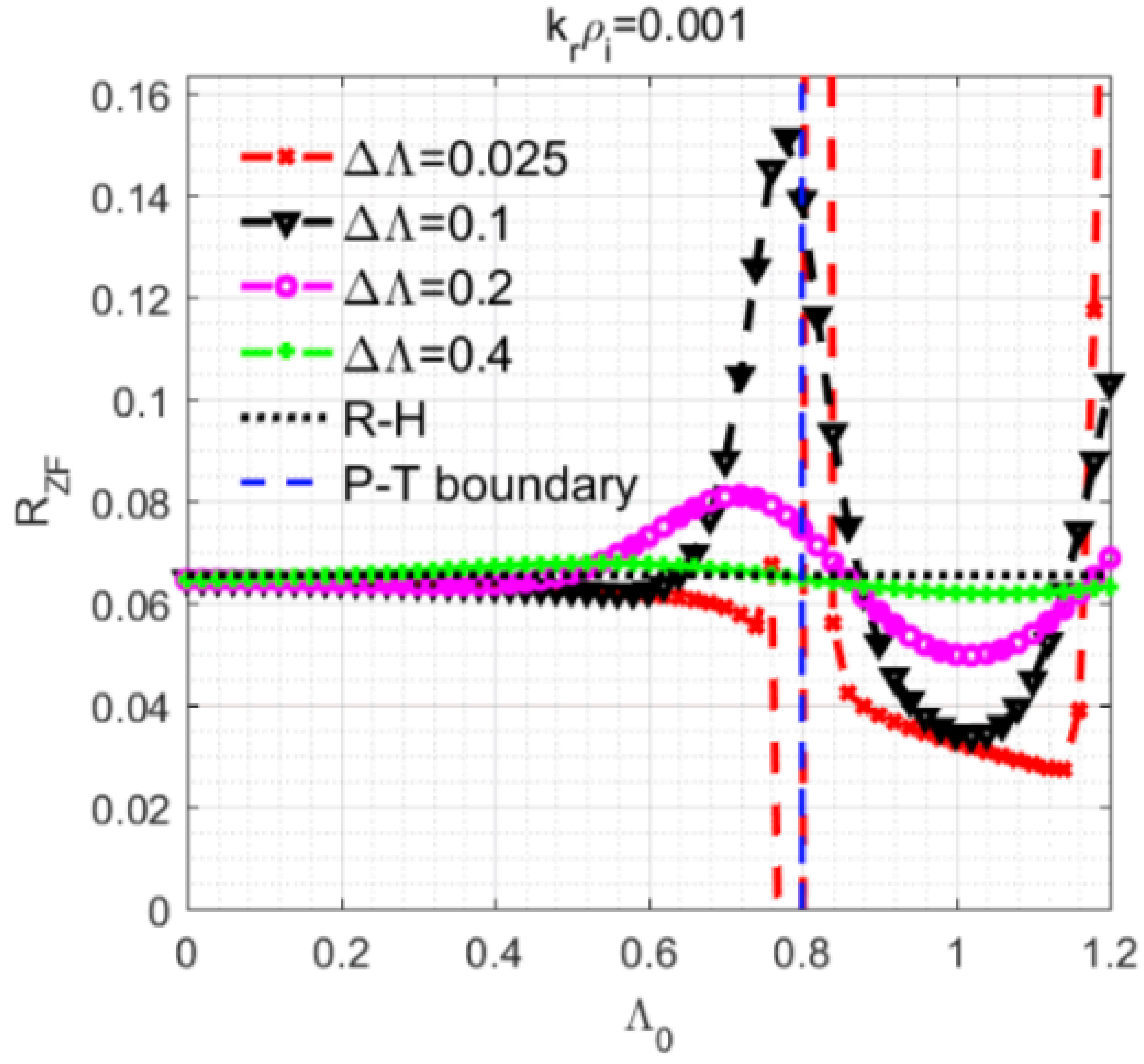


EGAM asymmetry of es. sidebands due to single-bump-on-tail EP distribution function

EP anisotropy effects on zonal flow residual

and

on particle, momentum and energy transport have been estimated



connections to most of the other WPs

can this be tested by experiments? (ICRH)

[Z Lu et al: PoP 2018, NF 2018, subm. to PPCF 2018]