

Status of Work on JOREK at IPP Garching

Matthias Hölzl

1 Numerics

2 ELMs

3 Resistive Wall

4 Summary

1 Numerics

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4 Summary

- Implicitness: Time step not restricted by CFL condition
- Convergence: Preconditioner inefficient with strong non-linearities
- Efficiency: Limited scalability and large memory consumption due to direct solver in preconditioner

What to do?

Linearization → Newton-iterations: Better convergence?

GMRES → Jacobian-free: Simpler and more flexible implementation?

Preconditioner → Matrix-free: Improve efficiency?

→ New IPP department on numerics: **Eric Sonnendrücker**

→ Postdoc working on JOREK: **Emmanuel Franck**

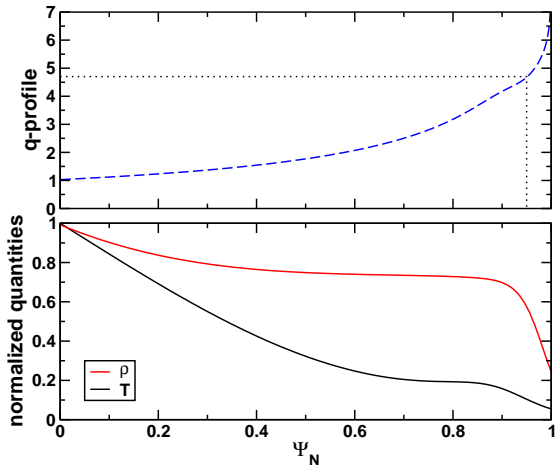
1 Numerics

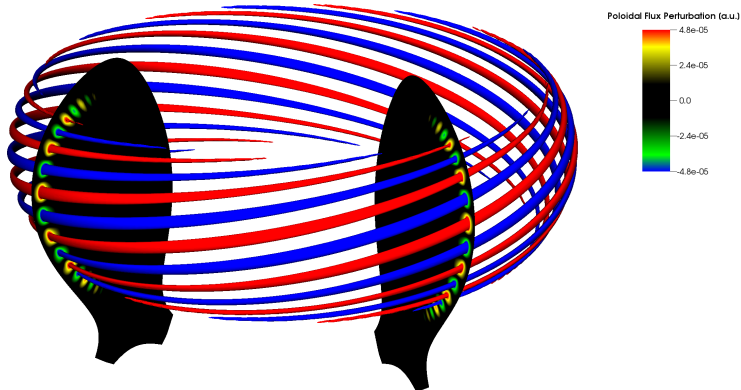
2 ELMs

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4 Summary

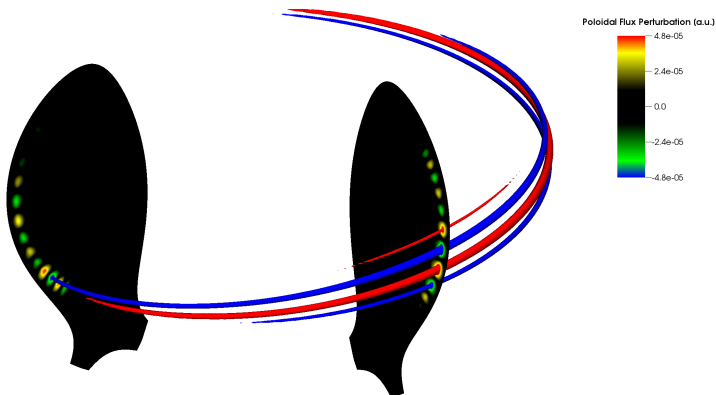
- ELMs in typical ASDEX Upgrade H-mode equilibrium
- Focus on early phase until non-linear saturation starts





$$n = 0, 8, 16$$

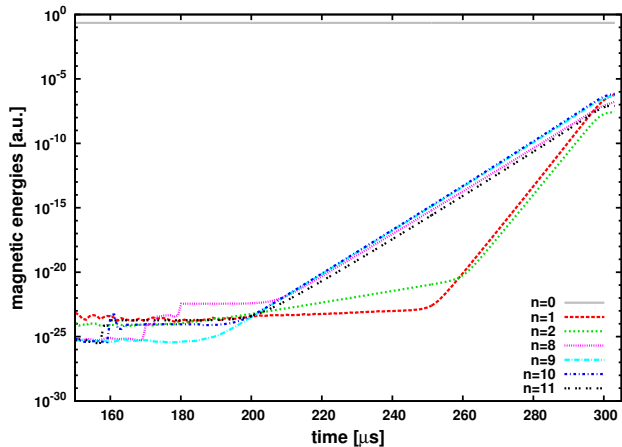
- Red/blue surfaces correspond to 70 percent of maximum/minimum values



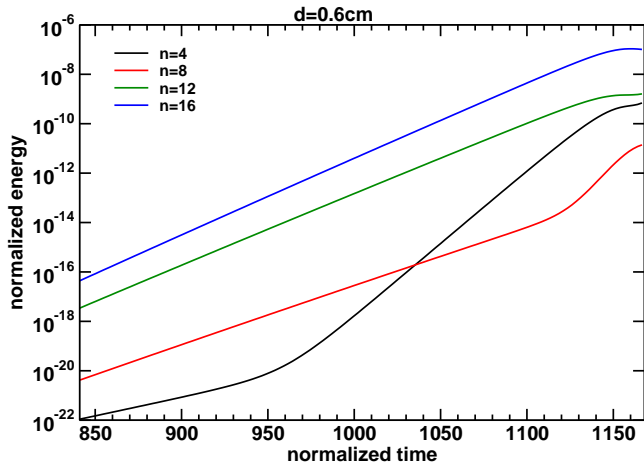
$$n = 0, 1, 2, 3, 4, \dots, 16$$

- Red/blue surfaces correspond to 70 percent of maximum/minimum values
- Localized due to several strong modes with adjacent n
- ~ Solitary Magnetic Perturbations in ASDEX Upgrade [Wenninger et al. \(2012\)](#)

- $n = 1$ (and others) driven non-linearly to large amplitude (Subdominant modes not shown for clarity)



- Consider a case with $n = 0, 4, 8, 12, 16$ for the start
- Can we reproduce and understand this with a simple model?



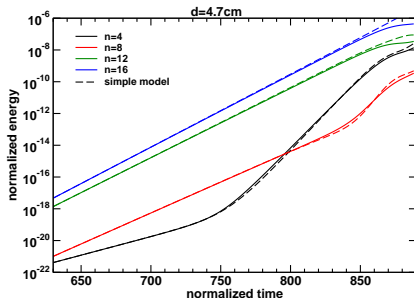
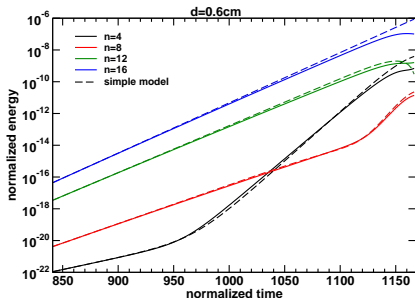
- Non-linear terms lead to mixing of toroidal modes
- Quadratic: $(n_1, n_2) \leftrightarrow n_1 \pm n_2$
- For instance: $(8, 4)$, $(12, 8)$, and $(16, 12)$ couple to $n = 4$
- Simple model (Mode rigidity, $n = 0$ fixed):

$$\dot{A}_4 = \underbrace{\gamma_4 A_4}_{\text{linear}} + \underbrace{\gamma_{8,-4} A_8 A_4 + \gamma_{12,-8} A_{12} A_8 + \gamma_{16,-12} A_{16} A_{12}}_{\text{non-linear interaction}}$$

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- Linear growth rates taken from JOREK simulation
- Energy conservation \Rightarrow Six remaining free parameters $\gamma_{i,j}$
- Determine free parameters numerically by minimizing quadratic difference



- Saturation not covered by the model (of course)
- Same set of interaction-parameters $\gamma_{i,j}$ for both wall-distances
- Non-linear growth described well
- Same mechanism brings up $n = 1$ in the simulations shown before with poloidally and toroidally localized ELMs!

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- Plasma instabilities interact with conducting structures via eddy currents
- For instance: External kink ($\sim \mu\text{s}$) \rightarrow Resistive wall mode ($\sim \text{ms}$)
- Aim: Non-linear resistive wall simulations
- Coupling of JOREK and STARWALL codes
- Showing status of implementation and benchmarking

- Developed by Peter Merkel ([Merkel and Sempf \(2006\)](#); [Strumberger et al. \(2011\)](#))
- Executed once for a JOEREK simulation
- Conducting structures represented by triangles (3D structures including holes possible)
- Wall currents described by current potentials Y_k at triangle nodes
- Vacuum field equation solved outside JOEREK domain (for set of unit Ψ -perturbations corresponding to Bezier DOFs)
- Expression for B_{tan} in terms of Ψ at the interface (response matrices)

Resistive Wall Natural boundary condition

- Current definition equation $j = \Delta^* \Psi$ in weak form (test function v^*):

$$\int dV \frac{v^*}{R^2} j - \int dV v^* \nabla \cdot \left(\frac{1}{R^2} \nabla \Psi \right) = 0$$

- Partial integration:

$$\int dV \frac{v^*}{R^2} j + \int dV \frac{1}{R^2} \nabla v^* \cdot \nabla \Psi - \oint dA \frac{v^*}{R} \underbrace{(\nabla \Psi \cdot \hat{\mathbf{n}}/R)}_{\equiv B_{\text{tan}}} = 0.$$

- Ideal-wall boundary conditions: Boundary integral vanishes in “old” JOEREK
- Natural boundary condition: Replace B_{tan} by STARWALL response

- Ideal wall (algebraic expression):

$$B_{\text{tan}} = \sum_i b_i \sum_j \hat{M}_{i,j}^{\text{id}} \Psi_j$$

- Vacuum response: \hat{M}^{id}

Resistive Wall Vacuum Response

- Ideal wall (algebraic expression):

$$B_{\text{tan}} = \sum_i b_i \sum_j \hat{M}_{i,j}^{\text{id}} \Psi_j$$

- Resistive wall:

$$B_{\text{tan}} = \sum_i b_i \left(\sum_j \hat{M}_{i,j}^{\text{ee}} \Psi_j + \sum_k \hat{M}_{i,k}^{\text{ey}} Y_k \right)$$

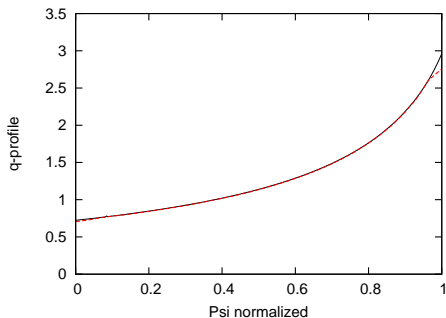
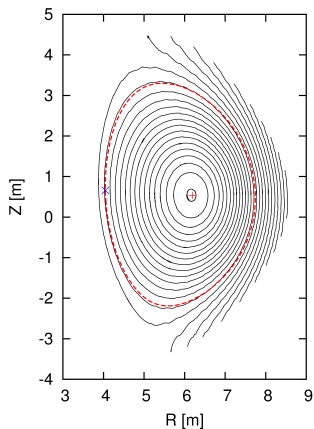
$$\dot{Y}_k = -\frac{\eta_w}{d_w} \hat{M}_{k,k}^{\text{yy}} Y_k - \sum_j \hat{M}_{k,j}^{\text{ye}} \dot{\Psi}_j$$

- Vacuum response: \hat{M}^{id} , \hat{M}^{ee} , \hat{M}^{ey} , \hat{M}^{ye} , \hat{M}^{yy}
- Discretize consistent with fully-implicit time evolution scheme

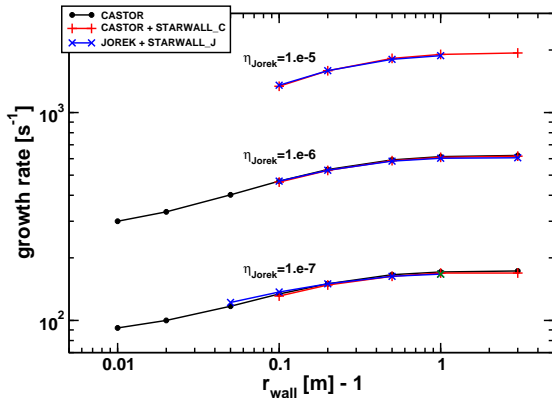
- Plug B_{tan} into boundary integral $\oint dA \frac{j_1^*}{R} \underbrace{(\nabla \Psi \cdot \hat{\mathbf{n}}/R)}_{\equiv B_{\text{tan}}}$

- Freeboundary equilibrium for ITER-like limiter case: Same boundary-integral
- Allows to test parts (no time-evolution, no wall-currents)
- Flux-surfaces and q -profile agree very well with CEDRES++

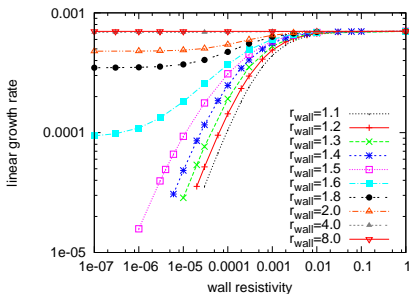
JOREK — CEDRES - - - limiter point ×



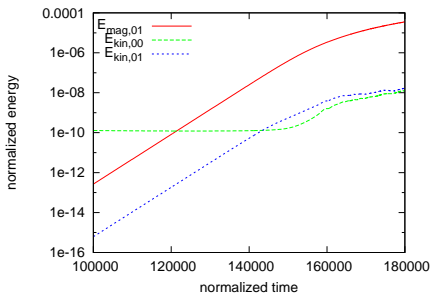
- 2/1 tearing mode in circular plasma ($R = 10$, $\alpha = 1$) with ideal wall
- Linear growth rates agree very well with CASTOR



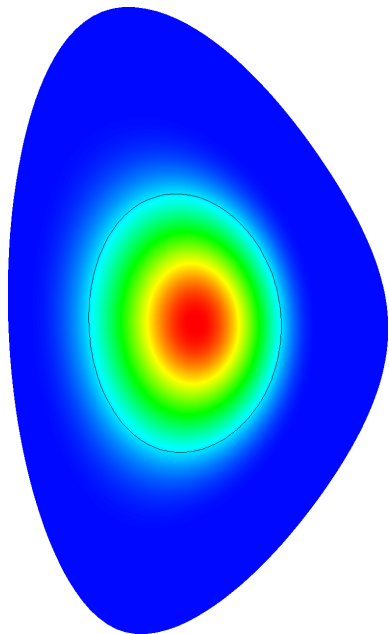
- 2/1 external kink mode in circular plasma with resistive wall
- To be compared to analytical theory and linear simulations (with R. McAdams, I. Chapman)



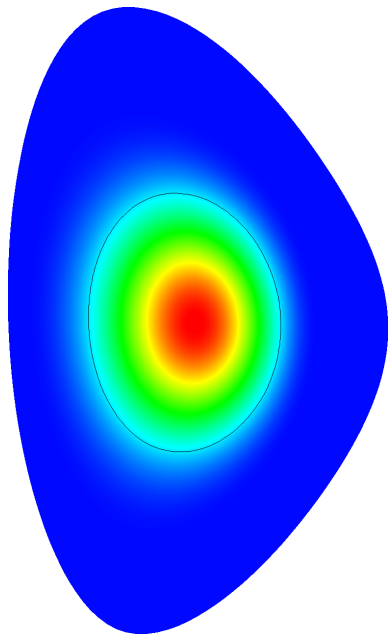
Linear growth rates for different wall radii and wall resistivities



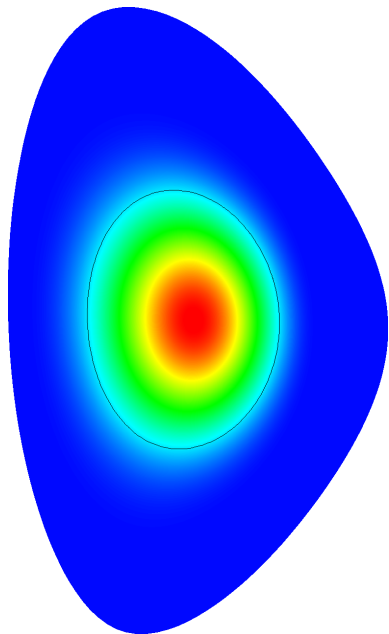
Example for non-linear saturation (stable, small time-steps for convergence)



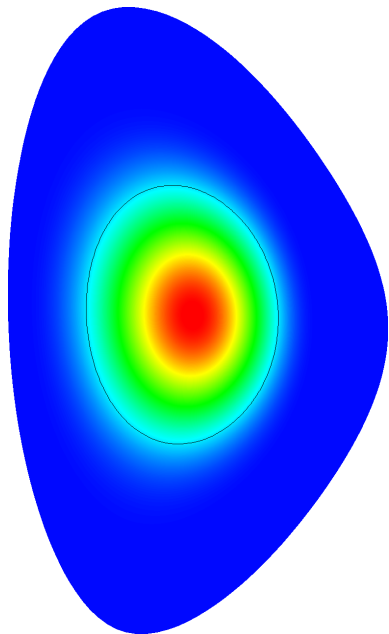
- Vertical displacement event (VDE)
- Start from ITER-like limiter case
- Compare to CEDRES (Eric Nardon)
- Problems at coordinate axis (already very high viscosity)!
- Gets worse for small timesteps



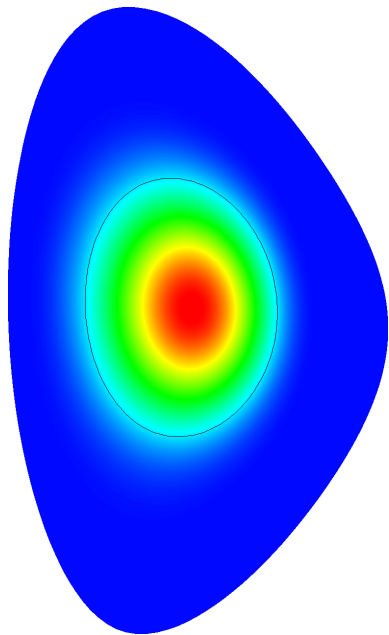
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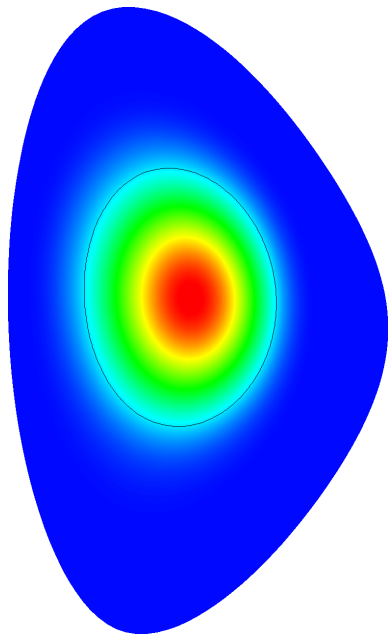
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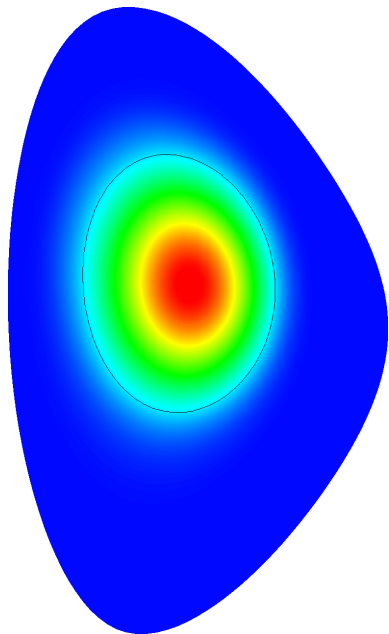
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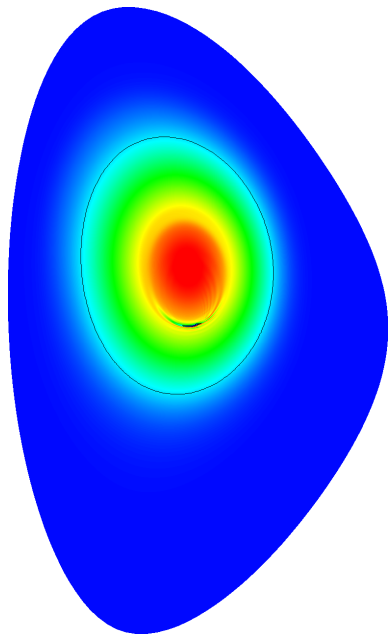
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Summary

Numerics

→ New Postdoc working on time-stepping / solver

ELM simulations for ASDEX Upgrade

- Poloidal and toroidal localization [Hözl et al. \(2012a\)](#)
- Low-n grow non-linearly [Krebs et al. \(2013\)](#)

→ ELM-types? Affected region? Heat-flux patterns?

Resistive-wall model [Hözl et al. \(2012b\)](#)

- Coupling to STARWALL
- Benchmarks ongoing

→ Non-linear simulations of RWMs, VDEs, ...

References

- O. Czarny and G. Huysmans. *Journal of Computational Physics*, 227, 7423 (2008).
- M. Hölzl, S. Günter, R. P. Wenninger, et al. *Physics of Plasmas*, 19, 082505 (2012a).
- M. Hölzl, P. Merkel, G. Huysmans, et al. *Journal of Physics: Conference Series*, 401, 012010 (2012b).
doi:10.1088/1742-6596/401/1/012010.
- G. Huysmans and O. Czarny. *Nuclear Fusion*, 47, 659 (2007).
- I. Krebs, M. Hölzl, K. Lackner, et al. to be published (2013).
- P. Merkel and M. Sempf. In *Proceedings of the 21st IAEA Fusion Energy Conference*. Chengdu, China (2006). TH/P3-8.
- E. Strumberger, P. Merkel, C. Tichmann, et al. In *Proceedings of the 38th EPS Conference on Plasma Physics*. Strasbourg, France (2011). P5.082.
- R. Wenninger, H. Zohm, J. Boom, et al. *Nuclear Fusion*, 42, 114025 (2012).

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Acknowledgements

P. Merkel
I. Krebs
G. Huysmans
K. Lackner
S. Günter
E. Nardon
E. Strumberger

- Non-linear MHD code
- Divertor tokamaks including X-point(s)
- Focus on plasma edge simulations
- Originally developed at CEA Cadarache [Huysmans and Czarny \(2007\)](#)
- Reduced MHD in toroidal geometry (next slide)
- Other models:
 - Two-fluid extensions (M. Becoulet, S. Pamela)
 - Neutrals (C. Reux)
 - Full MHD
- Fortran 90/95
- MPI + OpenMP hybrid parallelization

Reduced MHD Equations

$$\frac{\partial \Psi}{\partial t} = \eta j - R [\mathbf{u}, \Psi] - F_0 \frac{\partial \mathbf{u}}{\partial \phi}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) + \nabla \cdot (D_{\perp} \nabla_{\perp} \rho) + S_{\rho}$$

$$\frac{\partial (\rho T)}{\partial t} = -\mathbf{v} \cdot \nabla (\rho T) - \gamma \rho T \nabla \cdot \mathbf{v} + \nabla \cdot (K_{\perp} \nabla_{\perp} T + K_{\parallel} \nabla_{\parallel} T) + S_T$$

$$\mathbf{e}_{\phi} \cdot \nabla \times \left\{ \rho \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla p + \mathbf{j} \times \mathbf{B} + \mu \Delta \mathbf{v} \right\}$$

$$\mathbf{B} \cdot \left\{ \rho \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla p + \mathbf{j} \times \mathbf{B} + \mu \Delta \mathbf{v} \right\}$$

$$\mathbf{j} \equiv -\mathbf{j}_{\phi} = \Delta^* \Psi$$

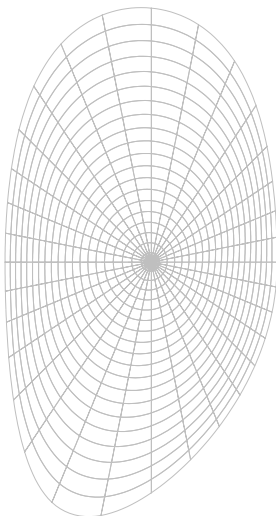
$$\boldsymbol{\omega} \equiv -\boldsymbol{\omega}_{\phi} = \nabla_{\text{pol}}^2 \mathbf{u}$$

Variables: $\Psi, \mathbf{u}, \mathbf{j}, \boldsymbol{\omega}, \rho, T, v_{\parallel}$

Ideal wall + Bohm boundary conditions

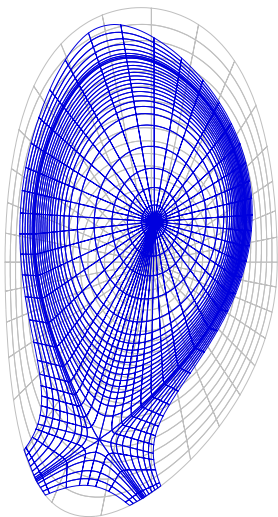
Definitions: $\mathbf{B} = \frac{R_0}{R} \mathbf{e}_{\phi} + \frac{1}{R} \nabla \Psi \times \mathbf{e}_{\phi}$ and $\mathbf{v} = -R \nabla \mathbf{u} \times \mathbf{e}_{\phi} + v_{\parallel} \mathbf{B}$

Typical code run



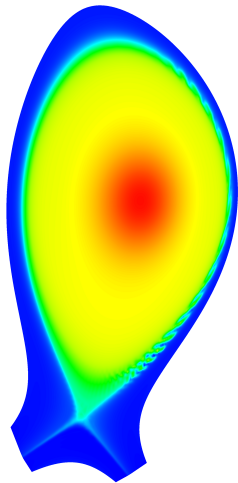
- Initial grid (Grids shown with reduced resolution)
- Equilibrium data (F_0 , Ψ_{bnd} , profiles for T , ρ , FF')
- Grad-Shafranov
- Flux aligned grid (may include X-points)
- Radial and poloidal grid meshing
- Grad-Shafranov
- Axisymmetric flows
- Time-integration
- Analysis of restart-files:
 - Poincare plots
 - 2D or 3D VTK files
 - ...

Typical code run



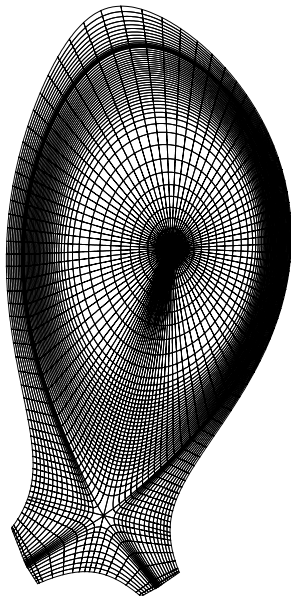
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Flux-aligned X-point Grid



$$\frac{\partial p}{\partial t} = -\mathbf{v} \cdot \nabla p - \gamma p \nabla \cdot \mathbf{v} + \text{Diffusion} + \text{Source}$$

$$\frac{\partial(\rho T)}{\partial t} = -\mathbf{v} \cdot \nabla(\rho T) - \gamma \rho T \nabla \cdot \mathbf{v} + \nabla \cdot (\mathbf{K}_{\perp} \nabla_{\perp} T + \mathbf{K}_{\parallel} \nabla_{\parallel} T) + S_T$$

Spatial Discretization

- Toroidal Fourier-decomposition
- 2D Bezier finite elements (isoparametric, C^0 and C^1)

Czarny and Huysmans (2008)

- Remaining problems:
 - Fourier basis-functions non-local (N^2 matrix entries)
 - Problems at axis and X-point (not C^1)
 - Positivity not guaranteed

Time evolution: Fully implicit

- Time-Discretization ($\mathbf{u}^{n+1} = \mathbf{u}^n + \delta\mathbf{u}^n$):

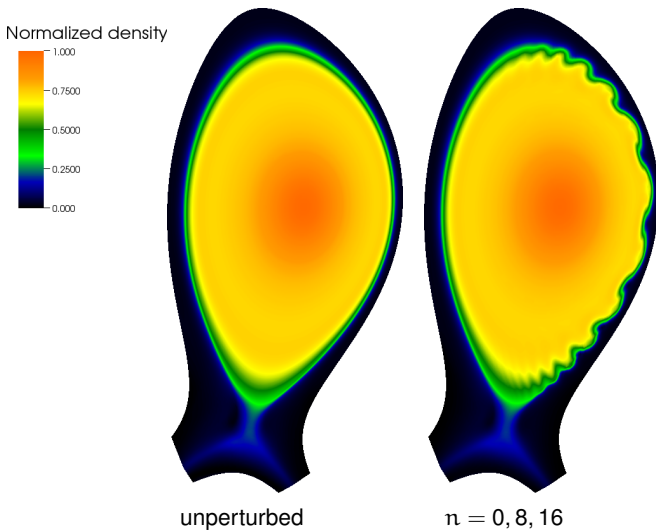
$$\dot{\mathbf{A}}(\mathbf{u}(t)) = \mathbf{B}(\mathbf{u}(t))$$

$$\left[(1 + \xi) \left(\frac{\partial \mathbf{A}}{\partial \mathbf{u}} \right)^n - \Delta t \theta \left(\frac{\partial \mathbf{B}}{\partial \mathbf{u}} \right)^n \right] \delta \mathbf{u}^n = \Delta t \mathbf{B}^n + \xi \left(\frac{\partial \mathbf{A}}{\partial \mathbf{u}} \right)^{n-1} \delta \mathbf{u}^{n-1}$$

- Crank-Nicholson: ($\theta = 0.5, \xi = 0$) or Gears: ($\theta = 1, \xi = 0.5$)
 - Involves a linearization (Newton iteration stopped after one step)
- ⇒ Large sparse system of equations solved iteratively with GMRES

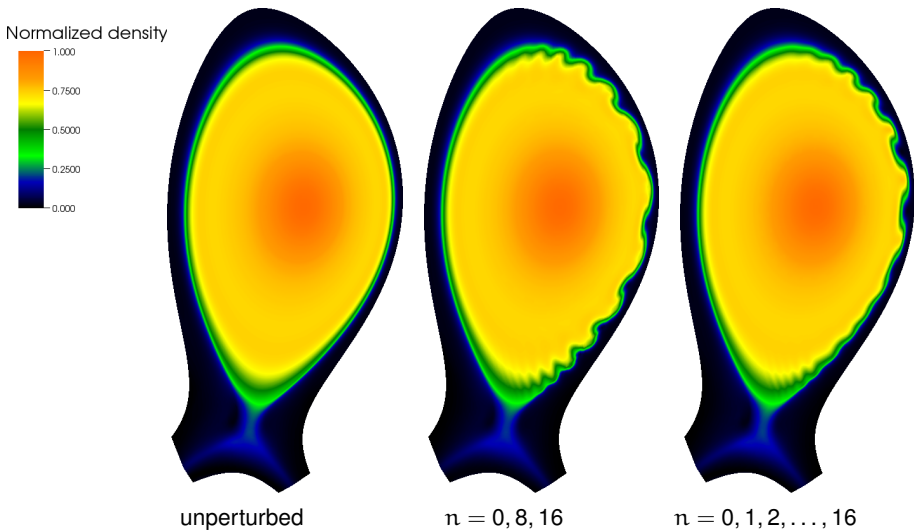
Ballooning Structure

- Mode-coupling causes localization of ballooning-filaments:



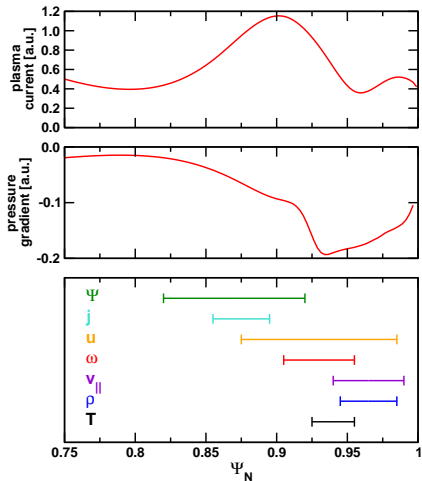
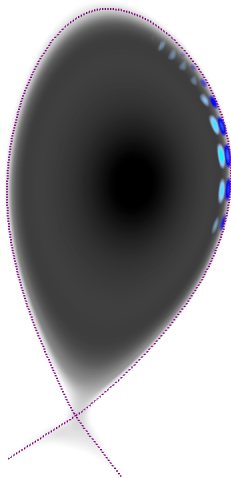
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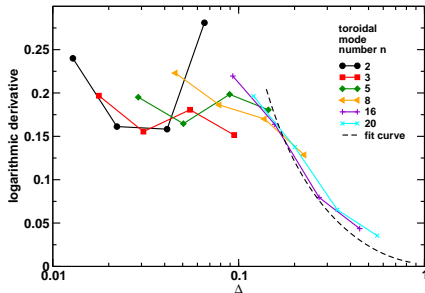
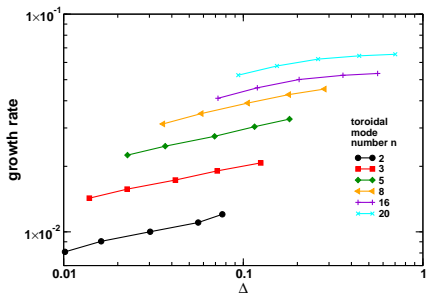


Position of Perturbations

- Radial perturbation positions differ between variables

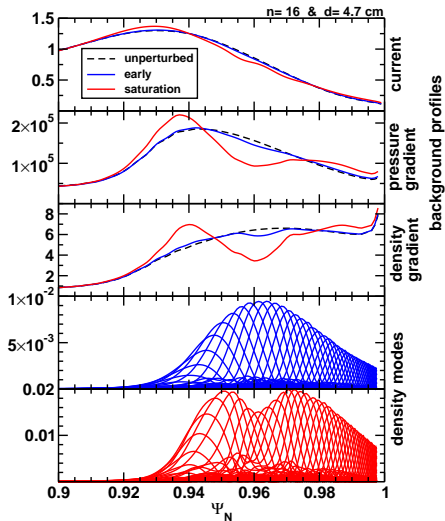


Influence of Wall



- Linear growth rate influenced by ideal wall boundary condition
- $\Delta = \text{wall distance} / \text{poloidal mode half-width}$
- Wall effects become negligible if $\Delta \gtrsim 0.4$

Saturation



Saturation mechanisms:

- Displacement ξ gets significant compared to wall distance
- Modification of background profiles by the instability

Time Discretization

- Discretize consistently with other JOEREK equations ($Y_k^{n+1} = Y_k^n + \delta Y_k^n$):

$$\begin{aligned}
 & (1 + \xi) \left[\delta Y_k^n + \sum_j \hat{M}_{k,j}^{ye} \delta \Psi_j^n \right] + \Delta t \theta \frac{\eta_w}{d_w} \hat{M}_{k,k}^{yy} \delta Y_k^n \\
 & = - \Delta t \frac{\eta_w}{d_w} \hat{M}_{k,k}^{yy} Y_k^n + \xi \left[\delta Y_k^{n-1} + \sum_j \hat{M}_{k,j}^{ye} \delta \Psi_j^{n-1} \right]
 \end{aligned}$$

- Crank-Nicholson: ($\theta = 0.5, \xi = 0$) or Gears: ($\theta = 1, \xi = 0.5$)
- Solve for δY_k^n and insert into B_{tan} at time-step $n + 1$:

$$B_{\text{tan}}^{n+1} = \sum_i b_i \left[\sum_j \hat{M}_{i,j}^{ee} \cdot (\Psi_j^n + \delta \Psi_j^n) + \sum_k \hat{M}_{i,k}^{ey} \cdot (Y_k^n + \delta Y_k^n) \right]$$

- Plug result into boundary integral $\oint dA \frac{j_1^*}{R} \underbrace{(\nabla \Psi \cdot \hat{\mathbf{n}}/R)}_{\equiv B_{\text{tan}}}$

formulation...

$$\begin{aligned}
 & \sum_{i_{\text{elem}}} \int \frac{dV}{R^2} (j_i^* \delta j^n + \nabla j_i^* \cdot \nabla \delta \Psi^n) - \sum_{i_{\text{bnd}}} \oint dA \frac{j_i^*}{R} \sum_i b_i \sum_j \hat{E}_{i,j} \delta \Psi_j^n \\
 &= - \sum_{i_{\text{elem}}} \int \frac{dV}{R^2} (j_i^* j^n + \nabla j_i^* \cdot \nabla \Psi^n) \\
 & \quad + \sum_{i_{\text{bnd}}} \oint dA \frac{j_i^*}{R} \sum_i b_i \left[\sum_k (\hat{F}_{i,k} Y_k^n + \hat{G}_{i,k} \delta Y_k^{n-1}) + \sum_j (\hat{H}_{i,j} \Psi_j^n + \hat{J}_{i,j} \delta \Psi_j^{n-1}) \right]
 \end{aligned}$$

and

$$Y_k^{n+1} = Y_k^n + \sum_j \hat{A}_{k,j} \delta \Psi_j^n + \hat{B}_{k,k} Y_k^n + \hat{C}_{k,k} \delta Y_k^{n-1} + \sum_j \hat{D}_{k,j} \delta \Psi_j^{n-1}$$

where

$$\begin{aligned}
 \hat{S}_{k,k} &= 1 + \xi + \Delta t \theta \frac{\eta w}{d_w} \hat{M}_{k,k}^{yy} & \hat{D}_{k,j} &= \xi \hat{M}_{k,j}^{ye} / \hat{S}_{k,k} & \hat{H}_{i,j} &= \hat{M}_{i,j}^{ee} \\
 \hat{A}_{k,j} &= -(1 + \xi) \hat{M}_{k,j}^{ye} / \hat{S}_{k,k} & \hat{E}_{i,j} &= \hat{M}_{i,j}^{ee} + \sum_k \hat{M}_{i,k}^{ey} \hat{A}_{k,j} & \hat{J}_{i,j} &= \sum_k \hat{M}_{i,k}^{ey} \hat{D}_{k,j} \\
 \hat{B}_{k,k} &= -\Delta t \frac{\eta w}{d_w} \hat{M}_{k,k}^{yy} / \hat{S}_{k,k} & \hat{F}_{i,k} &= \hat{M}_{i,k}^{ey} (1 + \hat{B}_{k,k}) \\
 \hat{C}_{k,k} &= \xi / \hat{S}_{k,k} & \hat{G}_{i,k} &= \hat{M}_{i,k}^{ey} \hat{C}_{k,k}
 \end{aligned}$$

and

$$\int dV = \sum_{i_{\text{elem}}} \int ds dt d\phi J_2 R \qquad \oint dA = \sum_{i_{\text{bnd}}} \int dt d\phi R \sqrt{\left(\frac{\partial R}{\partial t}\right)^2 + \left(\frac{\partial Z}{\partial t}\right)^2}$$

Generalized Minimum Residual Method (GMRES)

- $\hat{M} \mathbf{x} = \mathbf{b}$ solved iteratively with GMRES
- Requests the following operations via reverse communication (“black box”):
 - Matrix-vector product \rightarrow BLAS with OpenMP parallelization
 - Dot-product between two vectors \rightarrow BLAS
 - Left preconditioning (right currently not used)

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Left Preconditioning

- Need to solve $\hat{P} \mathbf{f} = \mathbf{g}$
 - \hat{P} : Matrix \hat{M} without coupling terms between toroidal harmonics
- \Rightarrow Block-diagonal matrix with few large sparse blocks
- Solve decoupled block-systems with direct solver PaStiX