EUROfusion

Advanced transport models for energetic particles

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ATEP team:

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and A. Bottino, M. Schneider, S.D. Pinches, O. Hoenen, TSVV10 team, ASDEX Upgrade team

MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

ENEA
ITALIAN NATIONAL AGENCY
FOR NEW TECHNOLOGIES, ENERGY AND
SUSTAINABLE ECONOMIC DEVELOPMENT

Center for Nonlinear Plasma Science
CNPS
Since 2020

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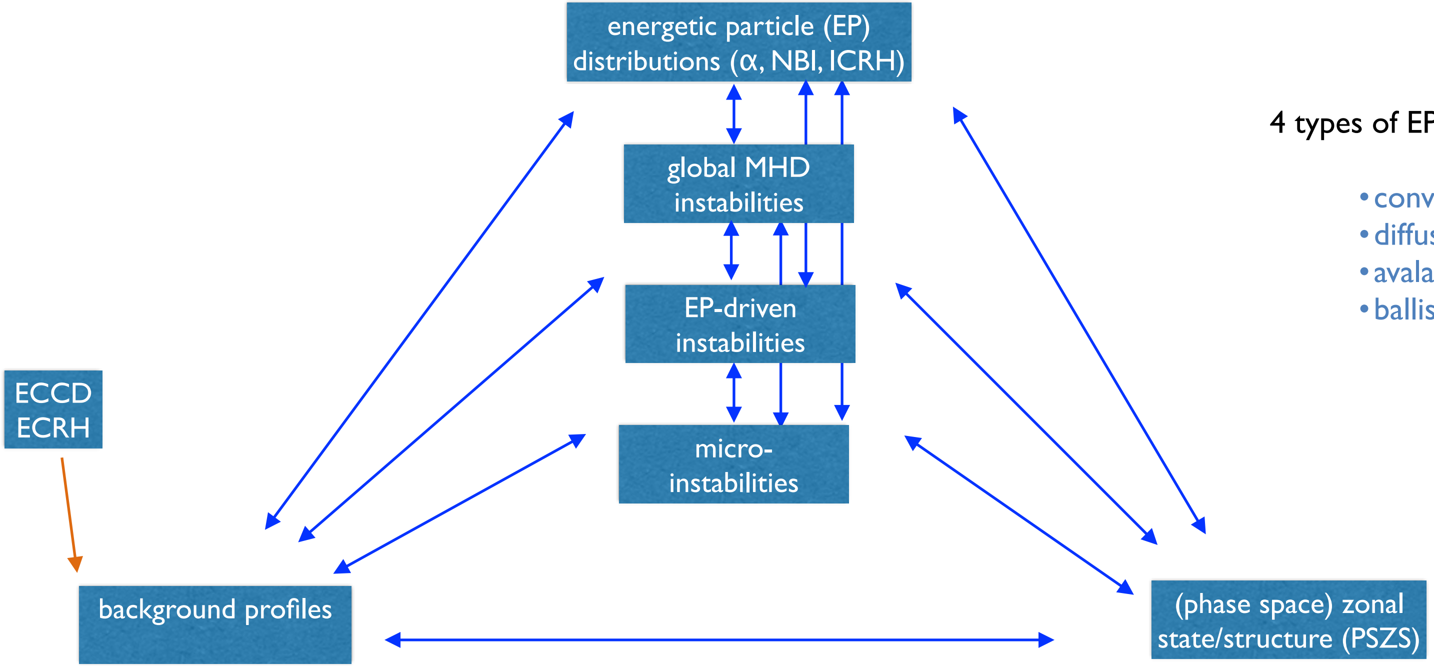
Energetic particle (EPs) transport is a key physics element of burning plasmas



4 types of EP transport:

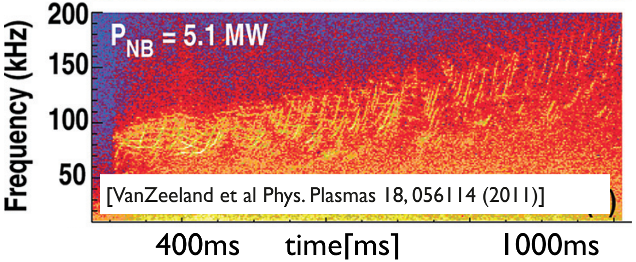
- convective
- diffusive
- avalanche
- ballistic

- final goal: predicting the self-organisation of a burning plasma
- challenge: complex interdependence on vastly different spatial and temporal scales

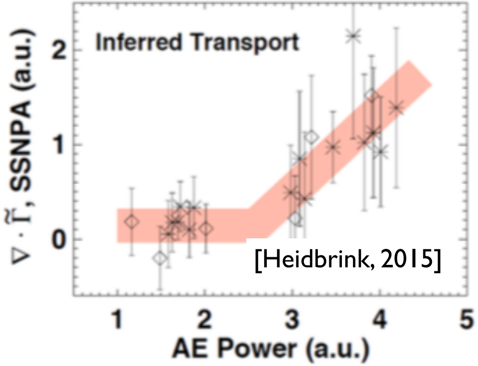
EP transport: experiment

- for multiple overlapping Alfvén eigenmodes (AEs) resonances: stiff EP transport found at DIII-D [Collins, Heidbrink 2015-2018], as predicted by QL theory [Sagedeev&Galeev, Kaufman 1972, …]; high q, large orbits, dominated by losses rather than redistribution
- in JET re-deposition of EPs (ICRH) was observed: core-localised TAEs redistribute EPs, redistributed EPs drive edge-TAE [Nabais et al, PPCF 2019]
- in ITER, both core and edge TAEs are weakly damped and can be driven non-linearly [Pinches, Lauber, Schneller 2014/2015, T Hayward 2019, ORB5]
- mode chirping and avalanches-type events found in many experiments [Kusama, Shinohara, JT-60U 1999+]
- bursting, non-linear mode-mode couplings and EP transport (FIDA, INPA) measured in ASDEX Upgrade EP super-shots [Lauber 2014+], .i.e. further development of AUG NLED benchmarks case [Vlad 2020-2023, Vannini 2019, Rettino 2021-23]

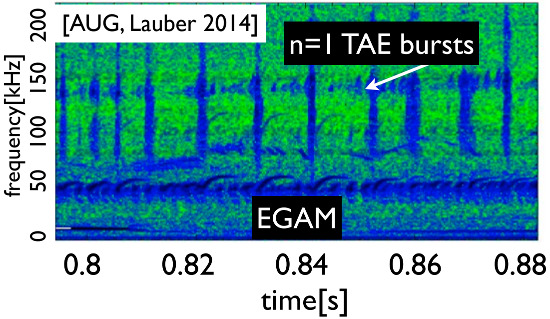
for a comprehensive review please refer to dedicated review articles, e.g. [NF ITPA special issue 2006, update 2023/24, Heidbrink 2008, Breizman& Sharapov 2011, Lauber 2013, Chen&Zonca RMP 2015, Gorelenkov&PinchesToi 2014, Todo 2019,…]



[VanZeeland et al Phys. Plasmas 18, 056114 (2011)]



[Heidbrink, 2015]



[AUG, Lauber 2014]

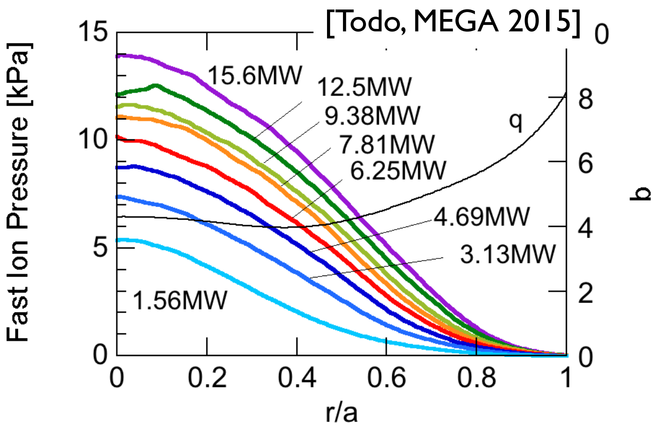
EP transport: modelling

- MHD-hybrid simulations of DIII-D case: transport due to steady and increasingly intermittent EP fluxes for higher power [MEGA, Todo 2015]
- multi-phase MEGA simulations for TAE- avalanches at JT-60U [Bierwage 2016,17];
- at increased EP pressure, so-called energetic particle modes start to exist: simulations as pioneered by (X)HMGC and HYMAGYC teams [Briguglio PoP1998, Bierwage 2012-16]; many dedicated diagnostics developed for phase space analysis
- chirping AE/EPMs: XHMGC simulations [X. Wang, S. Briguglio, 2021]: AE saturation level (and thus related EP transport) due to chirping modes is larger than standard quadratic scaling:

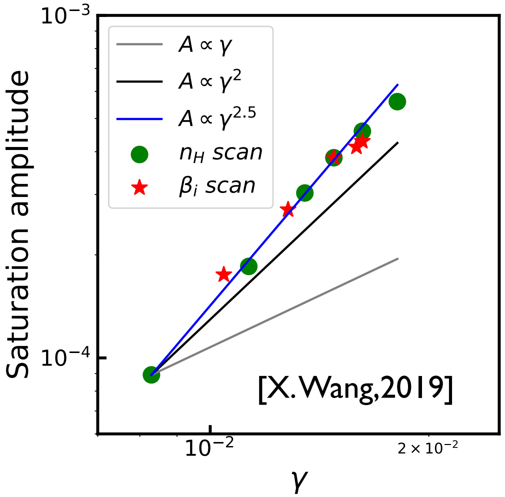
$$A \sim \gamma^{2.5}$$

- global GENE and GTC simulations highlight interaction with micro-turbulence [Citrin, diSiena 2019-2023, Brochard 2021-23]
- global ORB5 simulations with increasing complexity start to capture experimentally relevant regimes [A. Biancalani, T Hayward-Schneider, A. Bottino, F. Vannini, B. Rettino 2013-2023] and compare in with MHD-hybrid results [Vlad 2020-23]

- **difficult to disentangle various non-linearities in comprehensive codes- verify results?**
- **transport-time scales?**
- **vast parameter regime - sensitivity scans ?**
- **how to reduce to reasonably fast models?**



[Todo, MEGA 2015]



[X.Wang,2019]

EP transport: theory/reduced models

- diffusion coefficients for impurity transport by background turbulence, no e.m. EP-driven modes [Angioni 2009, Püschel, etc]

- critical gradient model [R. Waltz, E. Bass, 2014 -2023]: use local AE stability threshold, add upshift of transport threshold using $(ExB)_{turb}$ shearing rate; above threshold set D_{EP} to ad hoc values [e.g. $10 m^2/s$] to clamp EP's radial gradient to critical value

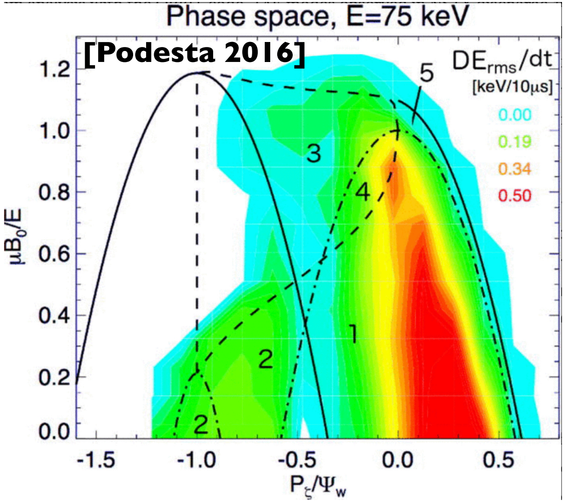
- kick model [M. Podesta, 2014-2022]: calculate probability density function of kick in Pz and E for given amplitude

- RBQ model, 1D, 2D [N. Gorelenkov 2015-2022]: use resonance broadening QL theory connected to NOVA-K to evolve mode amplitude consistently with evolution of F_{EP}

- gyrofluid model [D Spong, 2019-2022], TAEFL code: fluid closures simplify problem, runs on longer time scales

- GENE-Tango model [A. di Siena, 2022-23]: relies on global kinetic GENE runs + power balance

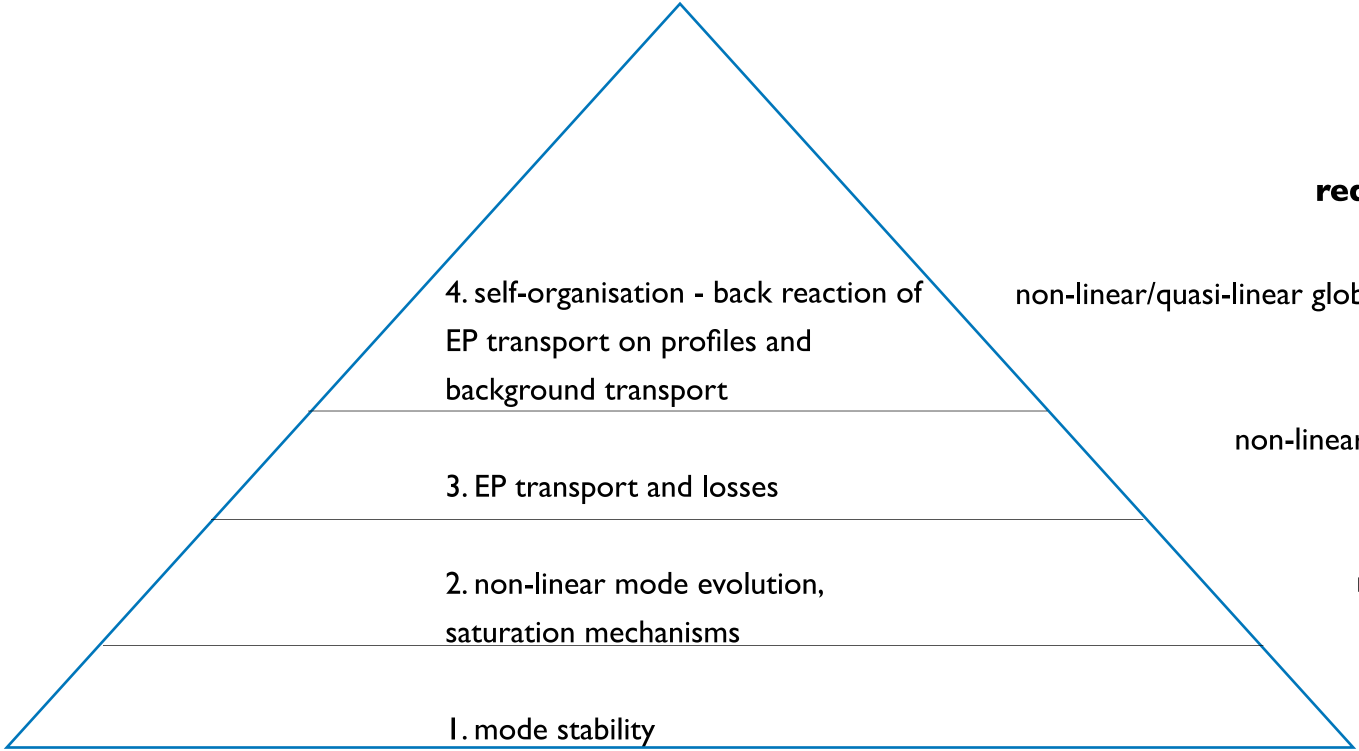
- transport models as derived from general non-linear gyrokinetic theory [Chen, Zonca RMP 2015, Z. Qiu et a 2017-2023] using phase space zonal structure (PSZS) transport theory [M-V. Falessi, F. Zonca, 2017-2023] *see talk M. Falessi at this conference*


[Podesta 2016]

within Eurofusion Enabling research project ATEP: based on general theoretical framework, develop and implement hierarchy of (reduced) phase space zonal structure (PSZS) transport models

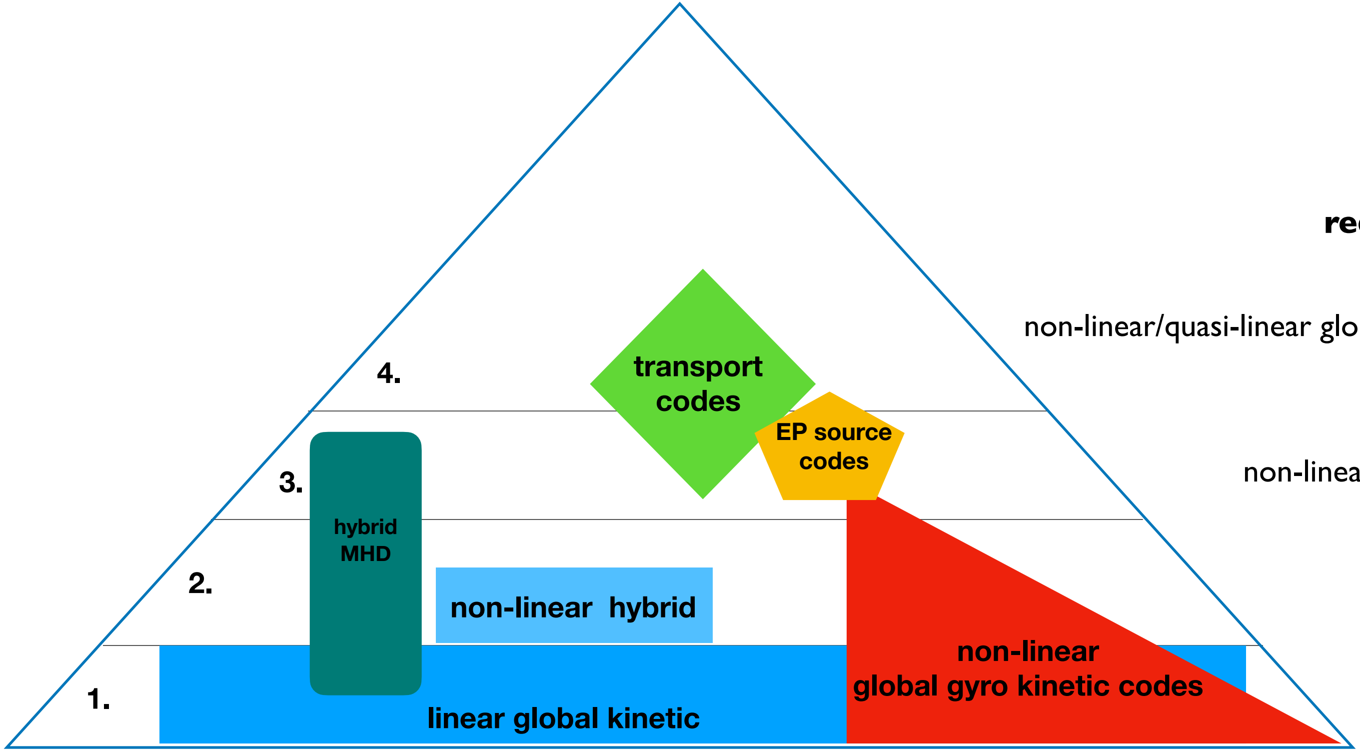
ingredients for reduced energetic particle (EP) transport models:

needed for scaling from TCV-AUG-JET, W7X… to JT-60SA-DTT-ITER-DEMO, in particular burning plasmas



	required models:
4. self-organisation - back reaction of EP transport on profiles and background transport	non-linear/quasi-linear global kinetic e.m.+ background transport
3. EP transport and losses	non-linear/quasi-linear global kinetic e.m. + long time scales (source +sink)
2. non-linear mode evolution, saturation mechanisms	non-linear global kinetic e.m.
1. mode stability	linear global kinetic e.m.

modelling hierarchy for plasmas with significant energetic particle pressure



required models:

non-linear/quasi-linear global kinetic e.m.+ background transport

non-linear/quasi-linear global kinetic e.m. + long time scales (source +sink)

non-linear global kinetic e.m.

linear global kinetic e.m.

modelling hierarchy for plasmas with significant energetic particle pressure

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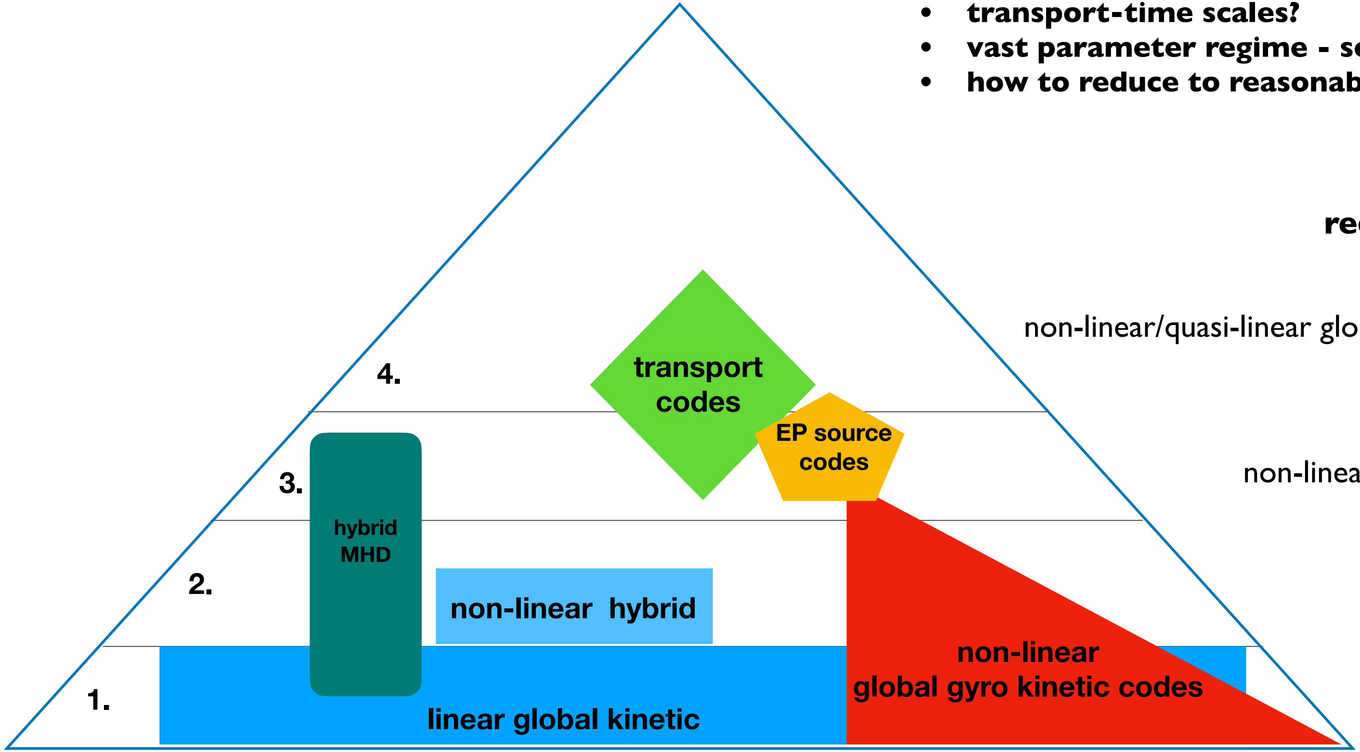

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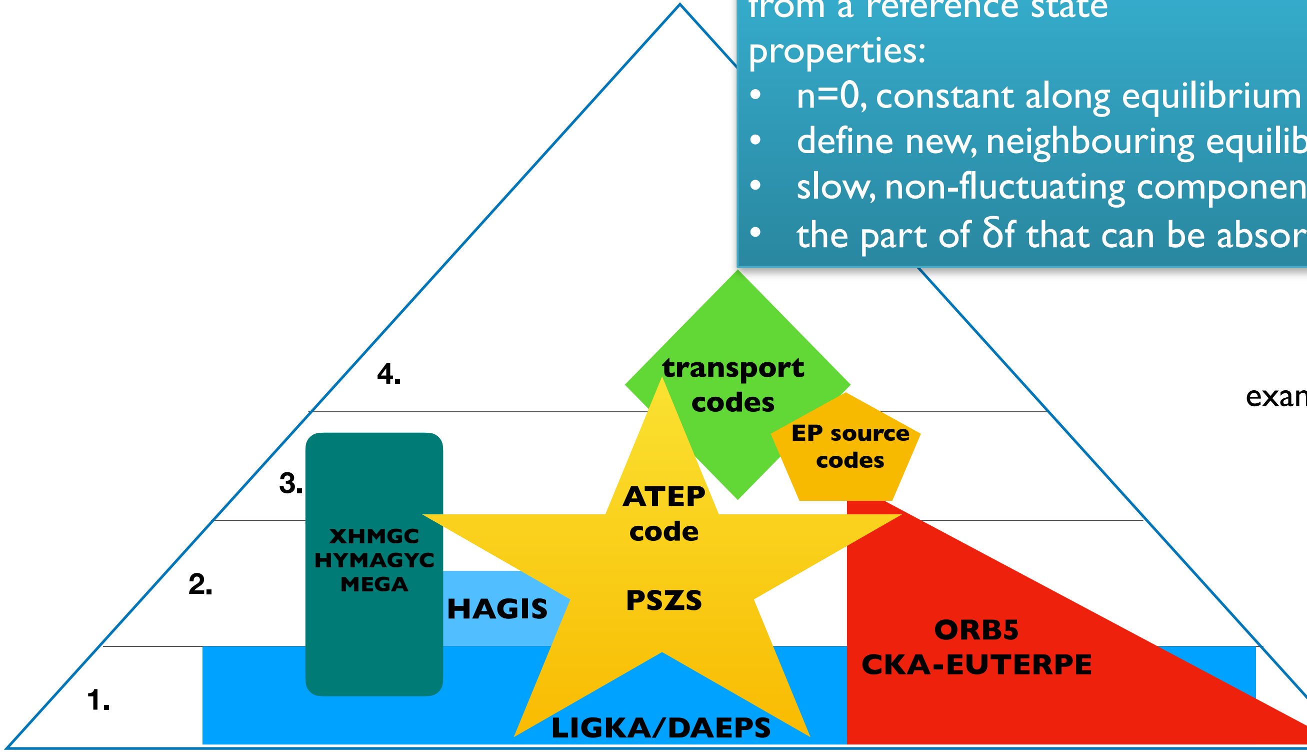
non-linear global kinetic e.m.

linear global kinetic e.m.

modelling hierarchy for plasmas with significant energetic particle pressure



4.

3.

2.

1.

transport codes

EP source codes

ATEP code

PSZS

XHMGC
HYMAGYC
MEGA

HAGIS

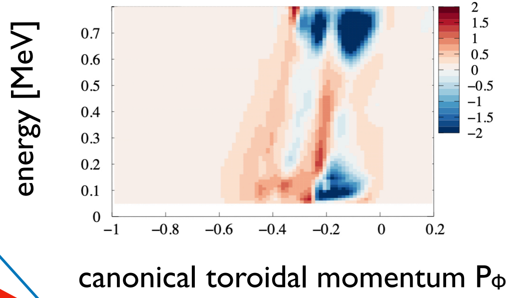
ORB5
CKA-EUTERPE

LIGKA/DAEPS

phase space zonal structures (PSZS) are
collision-less undamped, long-lived nonlinear deviations of the plasma from a reference state
properties:

- n=0, constant along equilibrium orbits
- define new, neighbouring equilibrium reference state
- slow, non-fluctuating component of F_{EP} evolution
- the part of δf that can be absorbed in new F_0

example in CoM space representation:



modelling hierarchy for plasmas with significant energetic particle pressure

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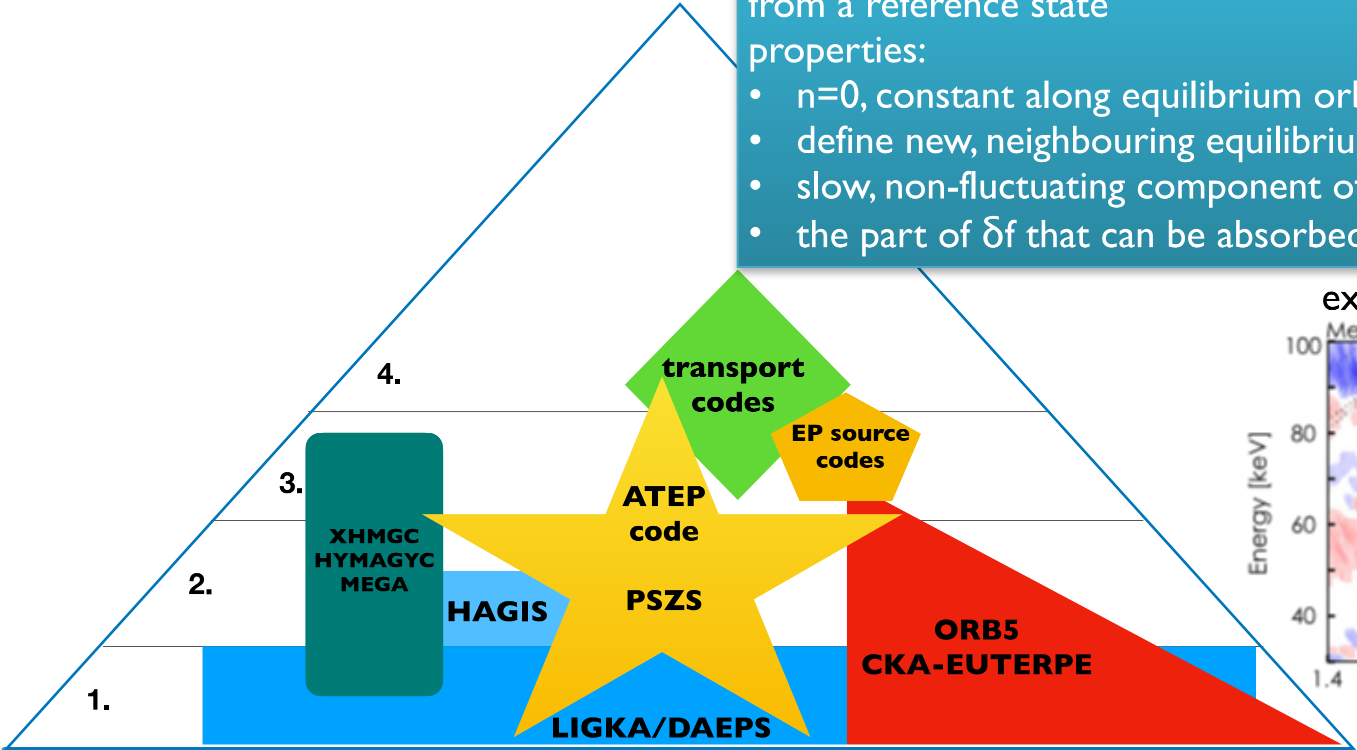
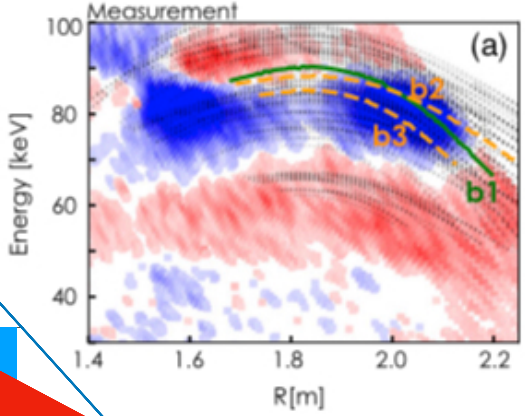


theory: explicit calculation of PSZSs for specific cases (*see talk M. Falessi*)

experiment:



DIII-D, INPA [Du et al PRL 2021]
AUG, INPA J. Rueda [FEC 2023]

outline

- PSZS theory and overall implementation strategy
- general distribution functions in constants of motion space (CoM)
- linear mode spectrum: the Energetic Particle Stability Workflow (EP-WF)
- phase space transport coefficients
- evolve transport equation in kick model and quasi-linear (QL) limit
- back mapping to real space and non-linear equilibria
- verification and validation - common effort of ENR ATEP team

PSZS transport theory and its connection to kick and QL limits

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b\delta\dot{P}_\phi\delta F\right)}_z + \frac{\partial}{\partial\mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right]+\mathcal{S}\right)}_{z\,S}$$

$\nabla_z \cdot \Gamma$

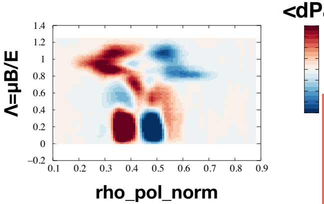
wave-induced phase space flux

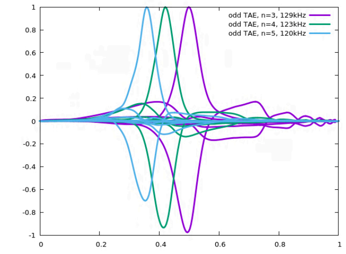
collisions + source

[M-V. Falessi, F. Zonca, 2017-2023]
continuity equation in phase space;
valid for single or multiple modes
in general valid for all regimes;
interactions with background fluctuations
can be consistently kept

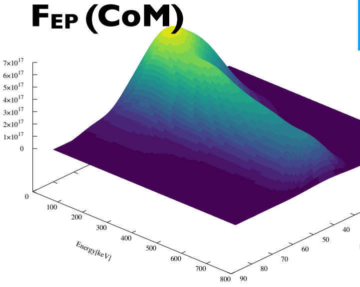
- kick limit: fix perturbations amplitudes for calculating <dP$_\phi$/dt> and evolve continuity equation in CoM space
- in the QL limit, assuming overlapping resonances, flux can be split into convective and diffusive component [L Chen, JGR 104, 1999]
- diffusion coefficients can be evaluated by determining $D_{P\phi P\phi}=|dP_\phi/dt|^2\,\tau_{ac}$, similar for D_{EE}, and off diagonal terms (if present), resonant and non-resonant contributions can be separated
- in [L Chen, JGR 104, 1999] also the importance of $E_{//}$ is discussed (KAW physics), leading to additional convective flux contributions (linear GK code LIGKA provides this information - see below)

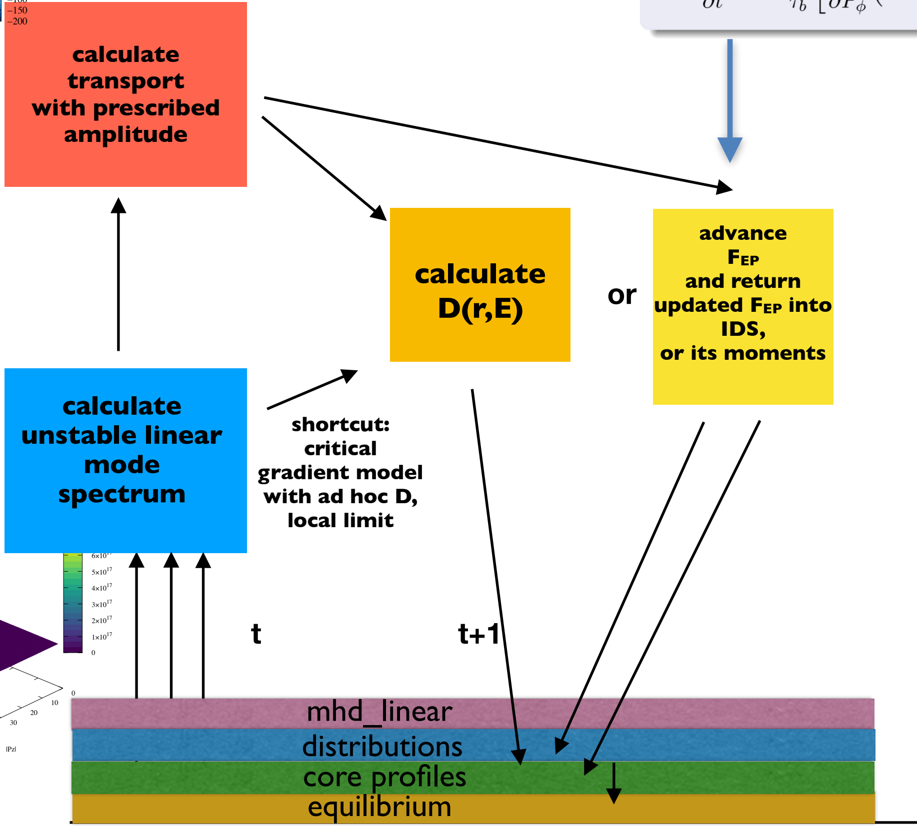
implementation using IMAS: kick model limit





F$_{EP}$ (CoM)





calculate transport with prescribed amplitude

calculate unstable linear mode spectrum

calculate D(r,E)

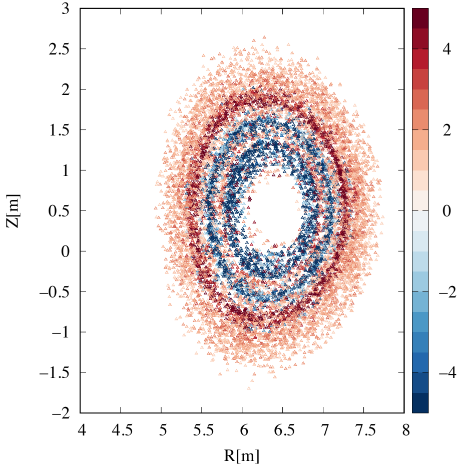
or

advance F$_{EP}$ and return updated F$_{EP}$ into IDS, or its moments

shortcut: critical gradient model with ad hoc D, local limit

PSZS transport theory [M. Falessi 2017-2022]

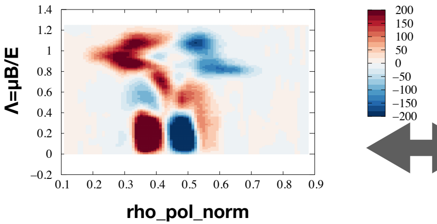
$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta\dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{z\,S}$$



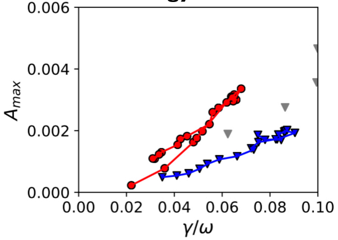
newly developed ATEP/ ATEP-3D code [Ph. Lauber, G. Meng,2022-23]

implementation: QL limit

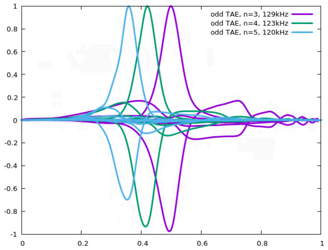
orbit+zonal averaging



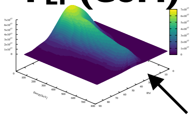
saturation rule/energy conservation (HAGIS model)

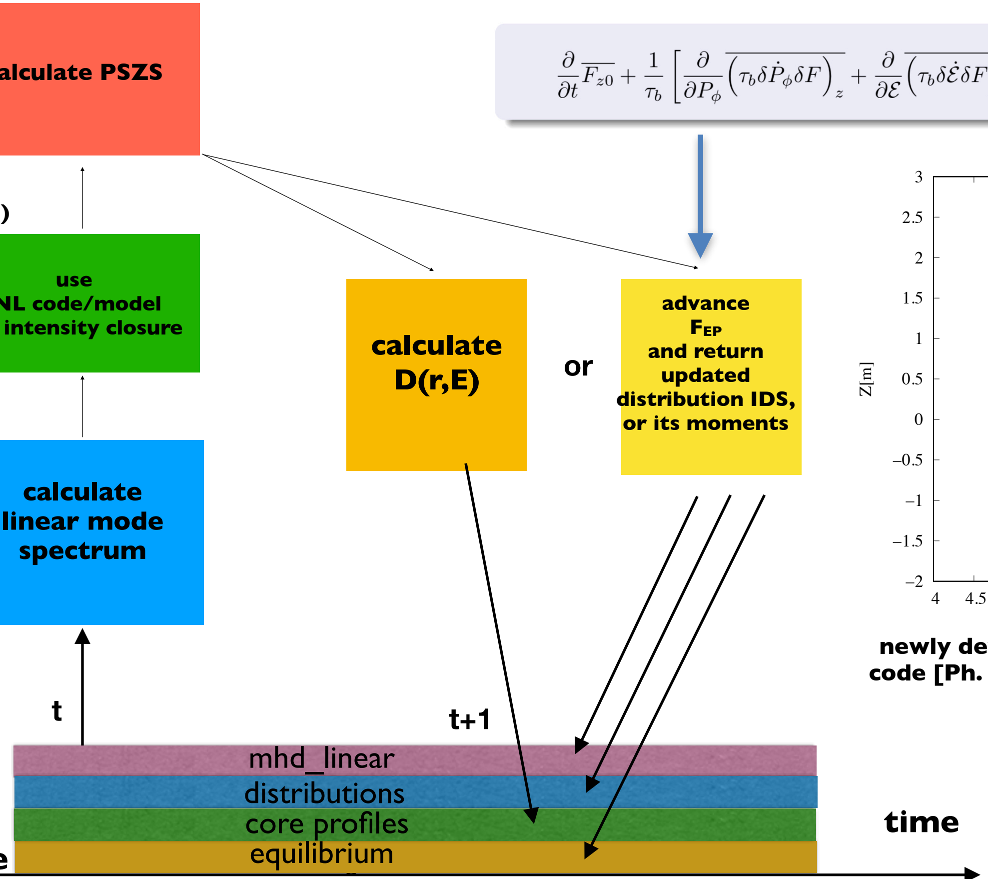


EP WF (LIGKA)



F_{EP} (CoM)





calculate PSZS

use NL code/model for intensity closure

calculate linear mode spectrum

calculate D(r,E)

or

advance F_{EP} and return updated distribution IDS, or its moments

t

t+1

mhd_linear

distributions

core profiles

equilibrium

transport code

time

PSZS transport theory [M. Falessi 2017-2022]

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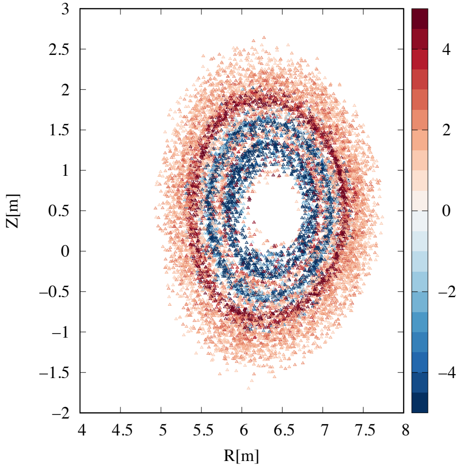


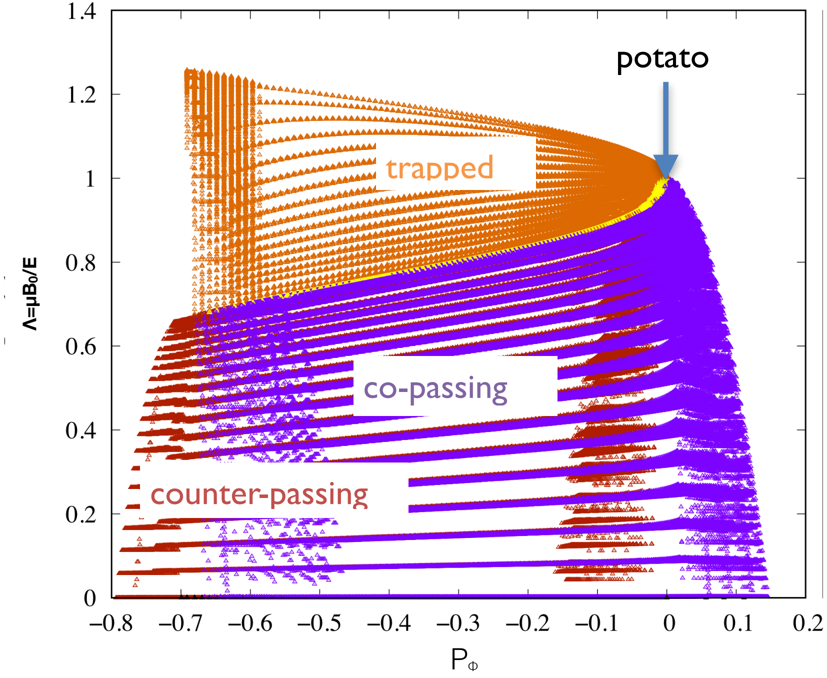
newly developed ATEP/ ATEP-3D code [Ph. Lauber, G. Meng,2022-23]

determining F_{EP} in constants of motion space (CoM)

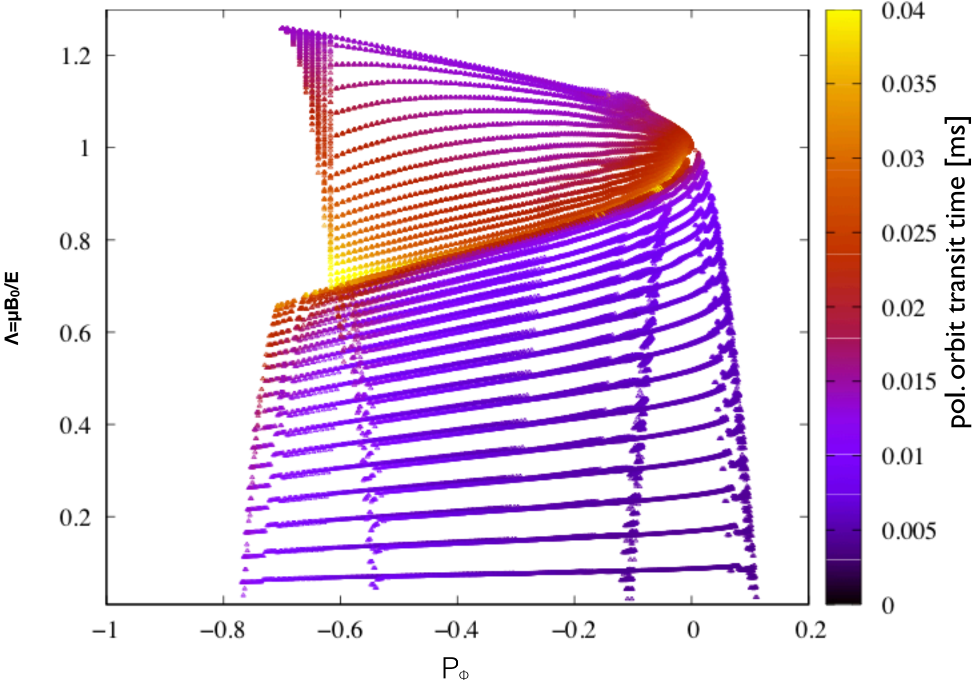
determining F_{EP} in constants of motion space (CoM)

several recent papers [Bierwage 2022, G. Brochard, FEC 2023, Salewski 2020-21] using a similar procedure:

- establish orbit database to classify particles
- determine CoM Jacobian (P_Φ,E,Λ,μB_0/E,σ)
- set up grid in CoM space
- bin markers as given by neoclassical physics codes [NEMO/Spot, ASCOT, RABBIT, etc…], here ITER H-pre-fusion case 100015,1 [M. Schneider, 2018]



pol. orbit transit time [ms] ~ CoM Jacobian, 500keV H ions in ITER



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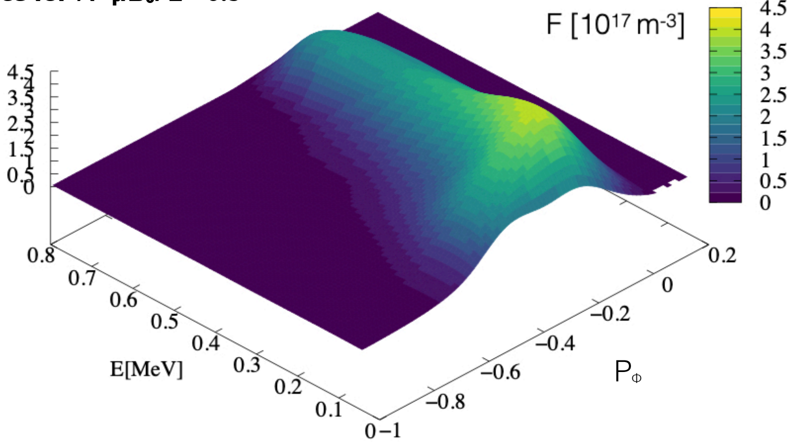
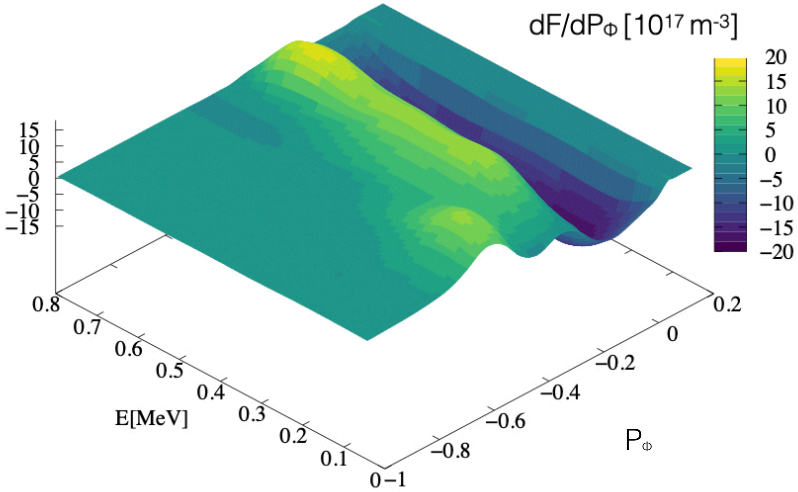
- use 2D cubic splines in each sub-space to create fine sub-grids, then create 3D spline for F_{EP}
- back-transform in other coordinate systems possible, if needed
- here, all calculations are using IMAS interfaces (equilibrium, transport code, orbit tracer (HAGIS [S.D. Pinches]), ATEP code)

slice for $\Lambda=\mu B_0/E=0.5$





Calculating the mode spectrum

Linear mode spectrum: Energetic particle stability workflow (LIGKA)

LIGKA [Qin 1998, Lauber 2003, JPC 2007, Lauber PLREP 2013, Bierwage&Lauber 2017, Lauber JPCS 2018]

https://git.iter.org/projects/STAB/repos/ligka/

- gyrokinetic moment equation (GKM):

shear Alfvén law

$$\boxed{-\frac{\partial}{\partial t}\left[\nabla\cdot\frac{1}{v_{\mathrm{A}}^2}\nabla_\perp\phi\right]+\boldsymbol{B}\cdot\nabla\frac{\nabla\times(\nabla\times(\frac{\nabla\psi}{i\omega})_\parallel\boldsymbol{b})}{B}}+(\boldsymbol{b}\times\nabla(\frac{\nabla\psi}{i\omega})_\parallel\boldsymbol{b})\cdot\nabla\frac{\mu_0 j_\parallel}{B}$$
$$=\boxed{-\sum_a\mu_0\int\mathrm{d}^2v\,e_a\left\{\boldsymbol{v}_\mathrm{d}\cdot\nabla J_0 f\right\}_a}+\sum_a\left[\boldsymbol{b}\times\nabla\left(\frac{\beta_{a\perp}}{2\Omega_a}\right)\right]\cdot\nabla\nabla_\perp^2\phi$$
$$+\sum_a\frac{3v_{th,a}^2}{8v_A^2\Omega_a^2}\nabla_\perp^4\frac{\partial\phi}{\partial t}+\boldsymbol{B}\cdot\nabla\frac{1}{B}\sum_a\frac{\beta_a}{4}\nabla_\perp^2(\frac{\nabla\psi}{i\omega})_\parallel\boldsymbol{b}$$

'pressure' tensor - curvature drift coupling

reduced MHD as limit;
various successful benchmarks with ORB5 [Hayward-Schneider, 2021-23]

- quasi neutrality (QN):

$$0=\sum_a e_a\int\mathrm{d}^2v\left\{J_0 f\right\}_a+\nabla_\perp\cdot\frac{m_i n_i\nabla_\perp\phi}{B^2}$$

- non-adiabatic response for perturbed distribution function:

propagator →resonance

$$\hat{h}=ie\sum_m\int_{-\infty}^{t}dt'\boxed{e^{i\left[n(\varphi'-\varphi)-m(\theta'-\theta)-\omega(t'-t)\right]}}e^{-im\theta}$$
$$\frac{\partial F_0}{\partial E}\left[\omega-\boxed{\hat{\omega}_*}\right]J_0^2(k_\perp\varrho_i)\left[\phi_m(r')-(1-\frac{\omega_d(r',\theta')}{\omega})\psi_m(r')\right]$$

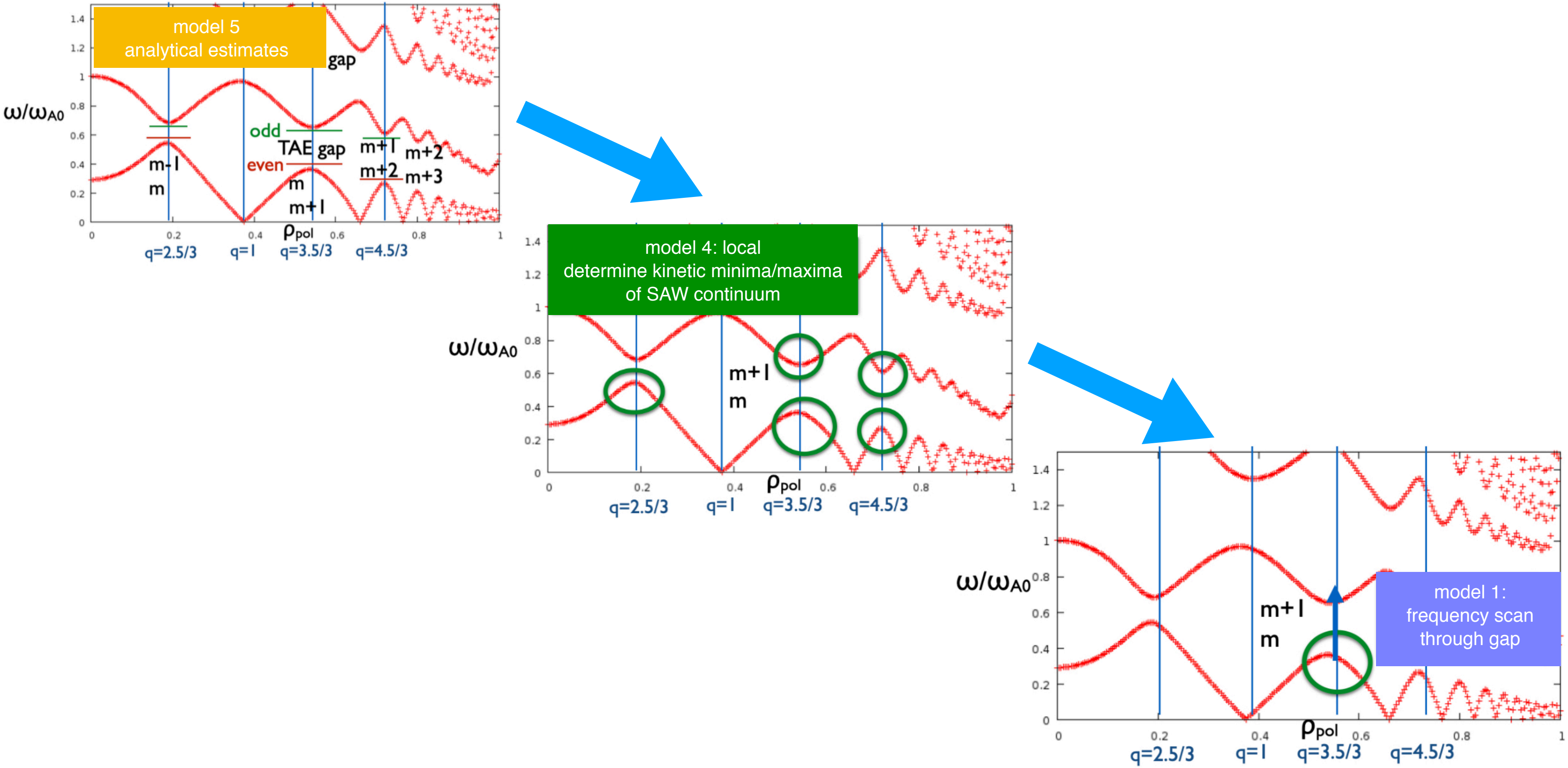
free energy

resonances (circ/trapped):

$$\begin{cases}\omega_{AE}-\omega_{prec}-(nq-m+k)\cdot\omega_t=0\\ \omega_{AE}-\omega_{prec}-k\cdot\omega_b=0\end{cases}$$

for all species , including electrons and energetic particles

Linear mode spectrum: Energetic particle stability workflow (EP-WF)



model 5
analytical estimates

model 4: local
determine kinetic minima/maxima
of SAW continuum

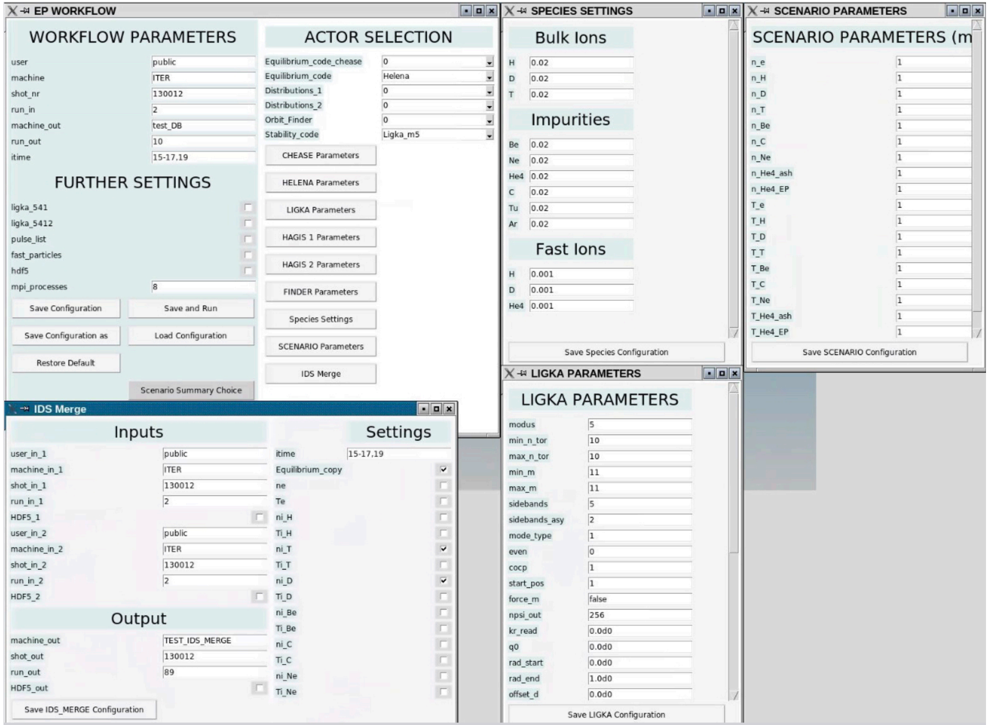
model 1:
frequency scan
through gap

EP-WF: available on Eurofusion Gateway and ITER cluster

[V.-A. Popa et al 2023 Nucl. Fusion 63]

training course & additional material: https://indico.euro-fusion.org/event/2729/

- fully IMAS compatible (python)
- git version control
- module installations available
- gui and non-gui versions
- batch job submission



WORKFLOW PARAMETERS

user	public
machine	ITER
shot_nr	130012
run_in	2
machine_out	test_DB
run_out	10
itime	15-17.19

FURTHER SETTINGS

ligka_541	☐
ligka_5412	☐
pulse_list	☐
fast_particles	☐
hdf5	☐
mpi_processes	8

ACTOR SELECTION

Equilibrium_code_chease	0
Equilibrium_code	Helena
Distributions_1	0
Distributions_2	0
Orbit_Finder	0
Stability_code	Ligka_m5

SPECIES SETTINGS

Bulk Ions	
H	0.02
D	0.02
T	0.02
Impurities	
Be	0.02
Ne	0.02
He4	0.02
C	0.02
Tu	0.02
Ar	0.02
Fast Ions	
H	0.001
D	0.001
He4	0.001

SCENARIO PARAMETERS

n_e	1
n_H	1
n_D	1
n_T	1
n_Be	1
n_C	1
n_Ne	1
n_He4_ash	1
n_He4_EP	1
T_e	1
T_H	1
T_D	1
T_T	1
T_Be	1
T_C	1
T_Ne	1
T_He4_ash	1
T_He4_EP	1

LIGKA PARAMETERS

modus	5
min_n_tor	10
max_n_tor	10
min_m	11
max_m	11
sidebands	5
sidebands_asy	2
mode_type	1
even	0
cocp	1
start_pos	1
force_m	false
npsi_out	256
kr_read	0.0d0
q0	0.0d0
rad_start	0.0d0
rad_end	1.0d0
offset_d	0.0d0

Inputs

user_in_1	public
machine_in_1	ITER
shot_in_1	130012
run_in_1	2
HDF5_1	☐
user_in_2	public
machine_in_2	ITER
shot_in_2	130012
run_in_2	2
HDF5_2	☐

Output

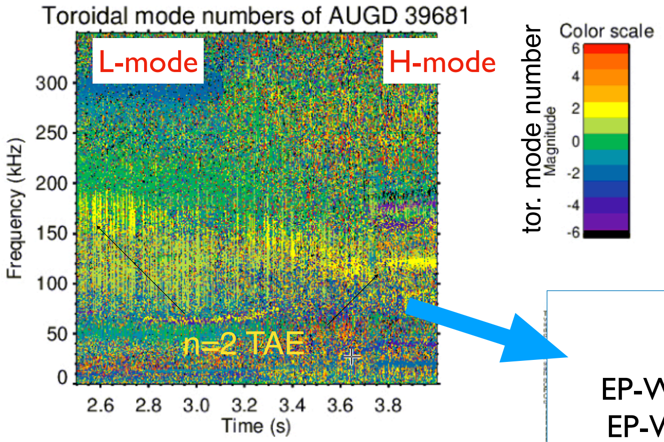
machine_out	TEST_IDS_MERGE
shot_out	130012
run_out	89
HDF5_out	☐

Settings

itime	15-17,19
Equilibrium_copy	☑
ne	☐
Te	☐
ni_H	☐
Ti_H	☐
ni_T	☑
Ti_T	☐
ni_D	☑
Ti_D	☐
ni_Be	☐
Ti_Be	☐
ni_C	☐
Ti_C	☐
ni_Ne	☐
Ti_Ne	☐

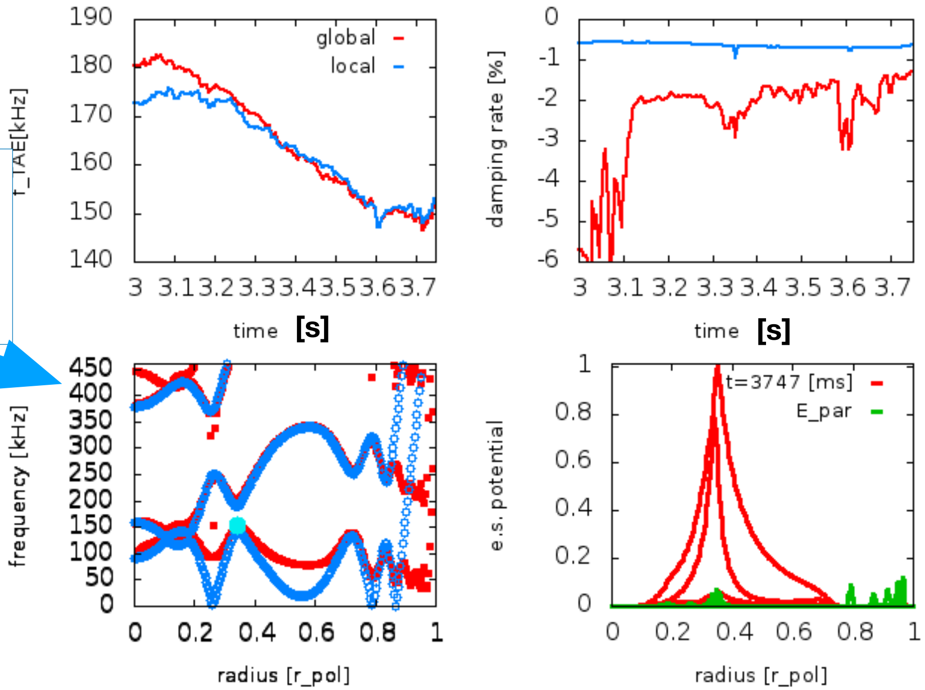
Energetic particle stability workflow: validation at ASDEX Upgrade

AUG EP ‘supershot’ scenarios: D NBI into D plasma
(further development of NLED AUG benchmark case)



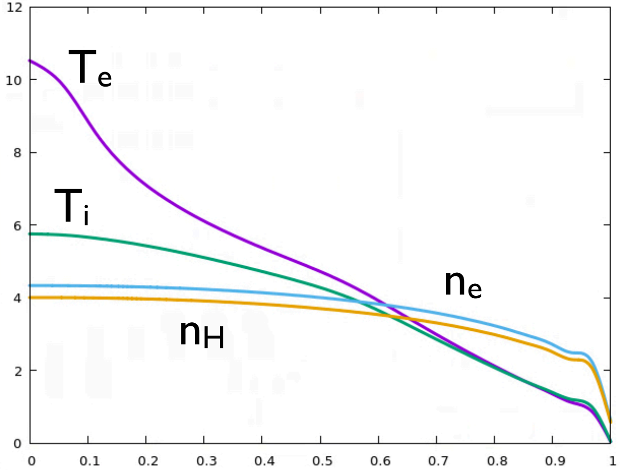
[Lauber, EPS 2022]

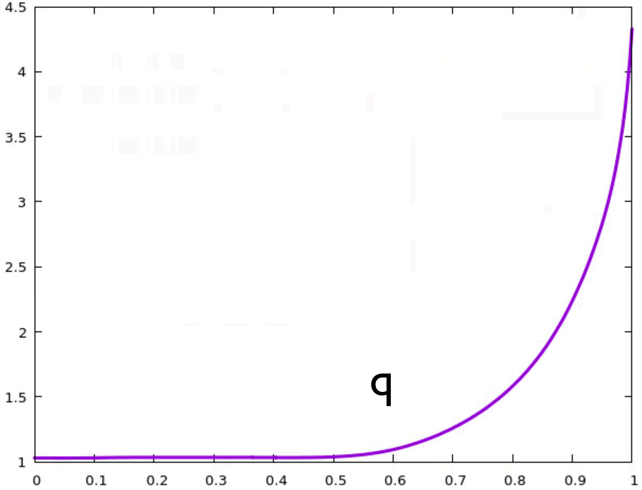
IDA +
TRVIEW +
EP-WF: LIGKA local +
EP-WF: LIGKA global



- analyse L-mode,H-mode and transition phase using
- systematic uncertainty quantification feasible
- bursty and steady-state phases visible, in agreement with damping analysis and drive
- speed up WF using ML methods [V.-A. Popa; in preparation]

ITER pre-fusion H scenario 100015,1 [Metis, M. Schneider NF (2021)]



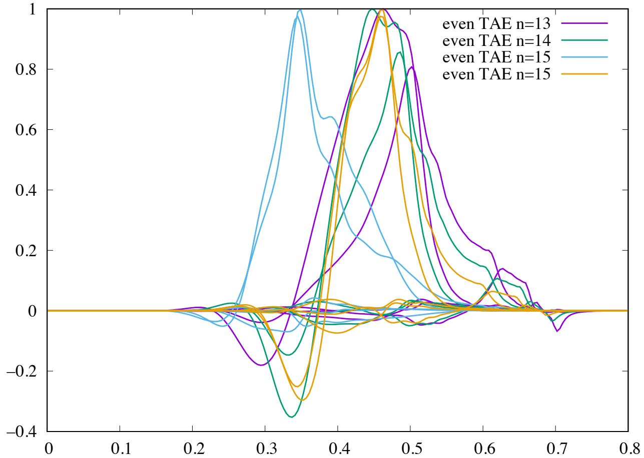


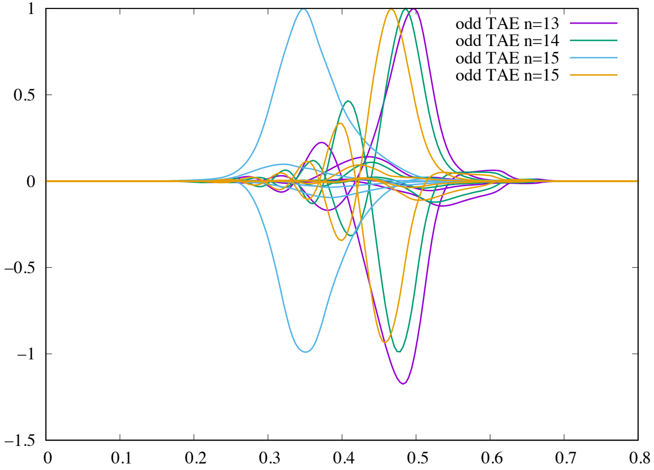
use set of mid-n TAEs
as test case
(as all data is consistently
available in
IMAS thx to M. Schneider)

B=-5 T
I= -1.8 MA

small damping rates:
1-5%

with H beam (870 keV)marginally
unstable

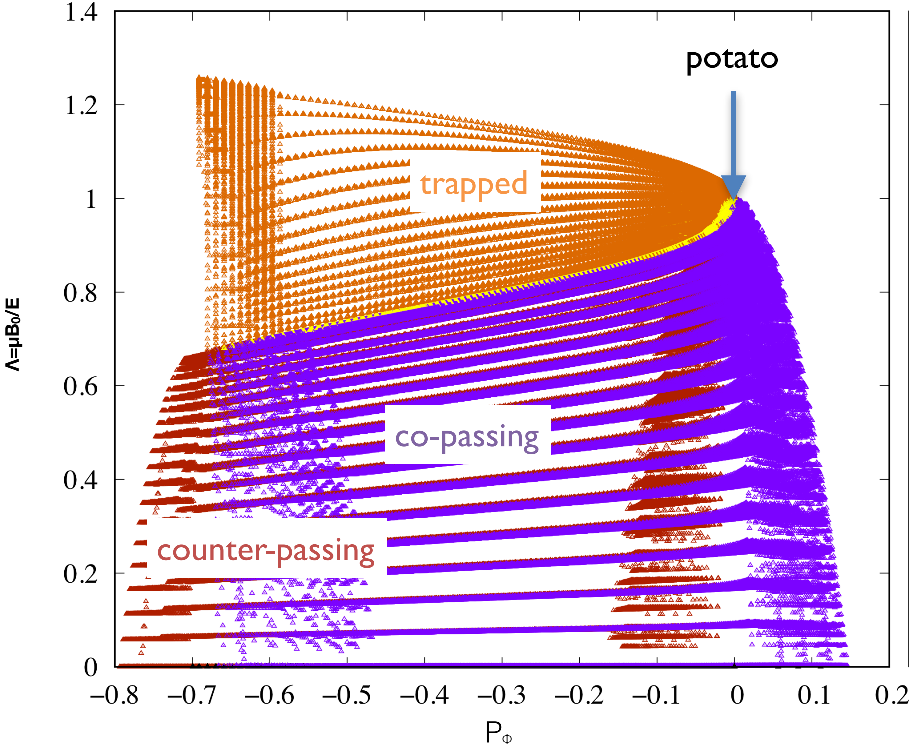




determine phase space transport coefficients

classify particles, calculate orbit properties with and without perturbation(s)

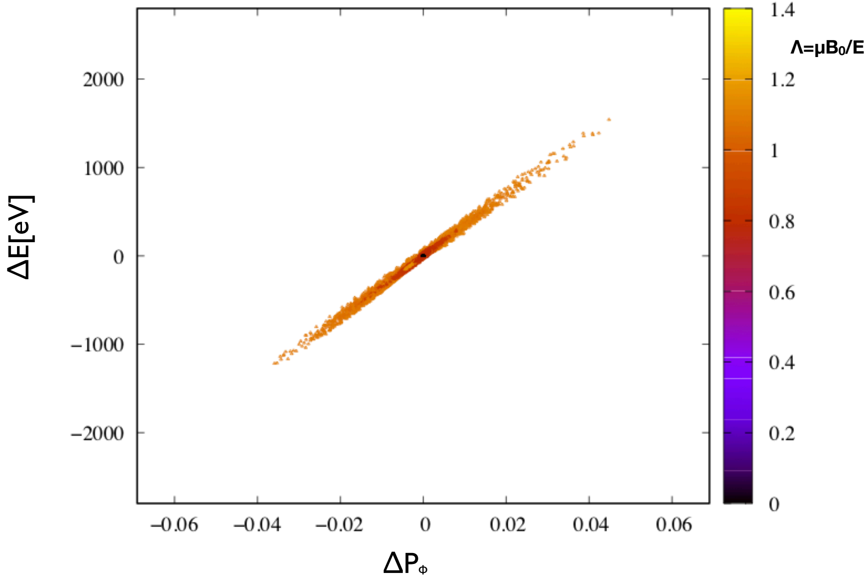
equilibrium orbits



use wrapper ('finder') for HAGIS [S.D. Pinches] to efficiently set up marker space

with perturbation: dP/dt and dE/dt of resonant particles in single wave are proportional to each other

n=13 TAE, δB/B=5.0d-4

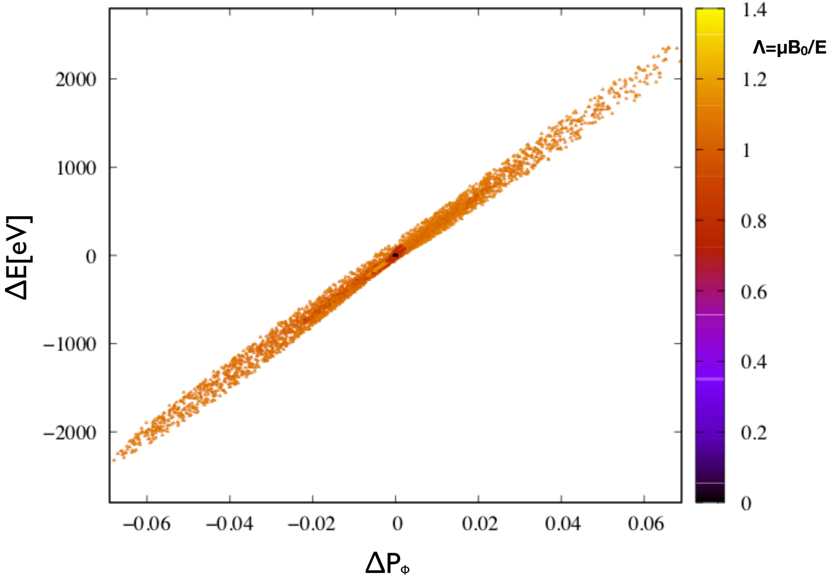


off-diagonal elements play a role in multi-mode cases, or with $E_{//}$

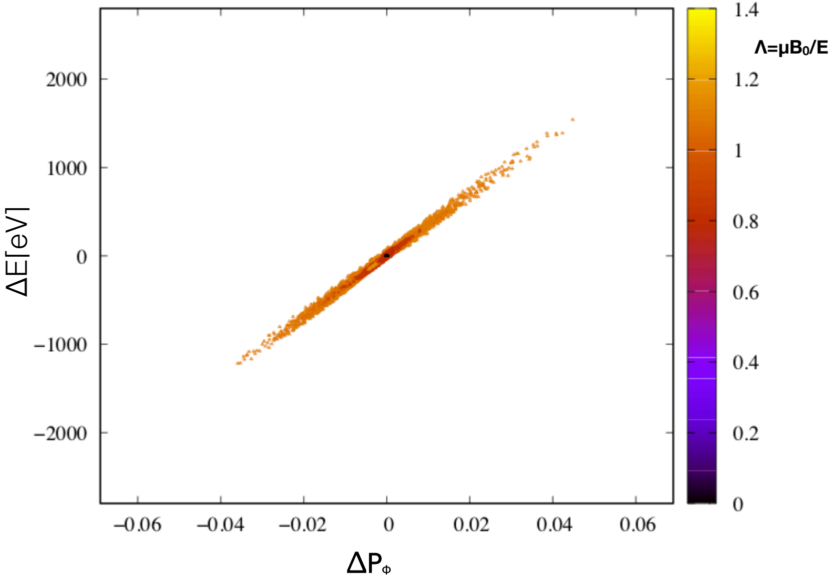
classify particles, calculate orbit properties with and without perturbation(s)

with perturbation: dP/dt and dE/dt of resonant particles in single wave are proportional to each other [Southwood, 1969]

n=13 + n=14 TAE, δB/B=5.0d-4



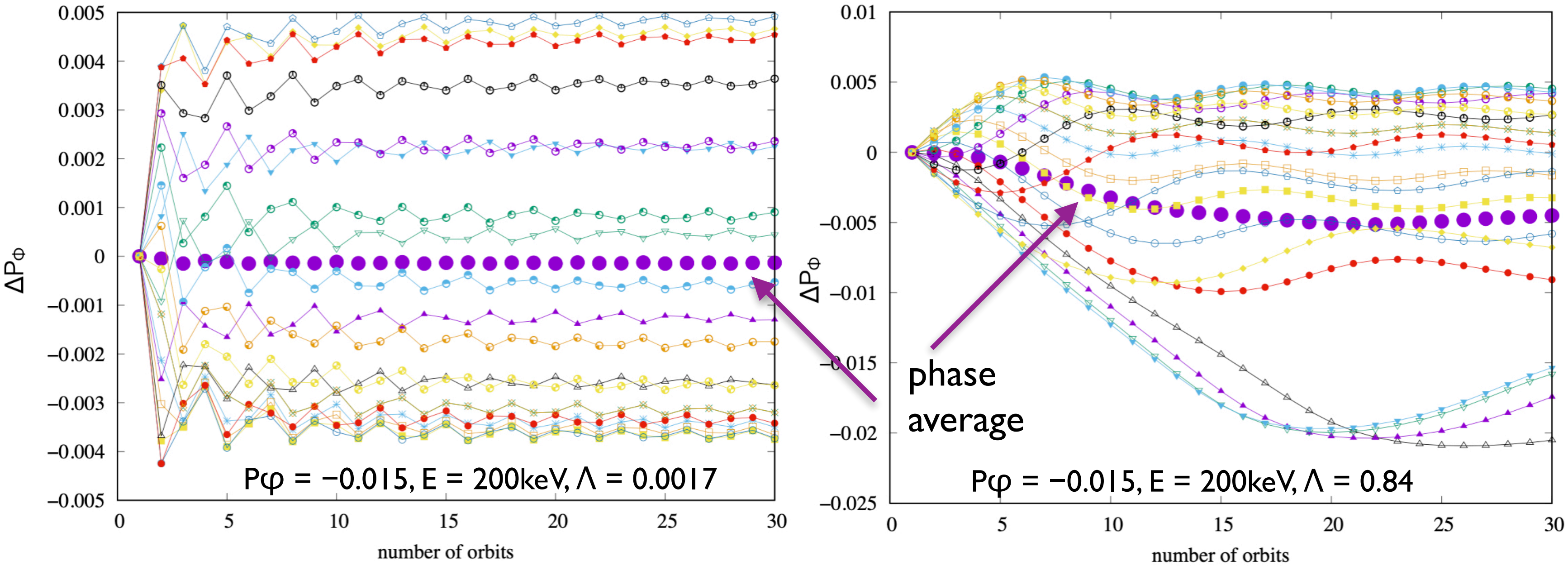
n=13 TAE, δB/B=5.0d-4



off-diagonal elements play a role in multi-mode cases, or with $E_{//}$

zonal and orbit averaging

start particles with different phase shifts with respect to wave: ($2\pi/n$, or random), follow typical 3-5 orbits to account for higher resonances, then average ($n=13$TAE; $\delta B/B = 5 \cdot 10^{-4}$)



non-resonant

resonant

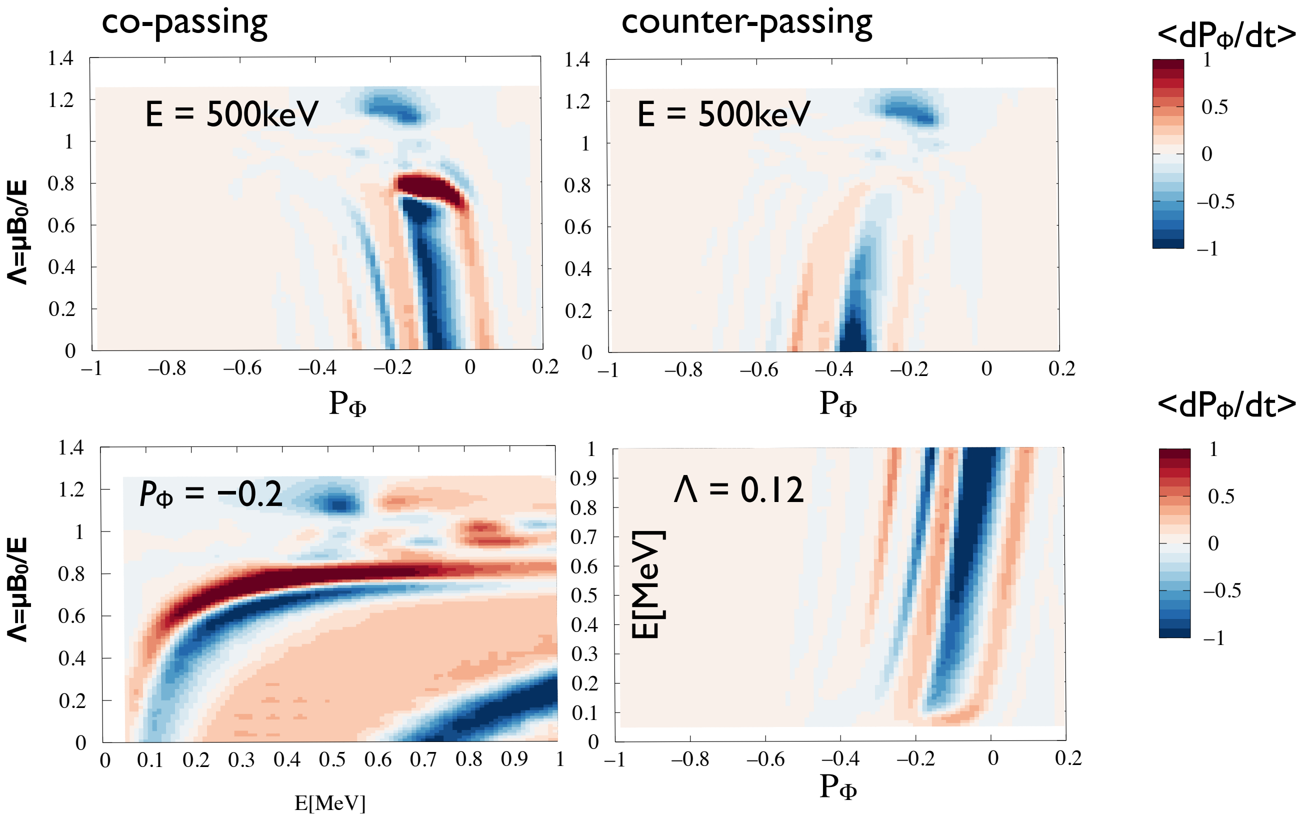
caveat: this procedure is reducing the full dynamics: valid in small-amplitude/QL/limit, transport time scales can be improved, relaxed if needed (ballistic transport cases);

note also close relation to P_Φ grid resolution/Courant criterion; accounts for resonance broadening consistently

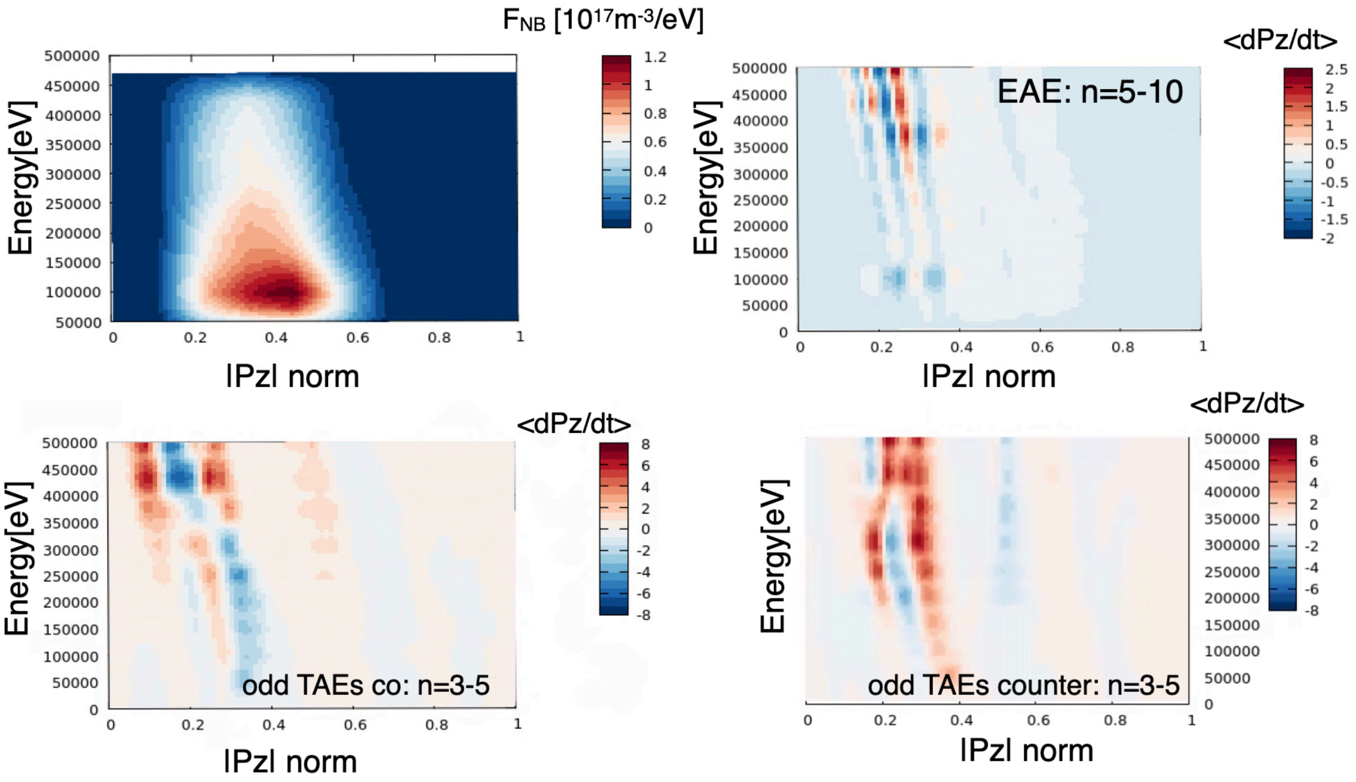
zonal and orbit averaging: <dP$_\Phi$/dt> in CoM representation

$\delta B/B = 5 \cdot 10^{-6}$

- typically follow 128x40x40x4 markers
- store in IDS (distributions)
- use multi-level spline interpolation [Lee 1997]
- use cartesian grid in CoM space (96x96x96)



PSZS for EAEs and odd TAEs



all plots for Λ=μB$_0$/E=0.24

resonances with both positive and negative gradients of F$_{EP}$ possible

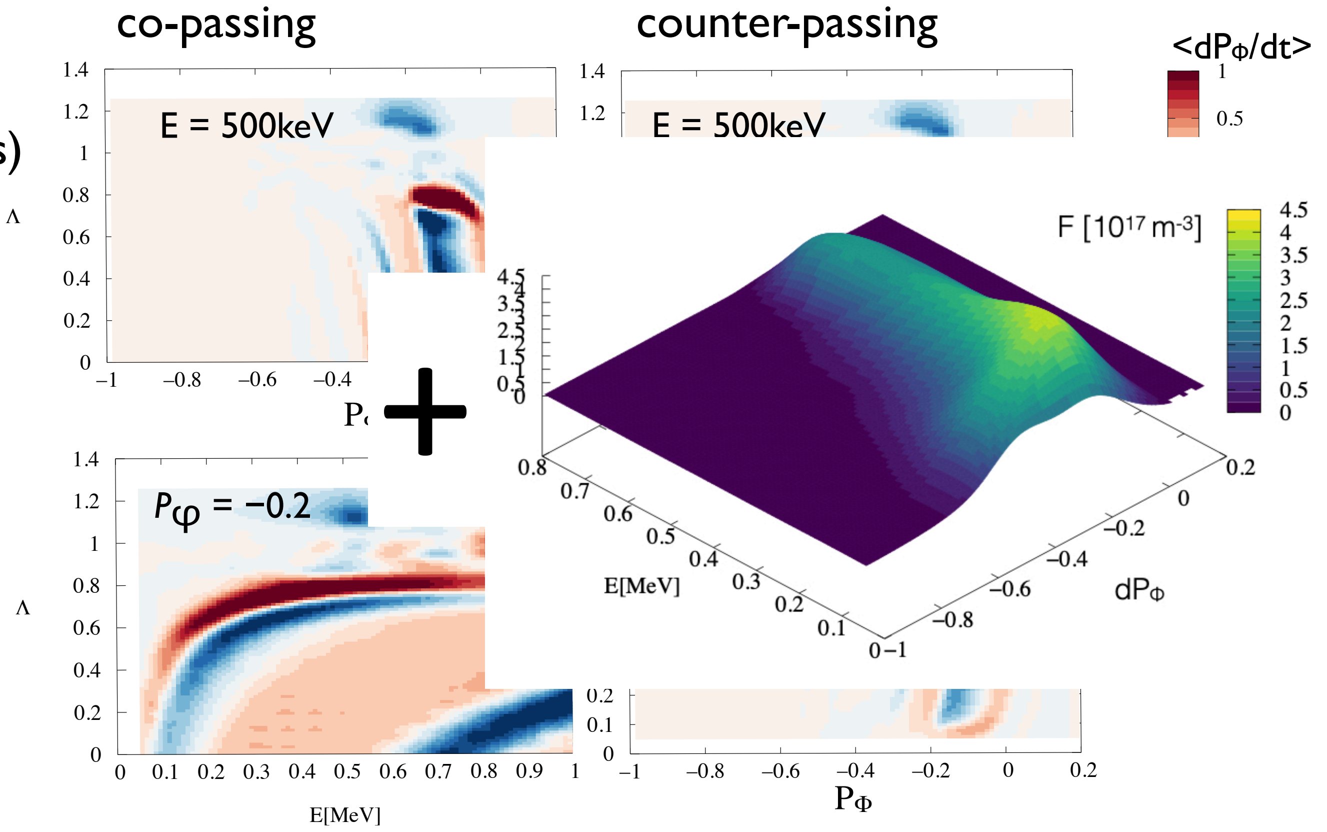
evolve transport equations in kick-model limit

zonal and orbit averaging: <dP$_\Phi$/dt> in CoM representation

$\delta B/B = 5 \cdot 10^{-6}$

- typically follow 128x40x40x4 markers
- store in IDS (distributions)

- use multi-level spline interpolation [Lee 1997]
- use cartesian grid in CoM space (96x96x96)

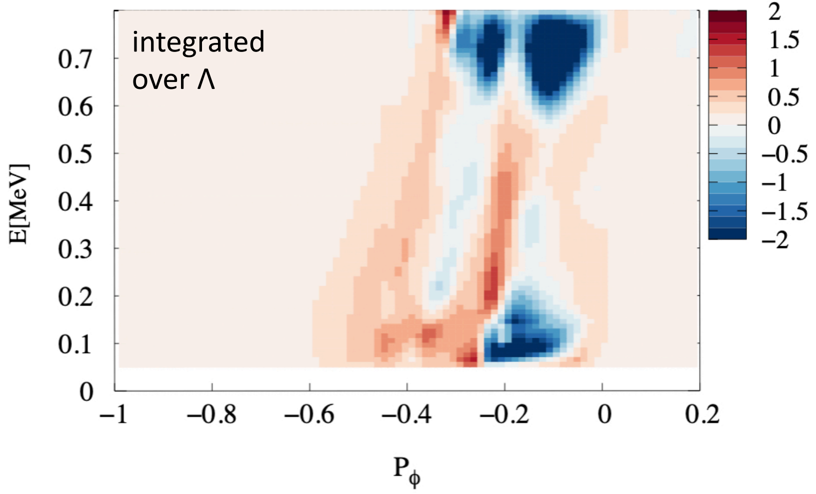


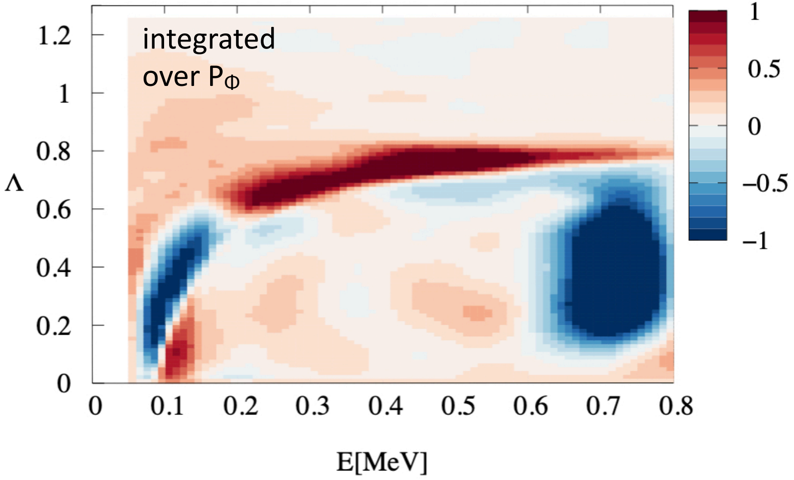
evolve continuity equation for Fz in CoM space

$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi}\left(\langle\overline{\frac{dP_\phi}{dt}}\rangle F_z\right) - \frac{\partial}{\partial E}\left(\langle\overline{\frac{dE}{dt}}\rangle F_z\right) \qquad \mathbf{v}_{P_\phi,E} = \left(\langle\overline{\frac{dP_\phi}{dt}}\rangle, \langle\overline{\frac{dE}{dt}}\rangle\right)$$

advection equation, assuming $\nabla \cdot \mathbf{v}_{P_\phi,E} = 0$ i.e. incompressible phase space flow is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

$\delta F_{EP} = F_{EP}(t = 700ms) - F_{EP}(t = 0)[10^{16} m^{-3}]$ with constant $\delta B(t)/B = 10^{-5}$





evolve continuity equation for Fz in CoM space

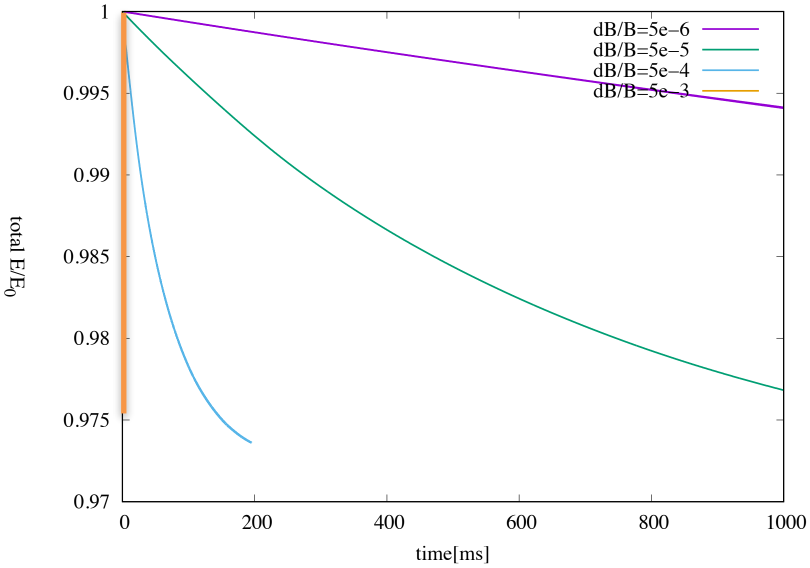
$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi}\left(\langle\overline{\frac{dP_\phi}{dt}}\rangle F_z\right) - \frac{\partial}{\partial E}\left(\langle\overline{\frac{dE}{dt}}\rangle F_z\right) \qquad \mathbf{v}_{P_\phi,E} = \left(\langle\overline{\frac{dP_\phi}{dt}}\rangle, \langle\overline{\frac{dE}{dt}}\rangle\right)$$

advection equation, assuming $\nabla \cdot \mathbf{v}_{P_\phi,E} = 0,$ i.e. incompressible phase space flow is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

phase space density is conserved
add energy diagnostic:

$$\mathscr{E}(t) = \int dv_{P_\phi,E,\Lambda} E \cdot F_{EP}(t)/E_0$$

if perturbations are consistently chosen
i.e. as unstable eigenfunctions of the equilibrium,
energy stored in gradients of F_{EP} is depleted



evolve continuity equation for Fz in CoM space

$$\frac{\partial F_z}{\partial t} = -\frac{\partial}{\partial P_\phi}\left(\langle\overline{\frac{dP_\phi}{dt}}\rangle F_z\right) - \frac{\partial}{\partial E}\left(\langle\overline{\frac{dE}{dt}}\rangle F_z\right) \qquad \mathbf{v}_{P_\phi,E} = \left(\langle\overline{\frac{dP_\phi}{dt}}\rangle, \langle\overline{\frac{dE}{dt}}\rangle\right)$$

advection equation, assuming $\nabla \cdot \mathbf{v}_{P_\phi,E} = 0$ i.e. incompressible phase space flow is evolved with Lax-Wendroff scheme (explicit, adaptive time step - Courant limit)

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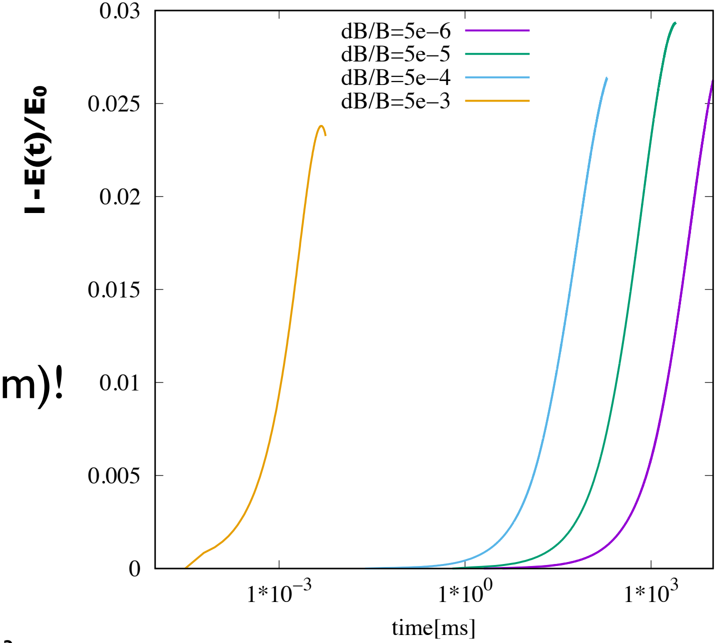
$$\mathscr{E}(t) = \int dv_{P_\phi,E,\Lambda} E \cdot F_{EP}(t)/E_0$$



note that energy can also increase (forced driven system)!

find minimum in energy, defining the maximally relaxed state of F_{EP} in presence of a fixed perturbation

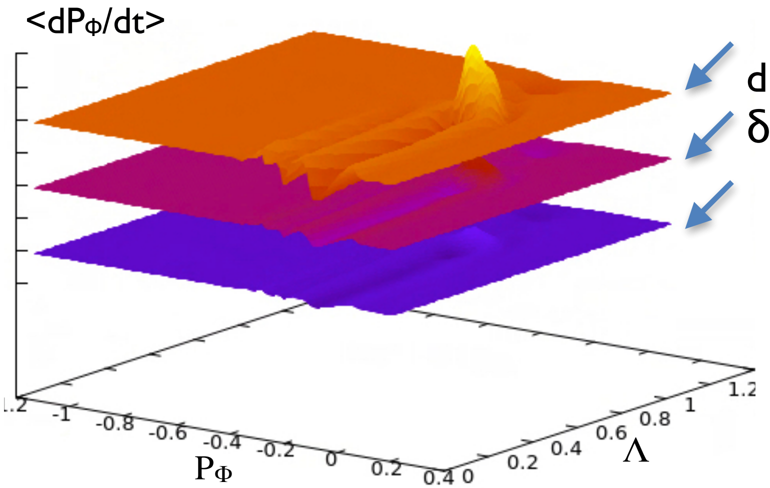
evolve transport equations in quasi-linear limit (QL)

quasi-linear model: add energy conservation equation

$$\frac{d}{dt}\left(\mathcal{E} + \sum_k W_k\right) = -2\sum_k \gamma_{d,k} W_k$$

$$\mathcal{E}(t) = \int dv_{P_\phi,E,\Lambda} E \cdot F_{EP}(t)$$

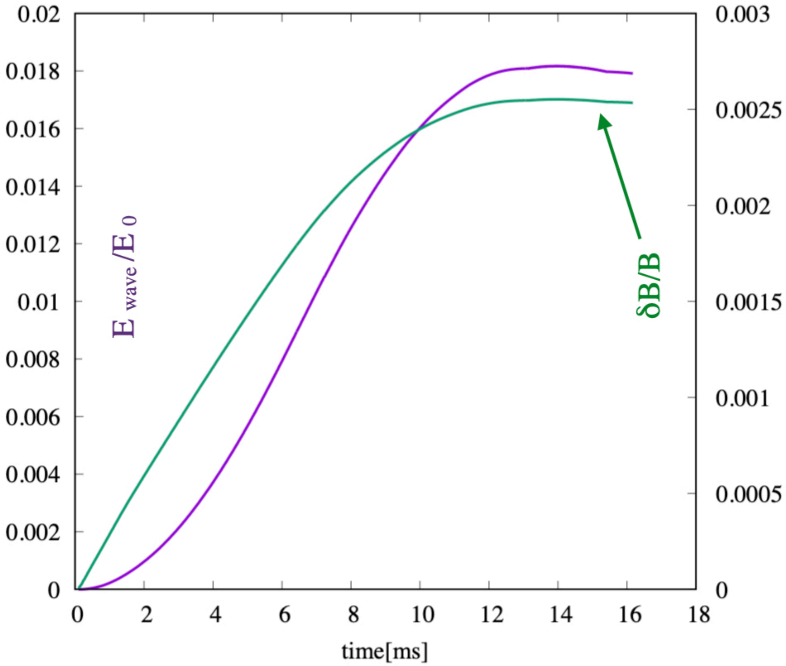
amplitude dependent $\langle dP_\Phi/dt\rangle$, $\langle dE/dt\rangle$ needed!



different $\delta B/B \rightarrow \Sigma_k W_k$

- run previously developed WF for calculating PSZS (FINDER/HAGIS) and store in different IDS occurrences
- import into ATEP code (typically 3-5 different amplitudes $\delta B/B = 5 \cdot 10^{-6}, 5 \cdot 10^{-5}, 5 \cdot 10^{-4}, 5 \cdot 10^{-3}$
- interpolate in CoM space, then construct 4D object
- it includes resonance broadening and transitions from isolated to overlapping modes
- it is NOT yet self-consistent, i.e. ratio of mode amplitudes is fixed (radial envelope equation not solved)
- use E-conservation of PSZS transport equation to determine energy transfer to mode and change mode amplitude(s) accordingly

quasi-linear model: δB/B and thus <dP$_\Phi$/dt> change while gradients of F_{EP} flatten

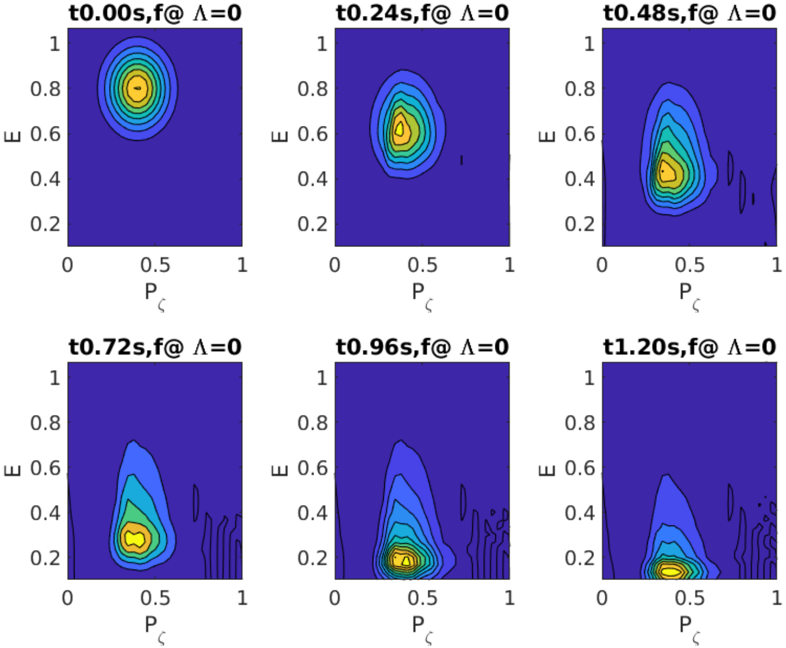


- energy conserving model - energy stored in F_{EP} gradients is converted into wave energy: non-linear hybrid model á la HAGIS (non-linear wave particle interaction Lagrangian)
- relative amplitudes of modes remain fixed, as given by linear growth rates (γ^2~A)
- here, no damping was used yet; mode growth stops after energy of F_{EP} has been exhausted
- for steady state, mode decay has to be balanced by collisions

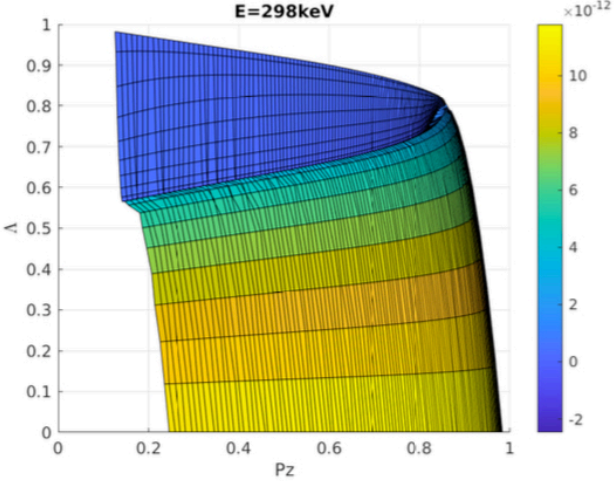
sources+ collisional transport in CoM space: ATEP-3D

[Guo Meng, poster at this conference, FEC 2023]

- collision operators are typically given in E, v// space (explicit pitch angle dependence)
- use framework above (IMAS based wrapper for HAGIS) with neoclassical HAGIS version [A. Bergmann, PoP 2001] to obtain orbit-averaged collision coefficients (linearised collision operator)
- use the same CoM grid as for PSZS part
- general 3D solver (implicit solver)
- details: poster *G. Meng, at this conference*

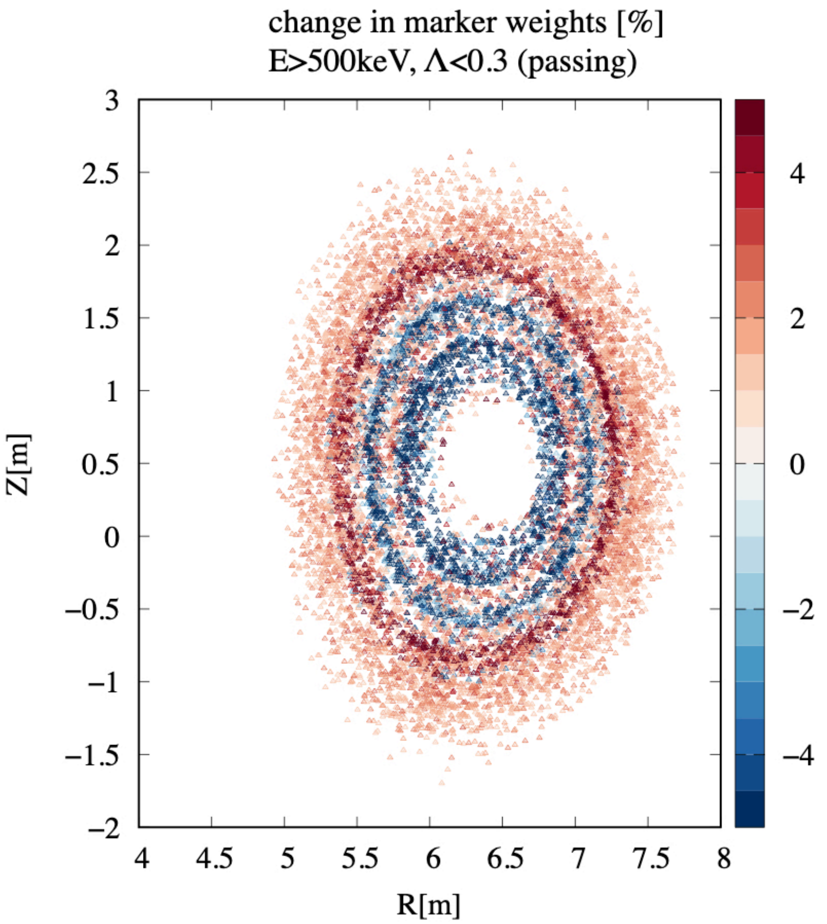


Collisional transport coefficient DPΦ



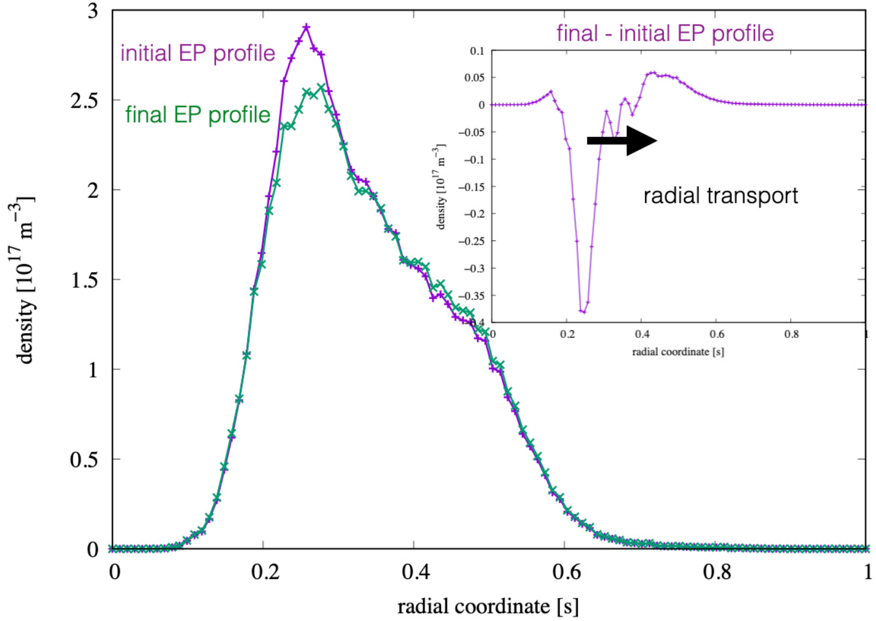
mapping back from CoM space to real space

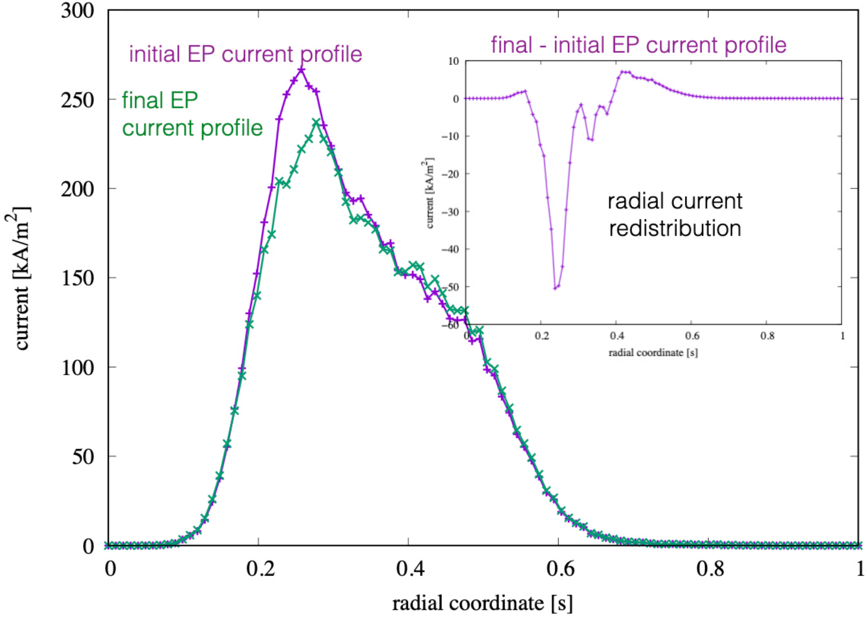
- use map created for setting up orbits quantities (see above) to assign new weights to markers as given by initial input from heating code or SD model
- only 'weights' in CoM are transported, not markers themselves
- transport is by construction 'zonal' - taking moments of evolved state allows us to define new non-linear equilibrium:



mapping back from CoM space to real space

back-mapping and calculating moments given EP transport in physical units:

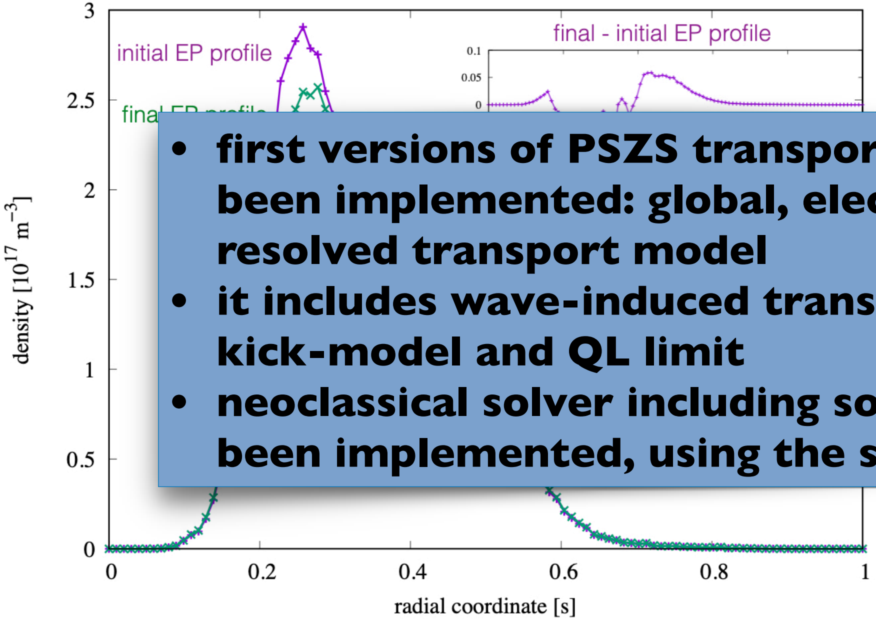


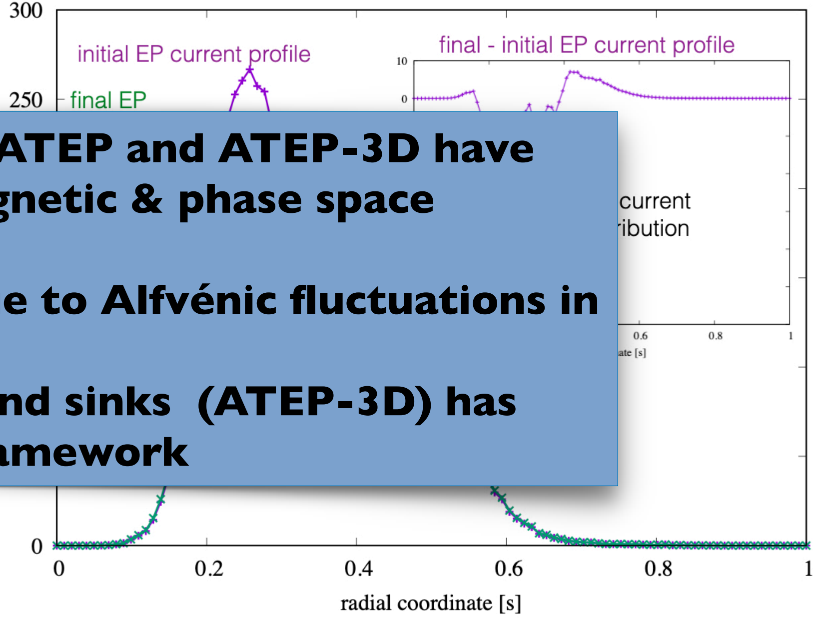


can be passed to transport/equilibrium code to calculate new consistent non-linear equilibrium

mapping back from CoM space to real space

back-mapping and calculating moments given EP transport in physical units:





- **first versions of PSZS transport code ATEP and ATEP-3D have been implemented: global, electromagnetic & phase space resolved transport model**
- **it includes wave-induced transport due to Alfvénic fluctuations in kick-model and QL limit**
- **neoclassical solver including sources and sinks (ATEP-3D) has been implemented, using the same framework**

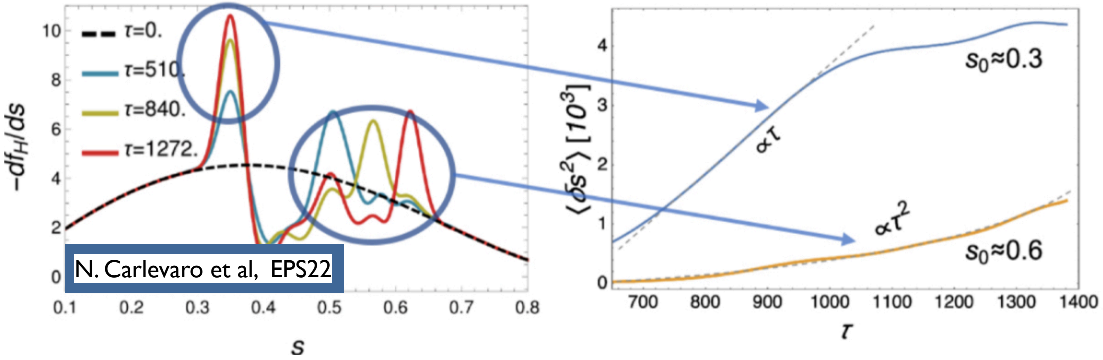
can be passed to transport/equilibrium code to calculate new consistent non-linear equilibrium

verify, validate and evolve models - ENR ATEP team effort

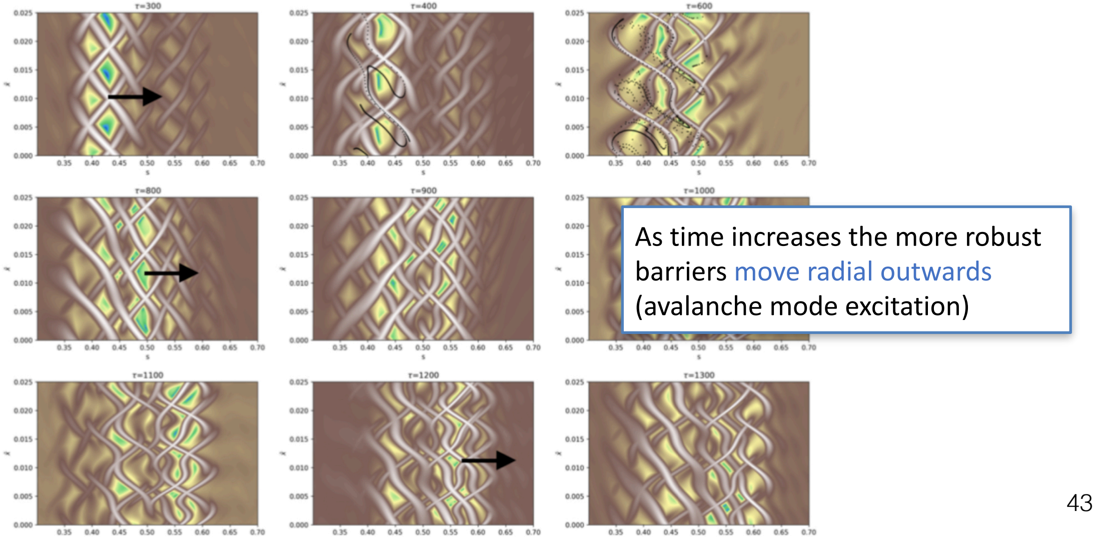
verify, validate and evolve models - ENR ATEP group

- benchmark with original HAGIS model
- benchmark with DAEPS code - calculates fluxes explicitly based on separation of radial and parallel mode structures
- started extension to 3D geometry [A. Zocco, 2023]
- benchmark with 1D beam-plasma system [N. Carlevaro, PPCF 2022]:
    - bump on tail model
    - partition phase space in slides of maximal power exchange
    - use LIGKA linear mode information
    - successful comparison with LIGKA-HAGIS model
- tracers dynamics studied with Lagrangian Coherent structures: relevant structures/barriers change during non-linear evolution: from inner to outer radial transport peak (see ITER case above):

add tracers to system an determine diffusive (τ) vs. convective (τ^2) scaling:



N. Carlevaro et al, EPS22



As time increases the more robust barriers move radial outwards (avalanche mode excitation)

verify, validate and evolve models - ENR ATEP group

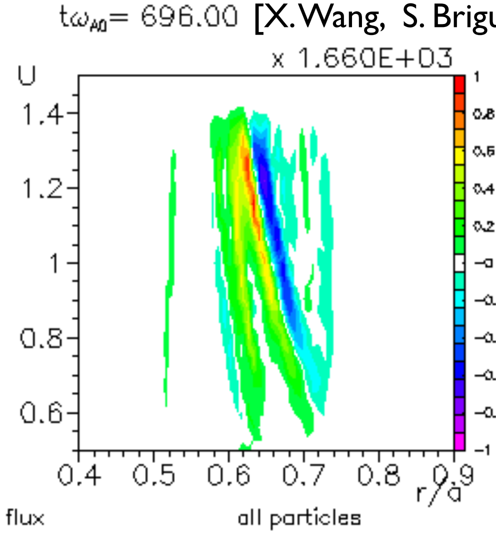
- benchmark with XHMGC calculations, featuring advanced features for transport analysis: Hamiltonian mapping diagnostics & explicit flux 'measurements'
- implemented also in HYMAGYC [G.Vlad, V. Fusco]

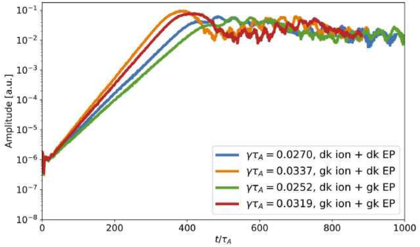
- benchmark with STRUPHY code: MHD-kinetic hybrid code based on new stringent mathematical formulation: structure preserving geometric finite elements + PIC ⇒ improved non-linear stability [F Holderried, S Possanner 2020-2023]

- compare with ORB5 PSZS diagnostics [A. Bottino Varenna 2022] *(see talk M. Falessi)*
  compare to various ORB5 results; e.g. use scaling for chirping modes
  ORB5 runs are available also in presence of turbulence

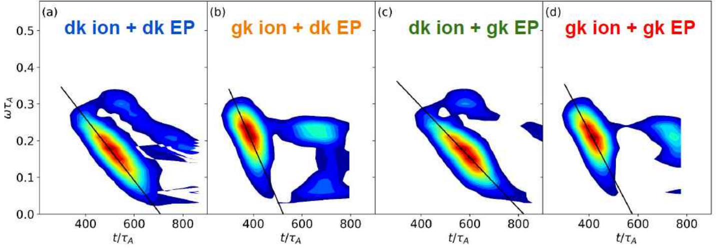
- analyse and plan new experiments based on AUG EP 'Supershots'
  INPA measurements of
  phase space transport !
  [J. R. Rueda, FEC 2023]

[X.Wang, S. Briguglio et al 2021]





[ORB5: X Wang et al, 2022-23]

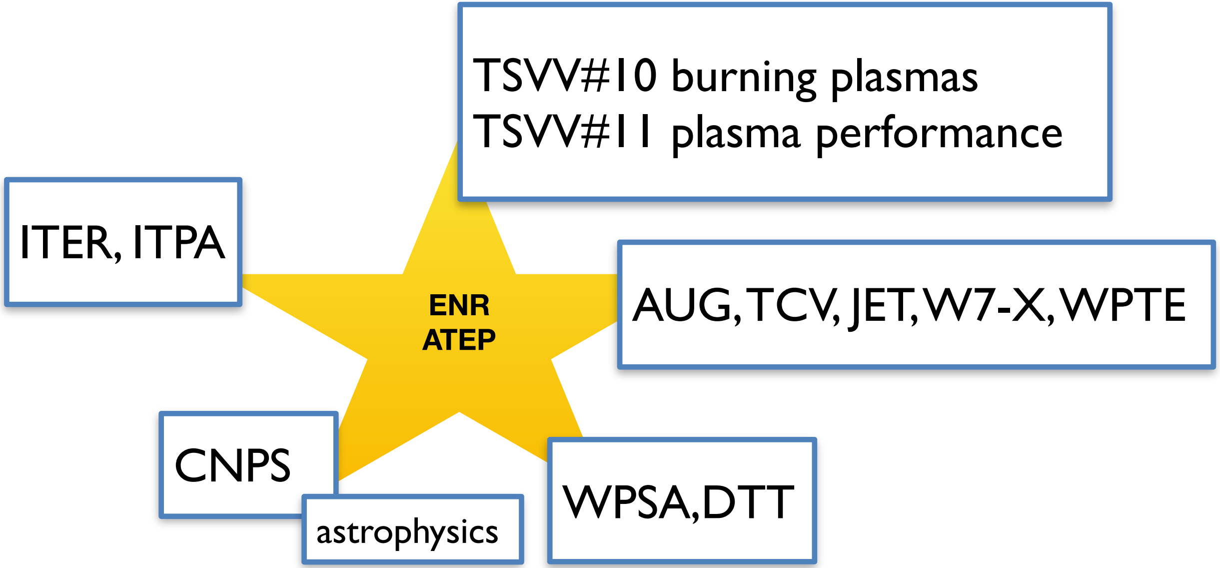


summary

started enable new routes to EP transport analysis and prediction via:

- new theoretical framework
- new common concept of connecting non-linear code results to reduced models (PSZS)
- new common EP transport code developments
- newly implemented analysis methods
- new IMAS based infrastructure

established and growing connections to other groups and experiments



ENR ATEP

TSVV#10 burning plasmas
TSVV#11 plasma performance

ITER, ITPA

AUG, TCV, JET, W7-X, WPTE

CNPS

astrophysics

WPSA, DTT