Some Fundamental Remarks on Momentum Balances

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Stationary (1D) momentum balance

parallel momentum balance eq. (in co-ordinate system at rest):

$$\frac{1}{V'}\frac{d}{dr}V'\left[B\,\dot{r}\,v_\parallel\,f\right] - \left[\frac{\underline{B}\cdot\underline{\nabla}B}{B}\,(v_\parallel^2 - \frac{1}{2}v_\perp^2)\,f\right] + \left[\frac{\underline{B}\times\underline{\nabla}\Phi}{B_0^2}\cdot\underline{\nabla}B\,v_\parallel\,f\right] = \left[B\,v_\parallel\,C(f)\right] + \left[B\,M_\parallel\right]$$

with the moments $[A] = \langle \int A\,d^3v \rangle$ and the flux-surface average $\langle \ldots \rangle$

momentum source: $M_\parallel$ (e.g. NBI and CX losses)

for shifted Maxwellian with $n(r)$, $T(r)$, $\Phi(r)$ and $U_\parallel(r)$:

$\rightarrow$ radial momentum transport and both viscosity effects vanish

"anomalous" contributions to radial momentum transport might exist

both viscous forces are parallel effects (like conductivity and bootstrap current) and will be dominated by neoclassical contributions

DKE: “neoclassical ordering” (r.h.s.)

assume for f_0 in 0th order a Maxwellian shifted by $U_{\|}(r)$

($U_{\|}(r)$ is determined from parallel momentum balance)

$$V(f_1) - C(f_1) =$$

$$- \underline{v}_{\nabla B|_r} \left\{ \frac{n'}{n} + \frac{q\Phi_0'}{T} + \left(\frac{v^2}{v_{th}^2} - \frac{3}{2}\right)\frac{T'}{T} \right\} f_0 + \frac{q}{TB} \langle \underline{E}_{\|} \cdot \underline{B} \rangle \left(pv - U_{\|}\right) f_0$$

$$- 2\frac{U_{\|}}{v_{th}^2} \left\{ \underline{v}_{\nabla B|_r} \left((v_{\|} - U_{\|}) \frac{U_{\|}'}{U_{\|}} - (v_{\|} + \frac{1}{2} U_{\|}) \frac{T'}{T} \right) + v_{\|} \frac{\underline{B} \times \underline{\nabla\Phi}}{B^3} \cdot \underline{\nabla} B - v_\perp^2 \frac{\underline{B} \cdot \underline{\nabla} B}{2B^2} \right\} f_0$$

$\rightarrow$ 2 traditional and 3 new ”thermodynamic” forces determine f_1

conservative ansatz for stellarators:

neglect the $U_{\|}$-terms and estimate the viscosity and momentum diffusion co-efficients only for the traditional thermodynamic forces

Simple picture: stellarators

$\rightarrow$ concentrate on the $v_{||}$ thermodynamic force (parallel conductivity)

(neoclassical) momentum transport:

no contribution from stellarator *lmfp* transport regimes
(symmetric f_1 with respect to $v_{||}$)

momentum transport coefficient $\propto E_r$ for small E_r, comparable to the Ware pinch coefficient at intermediate E_r, and $\propto 1/E_r$ for very large E_r

viscous damping:

in *lmfp*-regime, damping of parallel flow (passing particles) by friction with trapped particles (symmetric distribution)

"trapped particle fraction", $f_t(r)$, for small E_r

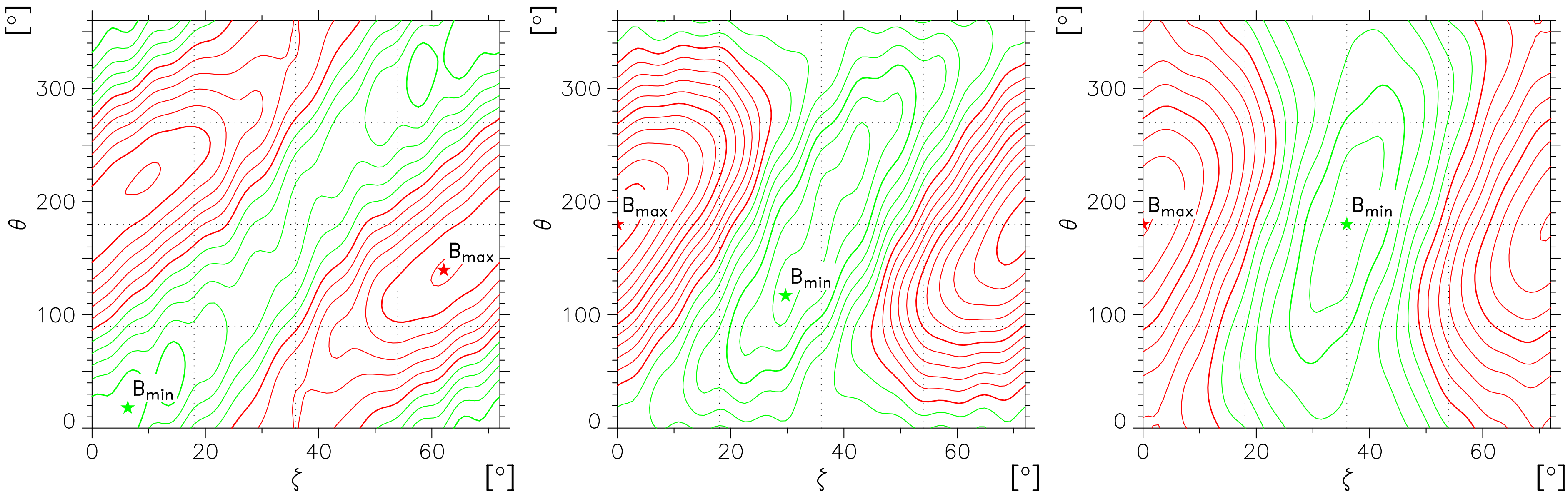
toroidal and helical resonances appear at intermediate E_r increasing the viscous damping

both viscosities become constant for very large E_r and small ν^*

in general, the viscous damping is strong in stellarators (small rotation)

W7-X: variation of toroidal mirror term - $|\boldsymbol{B}|$ contours

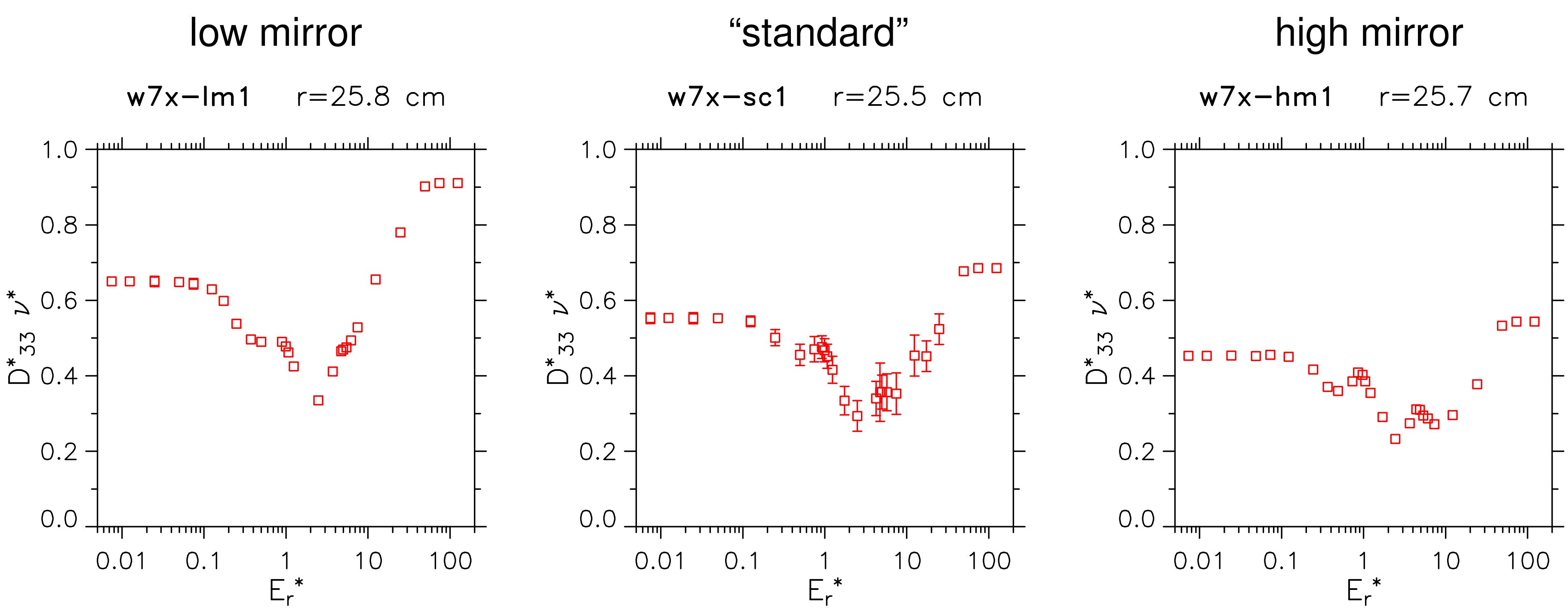
$r = \frac{a}{2}$ "low mirror" "standard" "high mirror"



W7-X mirror scan: momentum balance

parallel viscous damping (mono-energetic) from DKES code $(\nu/v = 10^{-6})$

depending on E_r (norm. $E_r^* = \frac{R}{\tau r}\frac{E_r}{vB}$)



Simple picture: tokamaks

special topic for ideal tokamaks:

for $E_r > \iota v B r / R$, no banana orbits exist
(corresponding to a purely toroidal rotation)

parallel viscosity vanishes for these large E_r,
$E \times B$ viscosity becomes constant (but small)

total viscosity decreases monotonically with E_r

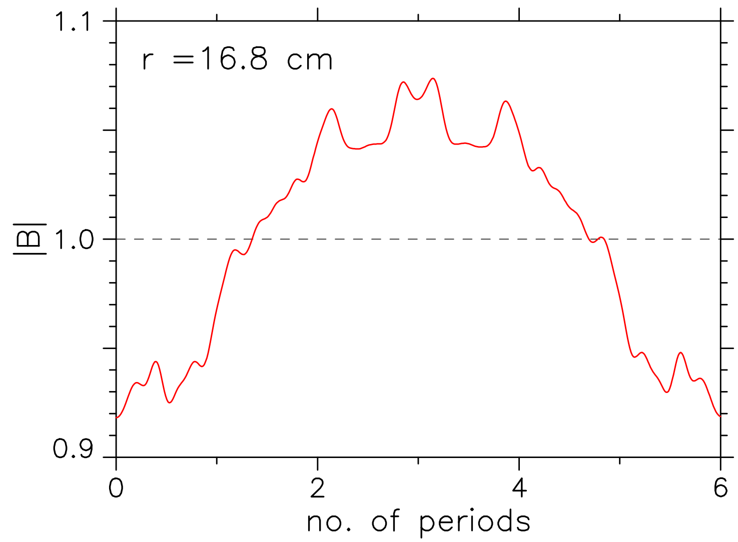
ideal picture without ion momentum transport and incompressible flow leads to a toroidal rotation larger than v_{th}

centrifugal and Coriolis forces become important (density variations)

this picture changes completely with even small coil ripples

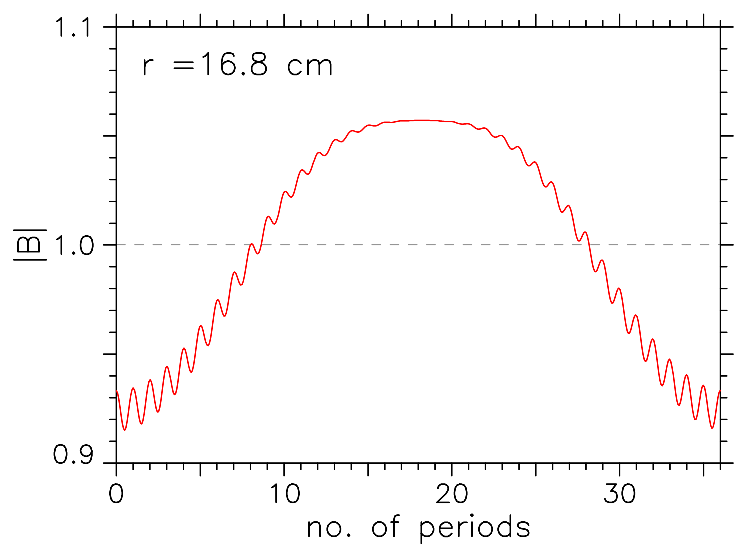
$\rightarrow$ NCSX-2 configuration compared to a tokamak with coil ripple

NCSX-2: momentum balance



$|B|$ along field lines:

NCSX-2 at $r = 16.8$ cm



"equivalent" tokamak with coil ripple:

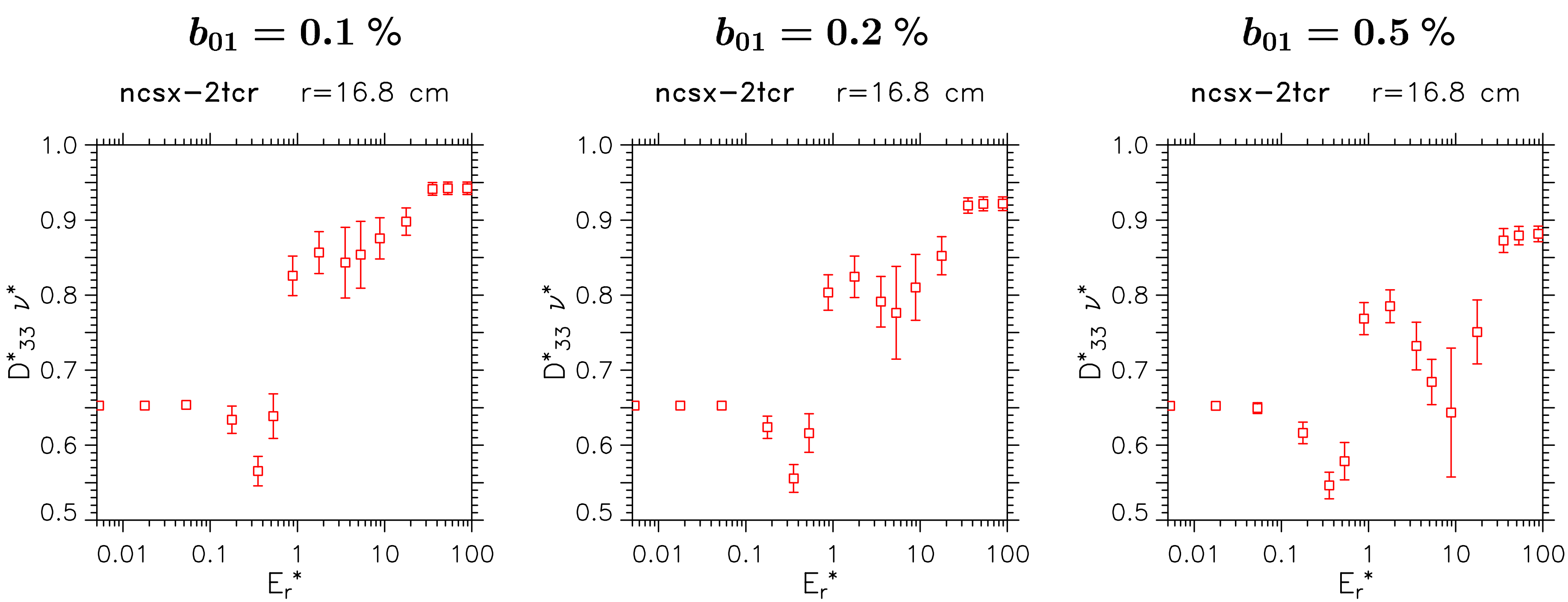
$$B/B_0 = 1 + \sum_m b_{m0} \cos m\theta + b_{01}\,(1 + \cos\theta) \cdot \cos\varphi$$

with $b_{01} \simeq 0.46\ \%$

Tokamak with coil ripple: momentum balance

parallel viscous damping (mono-energetic) from DKES code $(\nu/v = 10^{-6})$

depending on E_r (norm. $E_r^* = \frac{R}{\tau r}\frac{E_r}{vB}$)

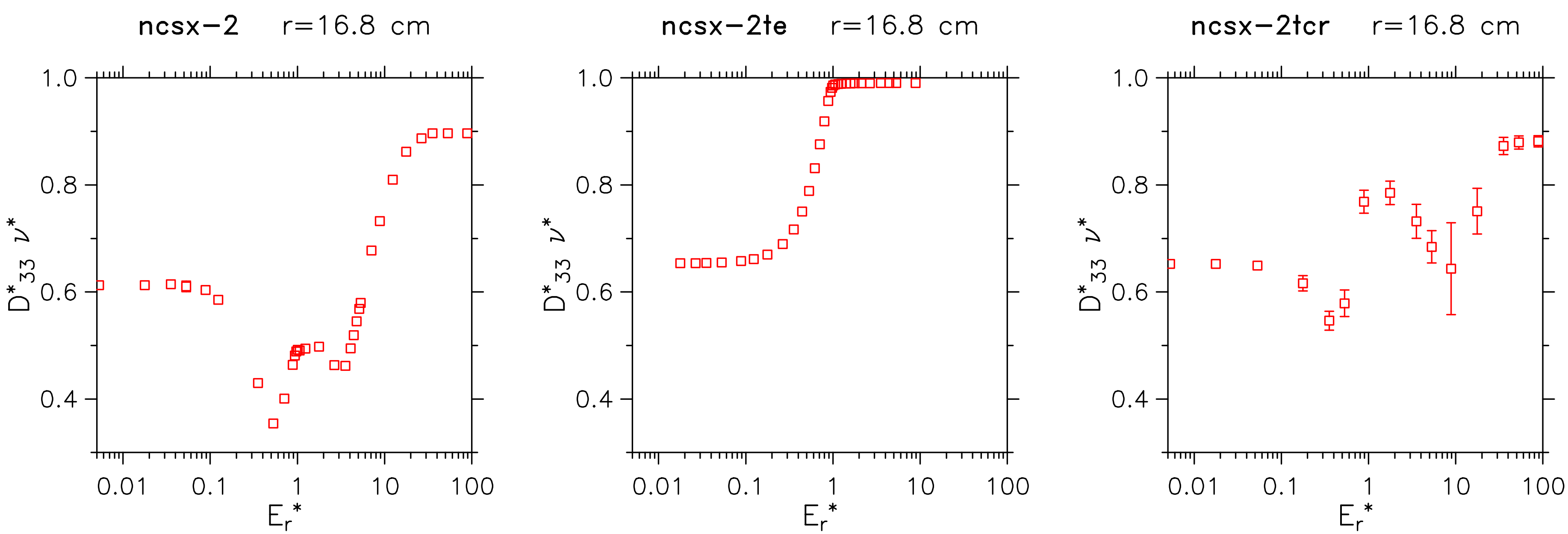


NCSX-2: momentum balance

parallel viscous damping (mono-energetic) from DKES code $(\nu/v = 10^{-6})$

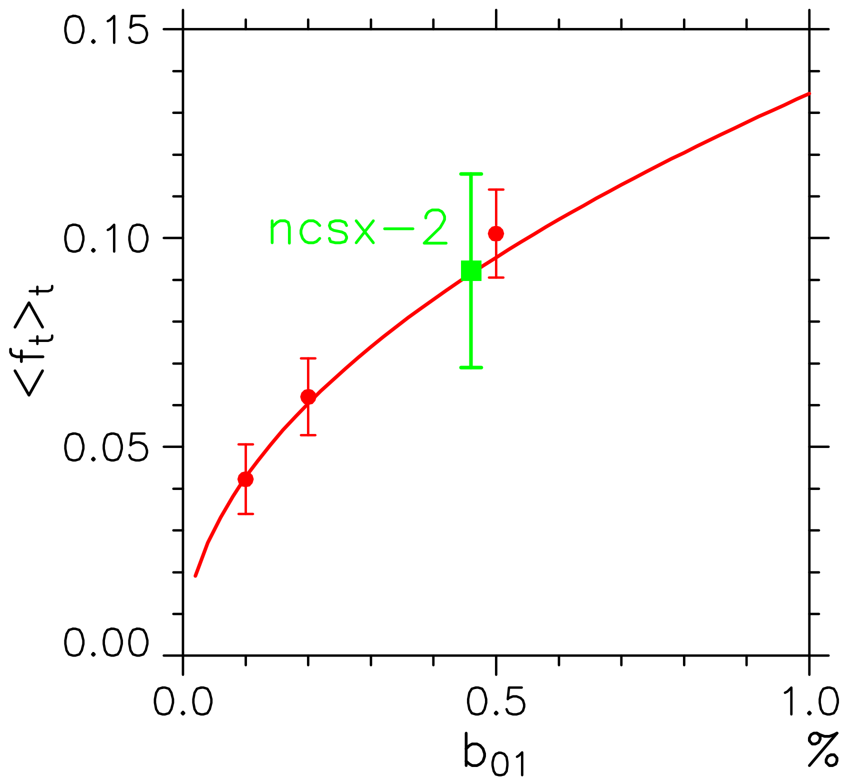
depending on E_r (norm. $E_r^* = \frac{R}{\tau r}\frac{E_r}{vB}$)

NCSX-2 | equiv. tokamak | rippled tokamak $b_{01} = 0.5$ %



NCSX-2: equiv. tokamak with coil ripple

damping of toroidal rotation for tokamaks with coil ripple, b_{01}



"toroidal" definition of trapped particle fraction for tokamaks with coil ripple:

$$\langle f_t \rangle_t = \int_0^{\pi} f_t(\theta) \frac{d\theta}{B} \Big/ \int_0^{\pi} \frac{d\theta}{B}$$

DKES:

NCSX-2 (16.8 cm) $\rightarrow$ $b_{01} \simeq 0.46$ %

DKES data: parallel viscosity and $\boldsymbol{E} \times \boldsymbol{B}$ contribution