



Modeling of Diffusive Heat Transport across Magnetic Islands and Stochastic Layers

Matthias Hölzl

Outline

- 1 Motivation and Introduction
- 2 Numerical Model
- 3 Magnetic Islands
- 4 Ergodic Layers
- 5 Comparison to Experiment
- 6 Conclusions and Outlook

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Heat Conduction

Perpendicular Transport

- Limited mobility perpendicular to magnetic field lines
- Dominated by gradient-driven turbulence

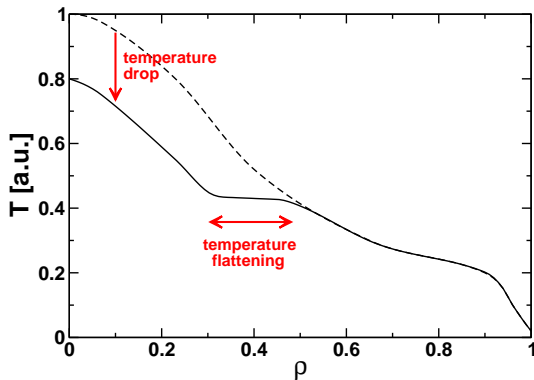
$$\Rightarrow \chi_{\perp} = \mathcal{O}(1 \text{ m}^2/\text{s})$$

Parallel Transport

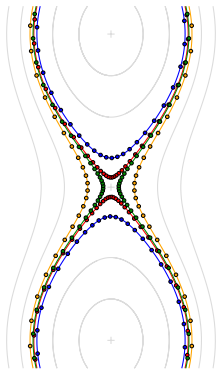
- High mobility along field lines
- **Spitzer Härm-conductivity**: Random walk-process of electrons with step width = mean free path
- But: Mean free path $\mathcal{O}(\text{ km })!$
- **“Heat-Flux-Limit”**: Limit to free streaming electrons

$$\Rightarrow \chi_{\parallel} = ?$$

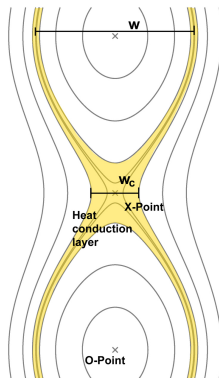
Transport across magnetic islands



Transport across magnetic islands



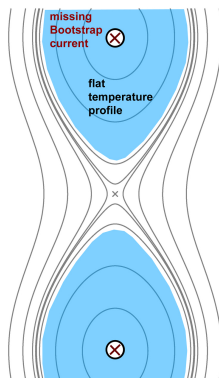
Transport across magnetic islands



[R. Fitzpatrick *Phys. Plasmas* 2 825 (1995)]

- Competition between parallel and perpendicular transport
- Scale island width $w_c \propto (\chi_{\parallel}/\chi_{\perp})^{-1/4}$ ◦
- Flattening of temperature profile depends on w/w_c only

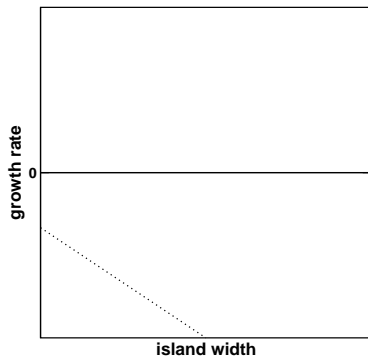
Neoclassical Tearing Modes



Consequences of temperature perturbation

- Resonant perturbation of Bootstrap current ($j_{bs} \propto \nabla p$)
 - Effective lack current causes further island growth
- ⇒ Neoclassical Tearing Mode (NTM)

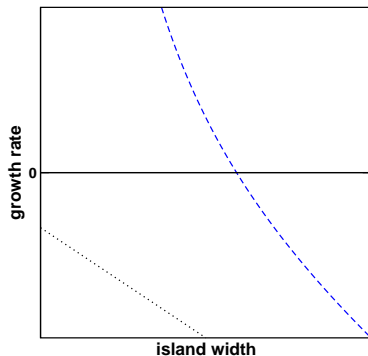
Evolution of (Neoclassical) Tearing Modes



Rutherford Equation

$$\frac{dw}{dt} \propto \Delta'(w)$$

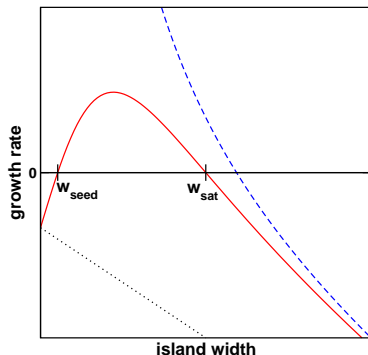
Evolution of (Neoclassical) Tearing Modes



Rutherford Equation

$$\frac{dw}{dt} \propto \Delta'(w) + \underbrace{\frac{C}{w}}_{\Delta'_{bs}}$$

Evolution of (Neoclassical) Tearing Modes

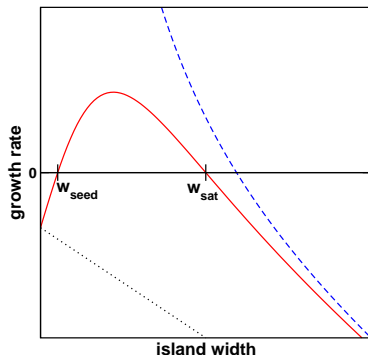


Rutherford Equation

$$\frac{dw}{dt} \propto \Delta'(w) + \underbrace{\frac{C}{w} \left(\frac{w^2}{w^2 + (1.8w_c)^2} \right)}_{\Delta'_{bs}}$$

$$w_c \propto \left(\frac{\chi_{||}}{\chi_{\perp}} \right)^{-1/4}$$

Evolution of (Neoclassical) Tearing Modes



Rutherford Equation

$$\frac{dw}{dt} \propto \Delta'(w) + \underbrace{\frac{C}{w} \left(\frac{w^2}{w^2 + (1.8w_c)^2} \right)}_{\Delta'_{bs}} + \Delta'_{pol} + \dots \quad w_c \propto \left(\frac{\chi_{||}}{\chi_{\perp}} \right)^{-1/4}$$

Rutherford Equation

Open questions

- Is Fitzpatrick's expression for Δ'_{bs} correct?

$$\Delta'_{bs} = \frac{C}{w} \left(\frac{w^2}{w^2 + (1.8w_c)^2} \right)$$

- How large is the heat diffusion anisotropy in experiments (determines w_c)?

Approach

- Compute diffusive heat transport numerically
- Compare to predictions and measurements

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Anisotropic heat conduction

Heat conduction equation

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} + \nabla \cdot \mathbf{q}_e = P_e$$

$$\mathbf{q}_e = -n_e [\chi_{||} \nabla_{||} T_e + \chi_{\perp} \nabla_{\perp} T_e]$$

n_e electron density

T_e electron temperature

\mathbf{q}_e heat flux density

P_e heat source

$\chi_{||}$ parallel- and

χ_{\perp} perpendicular heat diffusivity

$\chi_{||} / \chi_{\perp}$ heat diffusion anisotropy

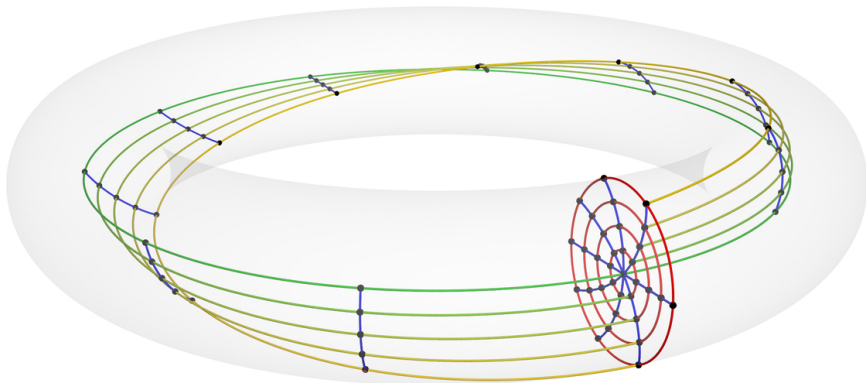
Numerical method

- Problem: Numerical diffusion
- Possibility: Align coordinates to field lines
Hard to do with dynamic equilibria or ergodization
- Other approach: Symmetric finite differences with staggered grids

[S. Günter et.al. *J. Comput. Phys.* **209** 354 (2005)]

⇒ No exact alignment of coordinates required

Helical coordinate system

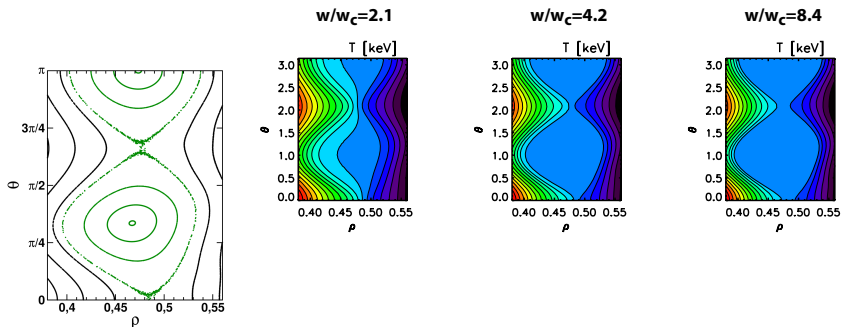


- Flux and Straight field line coordinates
 - Transformation of poloidal coordinate
- ⇒ **Unsheared helical coordinate system**

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Temperature flattening



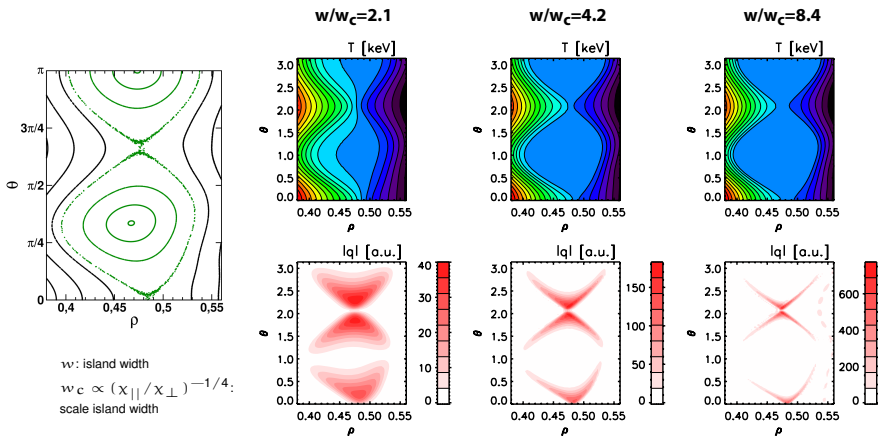
w : island width

$w_c \propto (\chi_{||} / \chi_{\perp})^{-1/4}$:
scale island width

[M. Hölzl et.al. *Phys. Plasmas* **15** 072514 (2008)]

- 3/2-island in ASDEX Upgrade
- Flattening for $w/w_c \gtrsim 2$

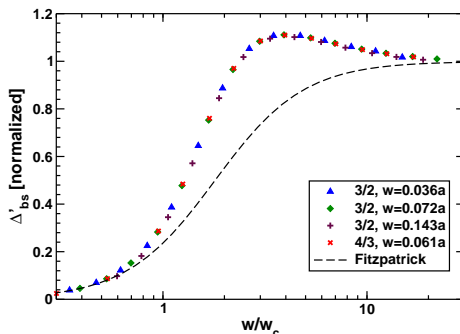
Temperature flattening



[M. Hölzl et.al. *Phys. Plasmas* **15** 072514 (2008)]

- 3/2-island in ASDEX Upgrade
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Neoclassical island drive



[M. Hölzl et.al. *Phys. Plasmas* **14** 052514 (2007)]

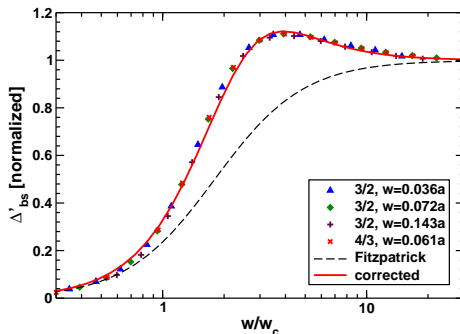
NTM stability

- Destabilization by temperature flattening underestimated by Fitzpatrick (limiting cases agree very well) ^o

$$\Delta'_{bs} = \frac{C}{w} \left(\frac{w^2}{w^2 + w_d^2} \right)$$

$$w_d = 1.8w_c$$

Neoclassical island drive



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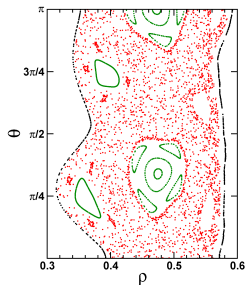
$$\Delta'_{bs} = \frac{C}{w} \left(\frac{w^2}{w^2 + w_d^2} \right) \left(1 + \frac{2.2}{(w/w_d)^2 + 3w_d/w} \right)$$

$$w_d = 1.8w_c$$

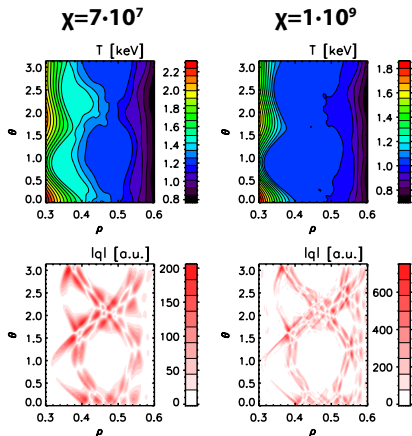
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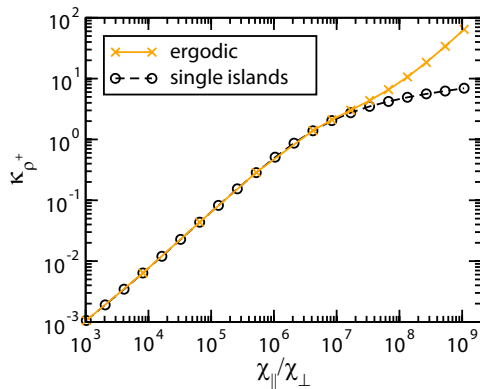


[M. Hölzl et.al. *Phys. Plasmas* 15
072514 (2008)]

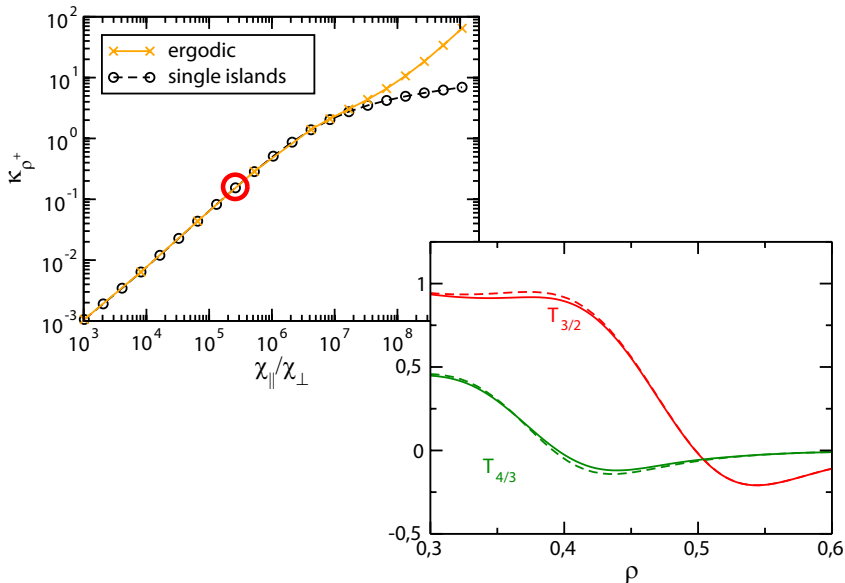


- Small to moderate anisotropies: Island effects dominate
- High anisotropies: Flattening of whole ergodic layer

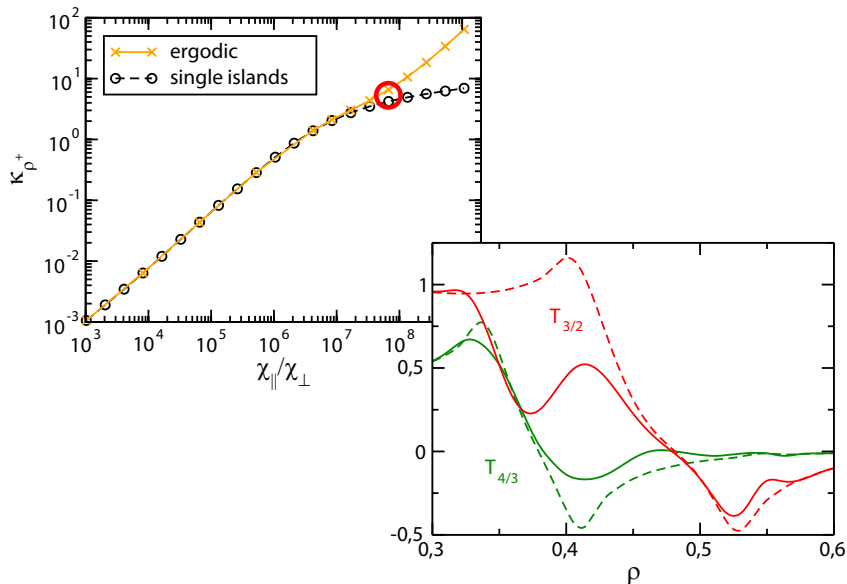
Island and ergodic effects



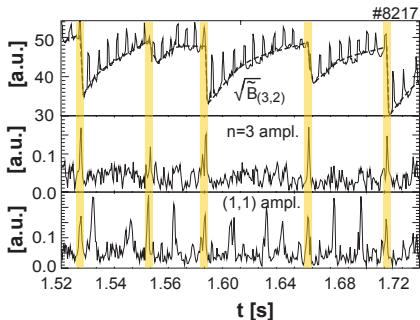
Island and ergodic effects



Island and ergodic effects

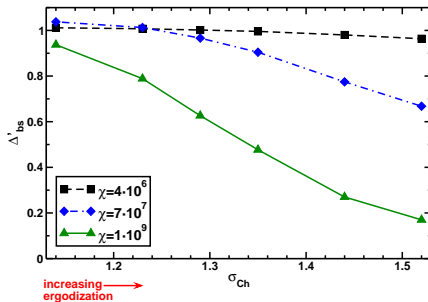


NTMs in the frequently interrupted regime (FIR-NTMs)



[A. Gude et.al. *Nucl. Fusion* **39** 127 (1999)]

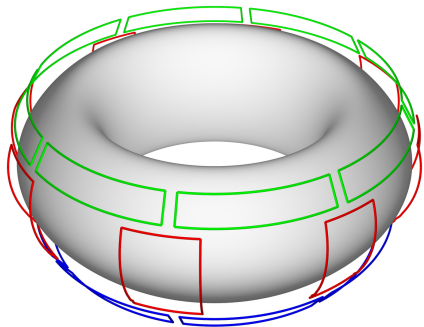
[S. Günter et.al. *Nucl. Fusion* **43** 161 (2003)]



[M. Hölzl et.al. *Phys. Plasmas* **15** 072514 (2008)]

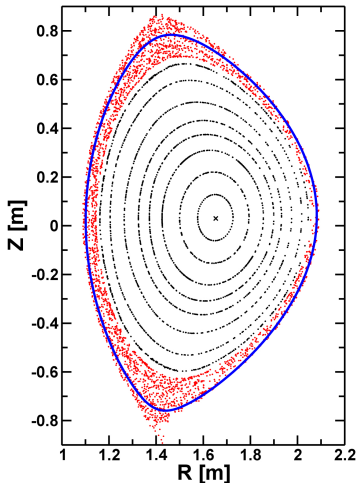
- High plasma pressure: Frequent amplitude drop
- \Rightarrow Reduced average amplitude
- Correlated with other mode activity
- Possible explanation: Ergodization

Ergodic plasma boundary

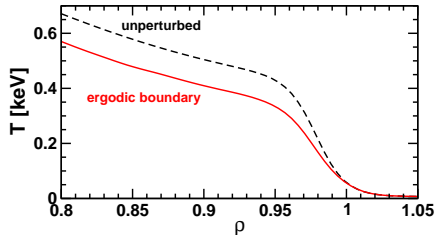
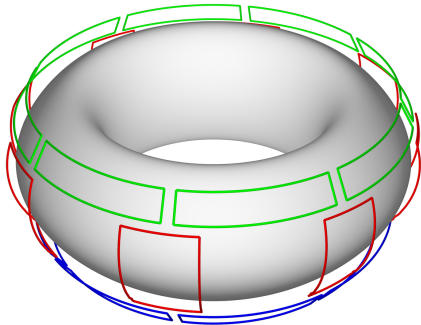


ASDEX Upgrade

- Ergodization of the plasma boundary by auxiliary coils
- Aim: Suppression of ELMs



Ergodic plasma boundary



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[M. Hölzl et.al. *Phys. Plasmas* **15** 072514 (2008)]

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Comparison to Experiment

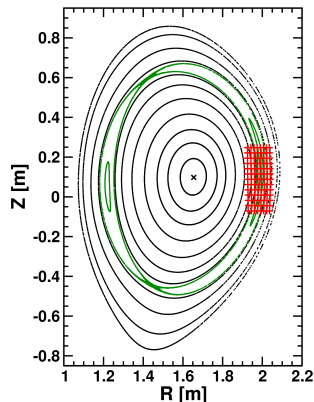
Approach

- Consider electron temperature at magnetic island
 - Simulations with several island widths, w , and heat diff. anisotropies, $\chi_{\parallel}/\chi_{\perp}$
 - Select simulation that reproduces measurements best (minimize quadratic differences)
- ⇒ Determine w and $\chi_{\parallel}/\chi_{\perp}$ independently for each transit

Comparison to Experiment

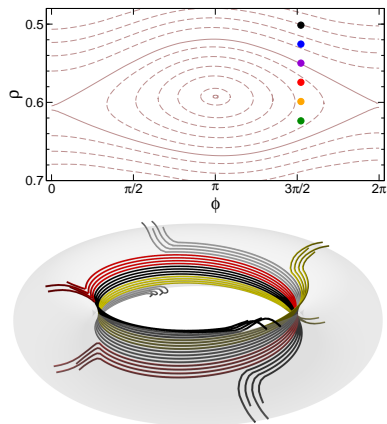
ECE-Imaging

- Electron cyclotron emission spectroscopy with several lines of sight
- Noise reduction by singular value decomposition ^o
- Select one line of sight:
 - Radial coverage
 - Channel quality
 - Channel positions



- Radial information: Several channels
- Toroidal information: Time-traces
- Calibration against 1D ECE
- Fine-calibration to ensure that T is approximately flat inside large island

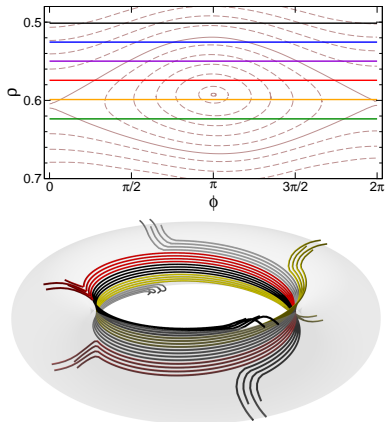
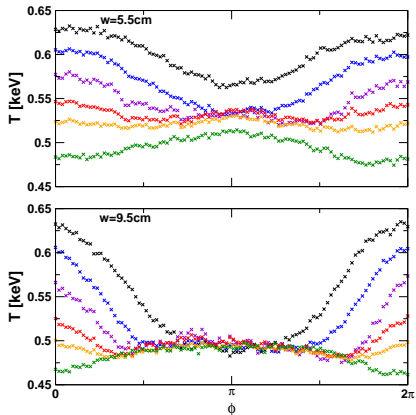
Comparison with 2/1 island in TEXTOR



[M. Hölzl et.al. *Nucl. Fusion* **49** 115009 (2009)]

- Produced by DED-coils; comparison while island grows ◦

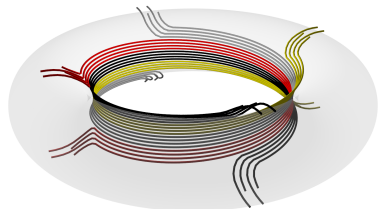
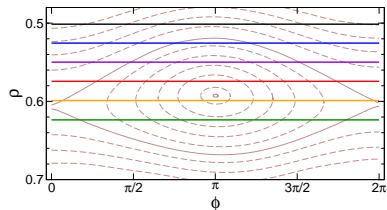
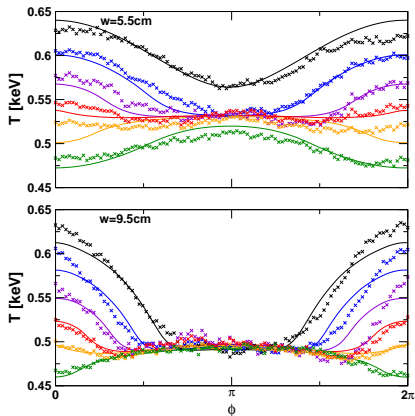
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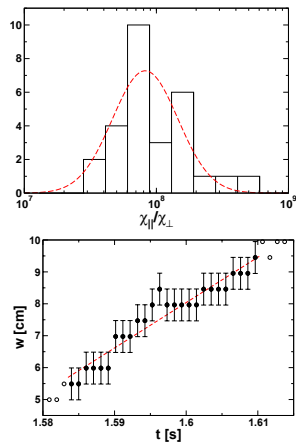
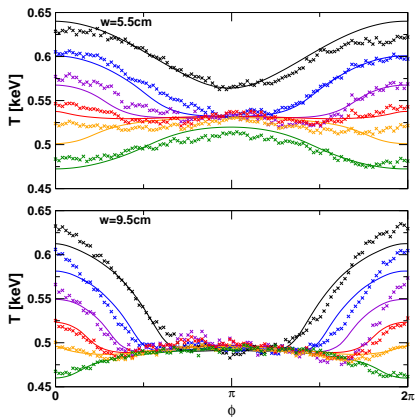
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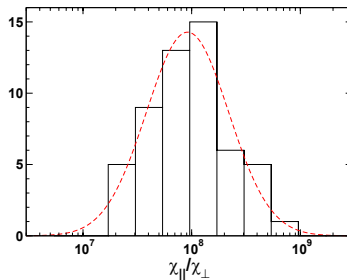
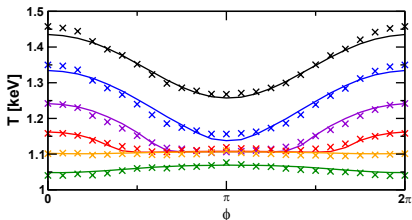
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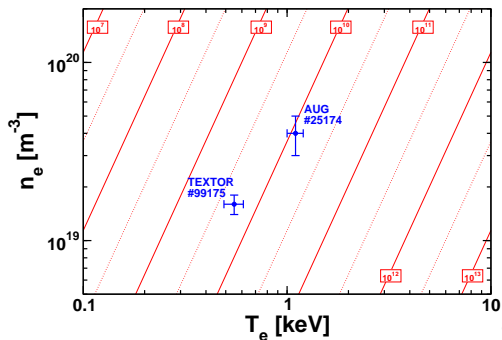
Comparison with 2/1 NTM in ASDEX Upgrade



Overview

- Discharge #25174, $t = 2.037 \text{ s} \dots 2.142 \text{ s}$
- 2/1 NTM ($\approx 5 \text{ kHz}$, 540 transitions)
- ECE-Imaging (100 kHz sampling frequency)
- Noise suppression: SVD + average of 10 transitions \circ

Summary: Comparison with Experiments



| | | | | | |
|---------|----------------|-----------------------|--|--|---------------------------|
| TEXTOR: | 2/1 RMP-island | $T_e = 0.6\text{keV}$ | $n_e = 1.6 \cdot 10^{19}\text{m}^{-3}$ | $\chi^{\text{SH}} = 10^{9.8 \pm 0.2}$ | $\chi = 10^{8.0 \pm 0.4}$ |
| AUG: | 2/1 NTM | $T_e = 1.1\text{keV}$ | $n_e = 4.0 \cdot 10^{19}\text{m}^{-3}$ | $\chi^{\text{SH}} = 10^{10.1 \pm 0.3}$ | $\chi = 10^{8.0 \pm 0.4}$ |

⇒ “Measurement” of heat diffusion anisotropy

⇒ Clear indication for heat-flux limit, more data needed!

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Conclusions and Outlook

Summary

- Heat transport across magnetic islands and stochastic layers
- Δ'_{bs} larger than analytically predicted
- Island effects at stochastic layers
- FIR-NTM: Reduction of Δ'_{bs} by ergodization
- Transport across ergodic plasma boundary
- Determination of $\chi_{||}/\chi_{\perp}$ by comparison to experiments

Outlook

- Work with and improve nonlinear MHD code JOREK
- But also continue the presented work. . .

Additional comparisons with experiments

Aims

- NTM with power ramp-down
 - ⇒ Verify heat flux limit theories
 - ⇒ Determine marginal w/w_c
- NTM with ECRH
 - ⇒ Investigate χ_{\perp} inside island

Requirements

$T_e(r)$: 1D and 2D ECE

$n_e(r)$: IDA

$T_i(r), v_{tor}(r)$: CXRS $\Rightarrow P_e(r)$: TRANSP

Separate discharge with 0.5% B_t -ramp in quiescent phase for calibration

Acknowledgments

Many thanks to . . .

Tokamak Theory Group
ASDEX Upgrade Team

especially to . . .

Sibylle Günter
Qingquan Yu
Marc Maraschek
Ivo Classen
Giovanni Tardini
Rainer Fischer
Erika Strumberger

Scale island width, w_c

$$w_c = \left(\frac{\chi_{\parallel}}{\chi_{\perp}} \right)^{-1/4} \left(\frac{8R_0 q_s}{n (\partial q / \partial r)_s} \right)^{1/2}$$

q_s Resonant value of the safety factor

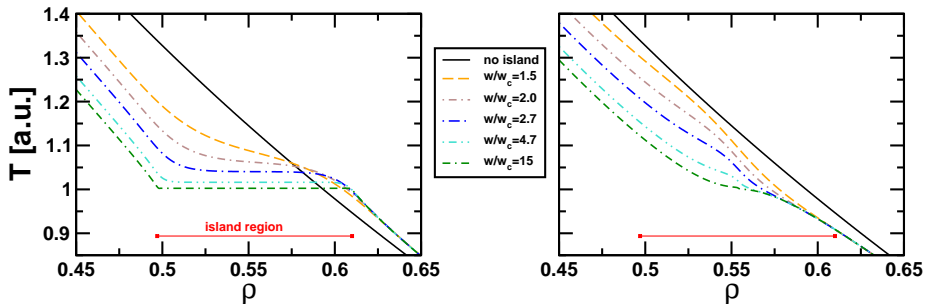
R_0 Major radius

n Toroidal mode number

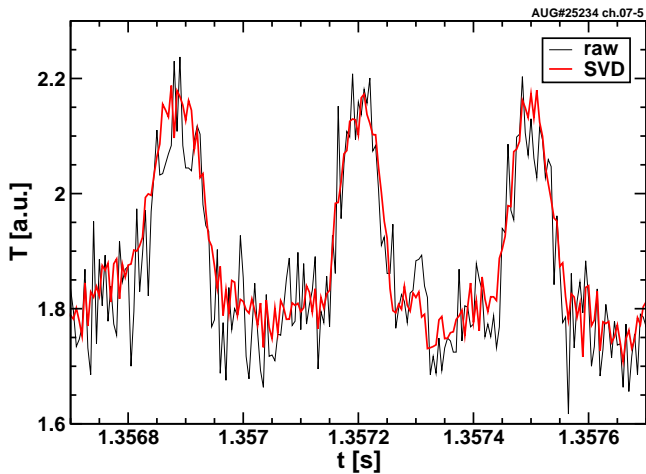
Spitzer Härm diffusivity

$$\chi_{\parallel}^{\text{SH}} = 3.16 \cdot v_{\text{th},e} \cdot \lambda_{\text{mfp},e} \approx 3.6 \cdot 10^{29} \text{ m}^2/\text{s} \cdot \frac{T_e^{5/2} [\text{keV}]}{n_e [\text{m}^{-3}]}$$

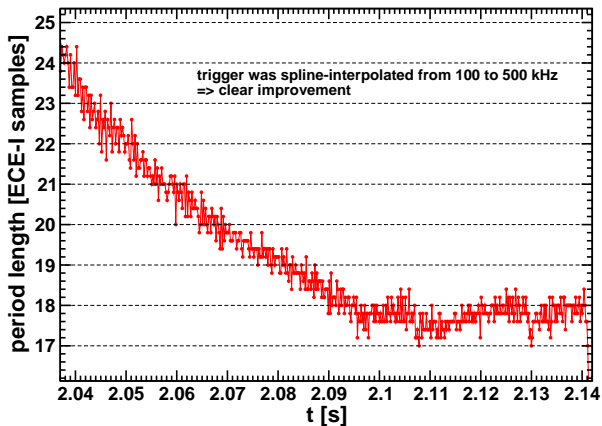
Temperature distribution at O- and X-Points



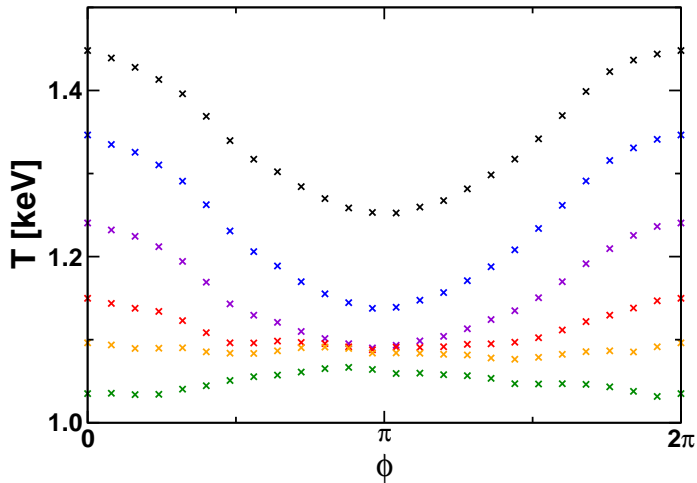
Singular Value Decomposition



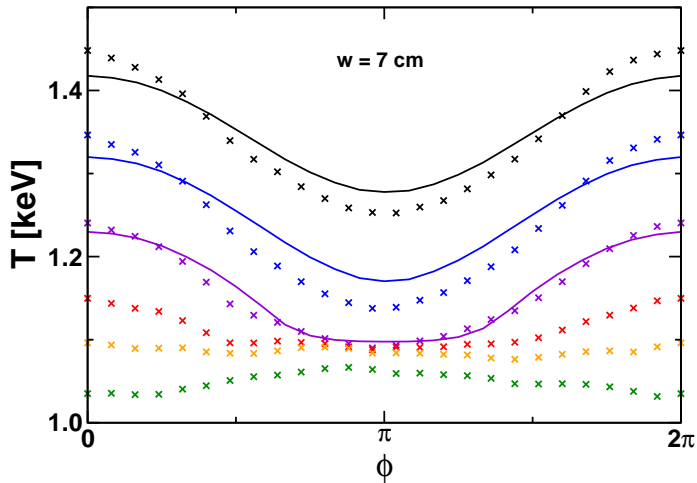
AUG: Period length of 2/1 NTM



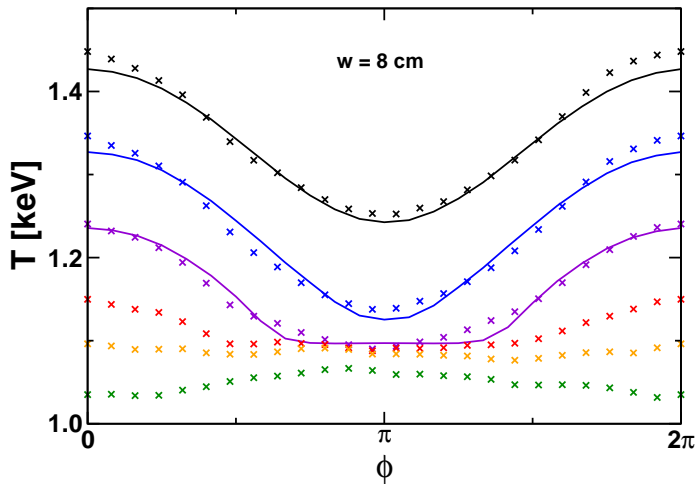
Sensitivity



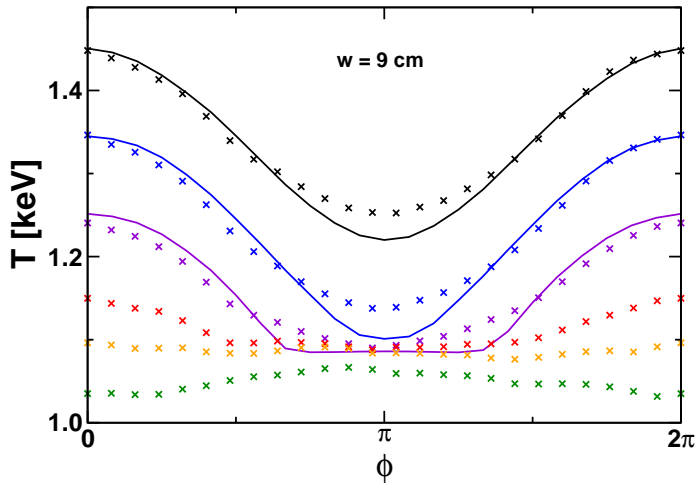
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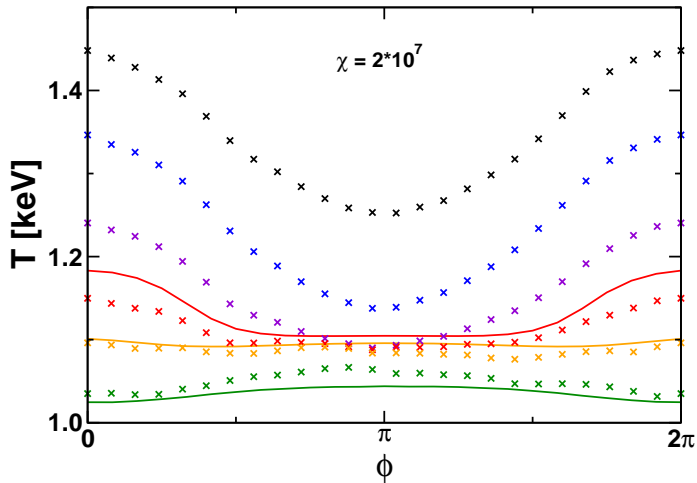
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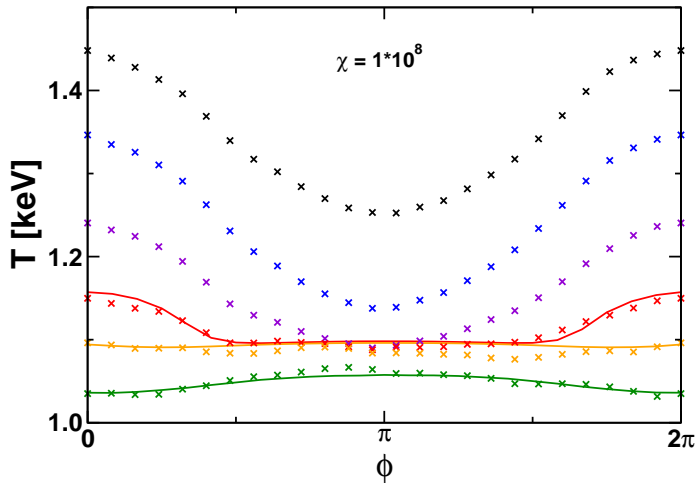
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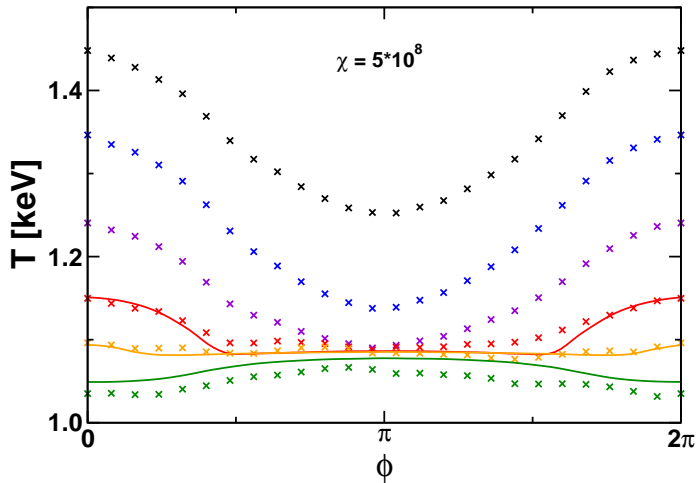
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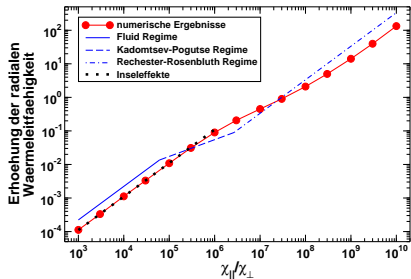
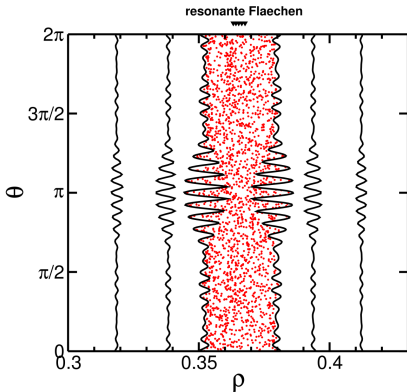
Sensitivity



Sensitivity



Strong ergodization



[M. Hölzl et.al. *Phys. Plasmas* 14 052514 (2007)]

Highly ergodic configuration

- Artificial case: Five helical perturbations, cylindrical geometry
- Allows for comparison to analytical theories