Heat Diffusion
in realistic Tokamak Geometry

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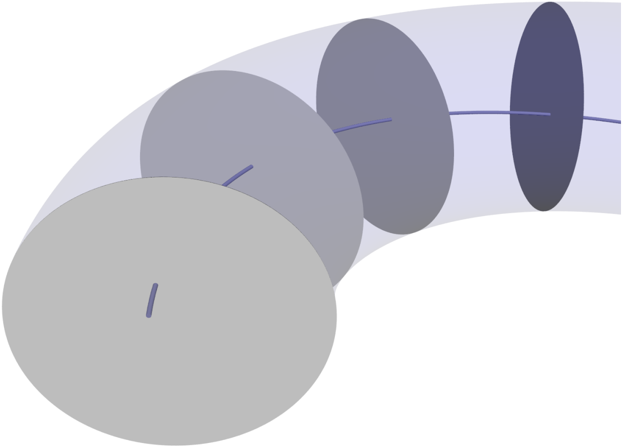
Ringberg Theory Workshop
Max-Planck-Institut für Plasmaphysik

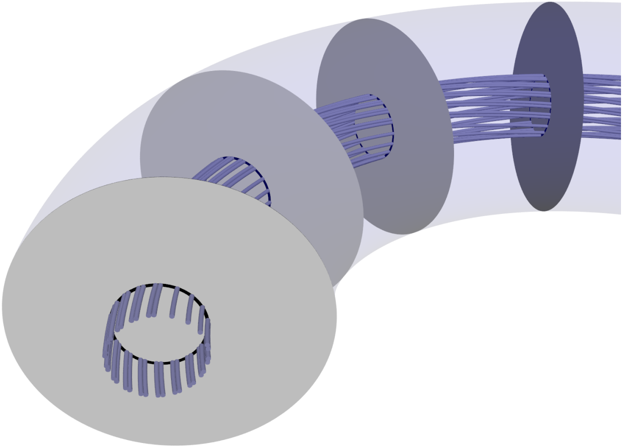
17.11.2008

IPP

1. Motivation
2. Model
    - Heat Diffusion Equation
    - Coordinate system
    - Coordinate alignment
3. Magnetic Islands
    - Temperature flattening
    - Comparison to TEXTOR
4. Ergodic Layers
    - Temperature flattening
    - FIR-NTMs
5. Edge Ergodization
6. Summary

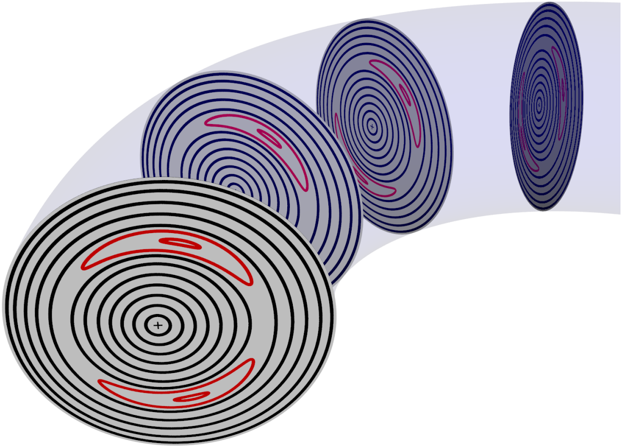


- “Safety factor” q: Number of toroidal turns per poloidal turn
- $\iota = 1/q$

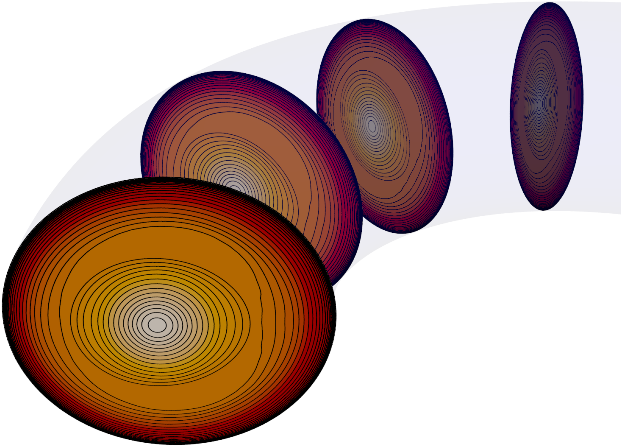


- Poincaré plot: Field lines traced for many toroidal turns

Magnetic island



- $2/1$ magnetic island at $q = 2$ surface



- Temperature profile flattens inside the magnetic island
- Bootstrap current $\propto \nabla p$ perturbed $\Rightarrow$ Island drive (NTM)

Steady State Heat Diffusion Equation

$$\boxed{\nabla \cdot \mathbf{q} = P, \text{ where } \mathbf{q} = -n_e \left(\chi_{||} \nabla_{||} T + \chi_{\perp} \nabla_{\perp} T \right)}$$

$\mathbf{q}$: heat flux, P: energy source, n_e: electron density, $\chi_{||}$ and $\chi_{\perp}$: heat diffusivities

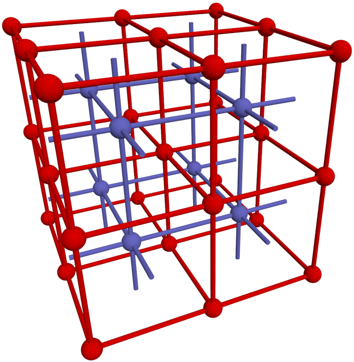
Anisotropy

$$\chi \equiv \chi_{||} / \chi_{\perp} \approx 10^8 \ldots 10^{10}$$

Finite Difference Scheme

see Günter et al. (2005)

- Two staggered grids
- Low numerical diffusion
- Coordinate alignment not required
- Realistic anisotropies



Curvilinear Coordinate System

- Heat diffusion eq. in tensor notation:

$$\boxed{\begin{aligned}&\frac{1}{\sqrt{g}}\frac{\partial}{\partial u^\alpha}\left(\sqrt{g}q^\alpha\right)=P\\ &q^\alpha=-n_e\chi_\perp\left[\chi b^\alpha b^\beta+g^{\alpha\beta}\right]\frac{\partial T}{\partial u^\beta}\end{aligned}}$$

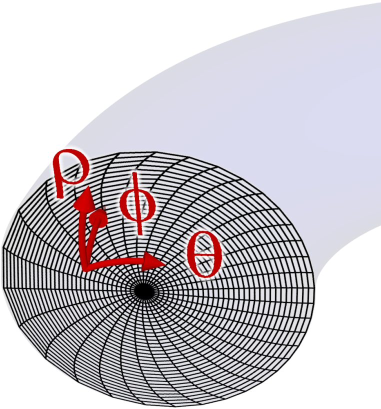
q^α: Contravariant heat flux components

u^α: Contravariant coordinates (ρ, θ, ϕ)

$g^{\alpha\beta}$: Metric tensor components

$g=\det[g_{\alpha\beta}]$: Determinant of the covariant metric tensor

- Axisymmetric straight field line coordinates



Coordinate Alignment to Unperturbed Magnetic Field

- Coordinate Transformation $\theta = \tilde{\theta} - \iota \cdot \phi$
  $\Rightarrow$ Sheared helical coordinates

  $\iota = 1/q$: Inverse safety factor
- Problems:
  - Grid deformation
  - Interpolation for toroidal periodicity condition $T_{\phi=0} \equiv T_{\phi=2\pi}$ increases numerical diffusion
  - Restriction $\chi_{||}/chi_{\perp} \lesssim 10^7$ ✗

Partial Coordinate Alignment

- Transformation $\theta = \tilde{\theta} - \iota_c \cdot \phi$ $\iota_c \equiv \mathrm{const}$
  $\Rightarrow$ Unsheared helical coordinates
- Realistic anisotropies ✓

Restrictions due to the Misalignment?

- Islands resolved well for: $N_\phi \gtrsim \Delta\iota \cdot N_\theta$

  $\Delta\iota = |\iota - \iota_c|$: misalignment at island

  N_ϕ, N_θ: toroidal and poloidal grid point numbers
- Suitable for magnetic perturbations with similar helicities
- Islands with very different helicities increase the numerical effort

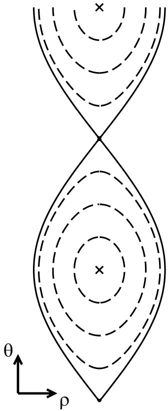
Heat Transport across Magnetic Islands

see Fitzpatrick (1995); Yu (2006); Hölzl et al. (2007)

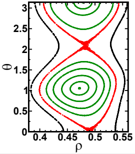
- Scale island width $w_c \propto (\chi_{||}/\chi_\perp)^{-0.25}$

$$\boxed{w/w_c \begin{cases} \ll 1 & \text{No perturbation} \\ \gtrsim 1 & \text{Temperature flattening} \end{cases}}$$

- Heat conduction layer at the separatrix
- Temperature flattening destabilizes NTMs (perturbation of the bootstrap current)
- This talk: Realistic tokamak geometry

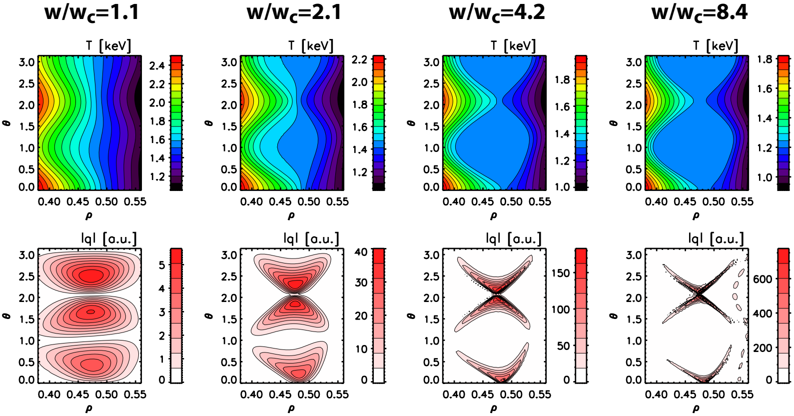


- ASDEX Upgrade equilibrium
- $3/2$ island with $w = 8.1\text{cm}$



$\theta = 0$: Low field side $\qquad$ $\theta = \pi$: High field side

Temperature flattening



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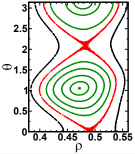

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Comparison to TEXTOR (preliminary)

Data provided by Ivo Classen (see Classen (2007))

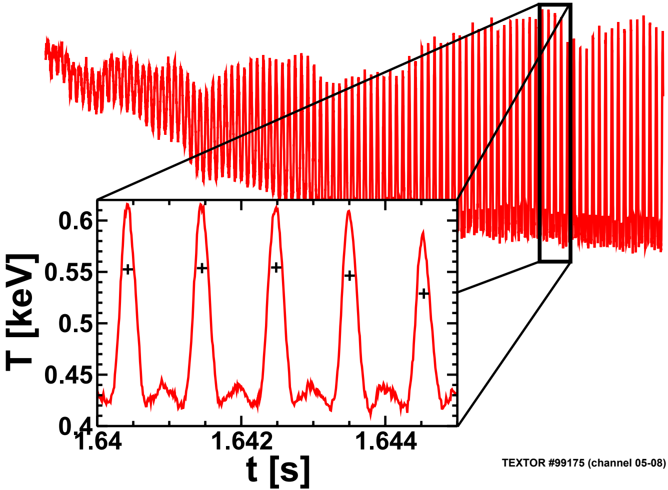
- 2/1 island triggered by DED coils
- Mode frequency 1 kHz
- ECE frequency 100 kHz
- Channels cover part of the island (including x-point)
- Channels not cross-calibrated
- Comparing during growth phase
- Aim: Draw conclusions for experimental $\chi_{||}/\chi_{\perp}$

TEXTOR: Tokamak experiment in Jülich with a circular plasma cross section

DED coils: Set of perturbation coils at TEXTOR (Dynamic ergodic divertor)

ECE: Diagnostic measuring the electron temperature (Electron cyclotron emission spectroscopy)

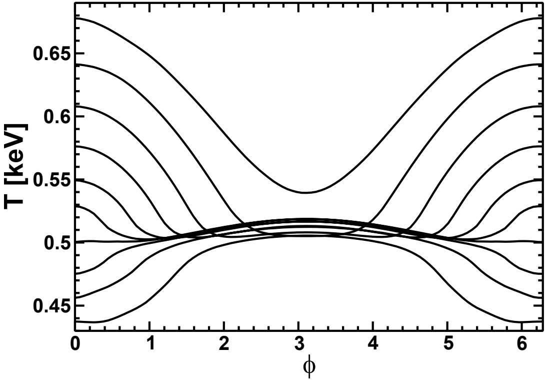
Timetrace of an ECE channel



TEXTOR #99175 (channel 05-08)

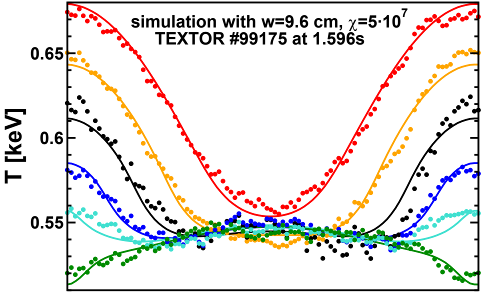
Numerical simulation

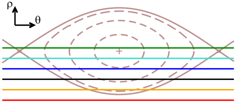
- Code runs with different $\chi_{||}/\chi_{\perp}$, power source, energy source, . . .
- Toroidal temperature cuts:



Automatic matching

- Adding calibration-summands to the ECE channel signals
- Best-fitting numerical code run for each experimental timepoint





Problems

- Sudden change of the island structure as the mode locks to the DED perturbation field
- Best fitting $\chi_{||}/\chi_{\perp}$ changes strongly

Reasons?

- Perturbation profile?
- Higher harmonics (4/2, . . .)?
- Different modes excited by DED coils (3/1, . . .)?

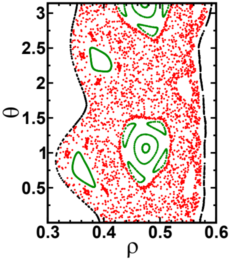
Additional Comparisons planned

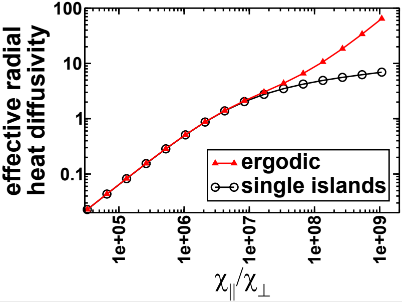
- ECRH heating at magnetic island
- ASDEX Upgrade with new ECE diagnostic

Heat Diffusion across an Ergodic Layer

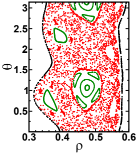
- Overlapping magnetic islands produce an ergodic layer
- Chaotic field line trajectories

$$\chi_{||}/\chi_\perp = \begin{cases} \text{small: single island effects dominate} \\ \text{large: ergodisation increases transport} \end{cases}$$





- Overlapping $3/2$ and $4/3$ islands
- Chirikov parameter $\sigma_{Ch} = 1.52$



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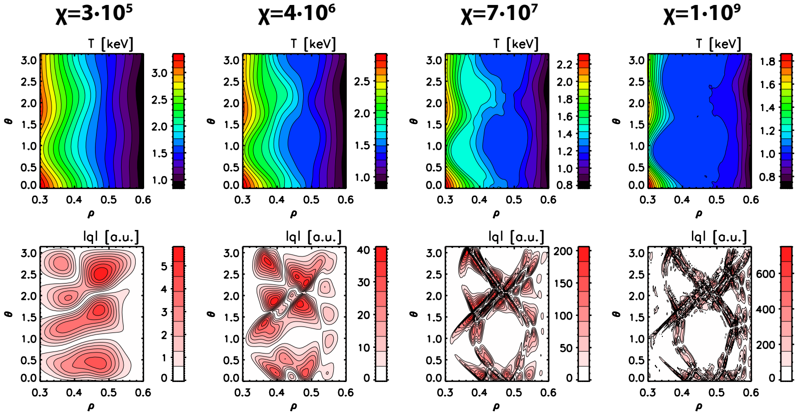

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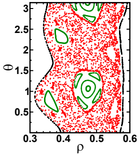

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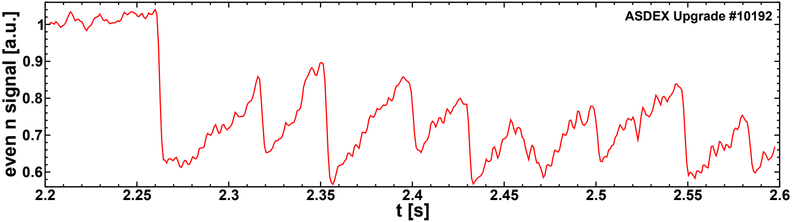
FIR-NTMs

FIR-NTM: Neoclassical tearing mode in the frequently interrupted regime

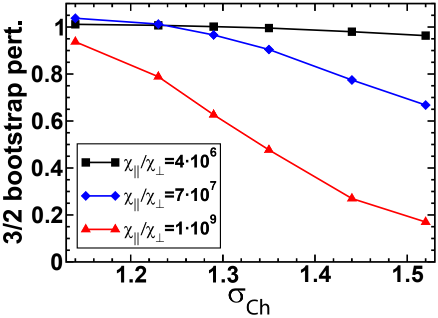
see Günter et al. (2001) and Gude et al. (2002)

- Frequent interruption of NTM growth by sudden amplitude drop
- Much faster than the resistive timescale
- Observed at large normalized plasma pressure β_N (i.e. large bootstrap current fraction)



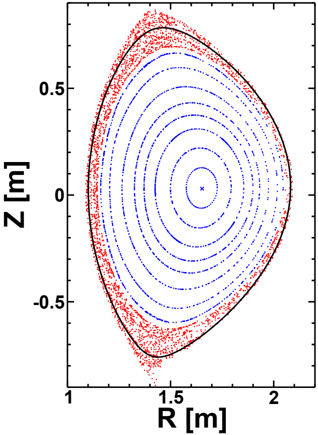
FIR-NTMs

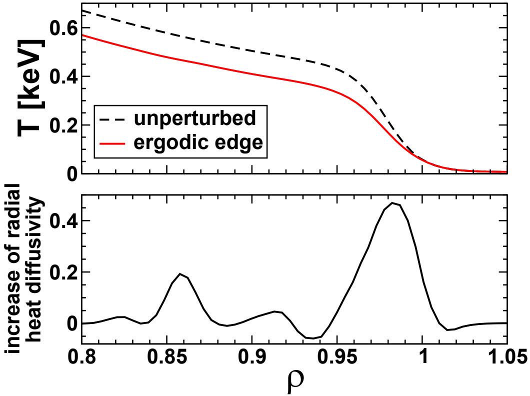
- Island ⇒ T flattening ⇒ bootstrap current perturbation ⇒ NTM
- Considering $3/2$ NTM and additional $4/3$ perturbation
- Resonant bootstrap current perturbation strongly reduced for $\chi_{||}/\chi_{\perp} \gtrsim 1 \cdot 10^9$ and $\sigma_{Ch} \gtrsim 1.4$ ⇒ Less island drive
- $4/3$ perturbation expected to be ideal (timescale!)



Edge ergodization

- Perturbation coils planned for ASDEX Upgrade
- Among others aimed at the mitigation of edge localized modes (ELMs)
- Ergodization of the plasma edge
- Increased heat conduction due to the transport of electrons along magnetic field lines





- Spitzer conductivity assumed
- Significant drop of edge temperature gradient observed
- Very sensitive to plasma parameters!

Summary

- Unsheared helical coordinates
- Realistic $\chi_{||}/\chi_{\perp}$ possible (islands, ergodic layers, ergodic edge)
- Magnetic islands: Temperature flattening for $w/w_c \gtrsim 2$
- Comparison to TEXTOR
  - ECE timetraces vs. toroidal cuts
  - Automatic matching
  - Problems with perturbation profile
- Ergodic layers
  - Temperature flattening at the ergodic layer for large $\chi_{||}/\chi_{\perp}$
- NTM
  - Resonant bootstrap current perturbation drives island
  - FIR-NTM: Frequent amplitude drop
  - Possible explanation: Ergodization reduces island drive
- Edge: Ergodization might increase radial heat transport

Thanks for your attention!

Acknowledgements

Prof. Dr. Sibylle Günter · Dr. Qingquan Yu · Dr. Erika Strumberger · Klaus Reuter

References

Most of the results shown in this talk have been published in Hölzl et al. (2008).

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