Global Gyrofluid Computations of Tokamak Core Turbulence

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Outline

- **Basic Ideas**

  — basic computations

  — tokamak core conditions

- **gyrofluid model GEM**

  — adiabatic electrons

  — “global” computation

  — “Cyclone” run campaign

- **Basic Results**

  — standard ITG turbulence

  — decaying profiles, long confinement time

  — zonal flows, effects of Landau damping (LD) model

  — transport scaling results, LD effects

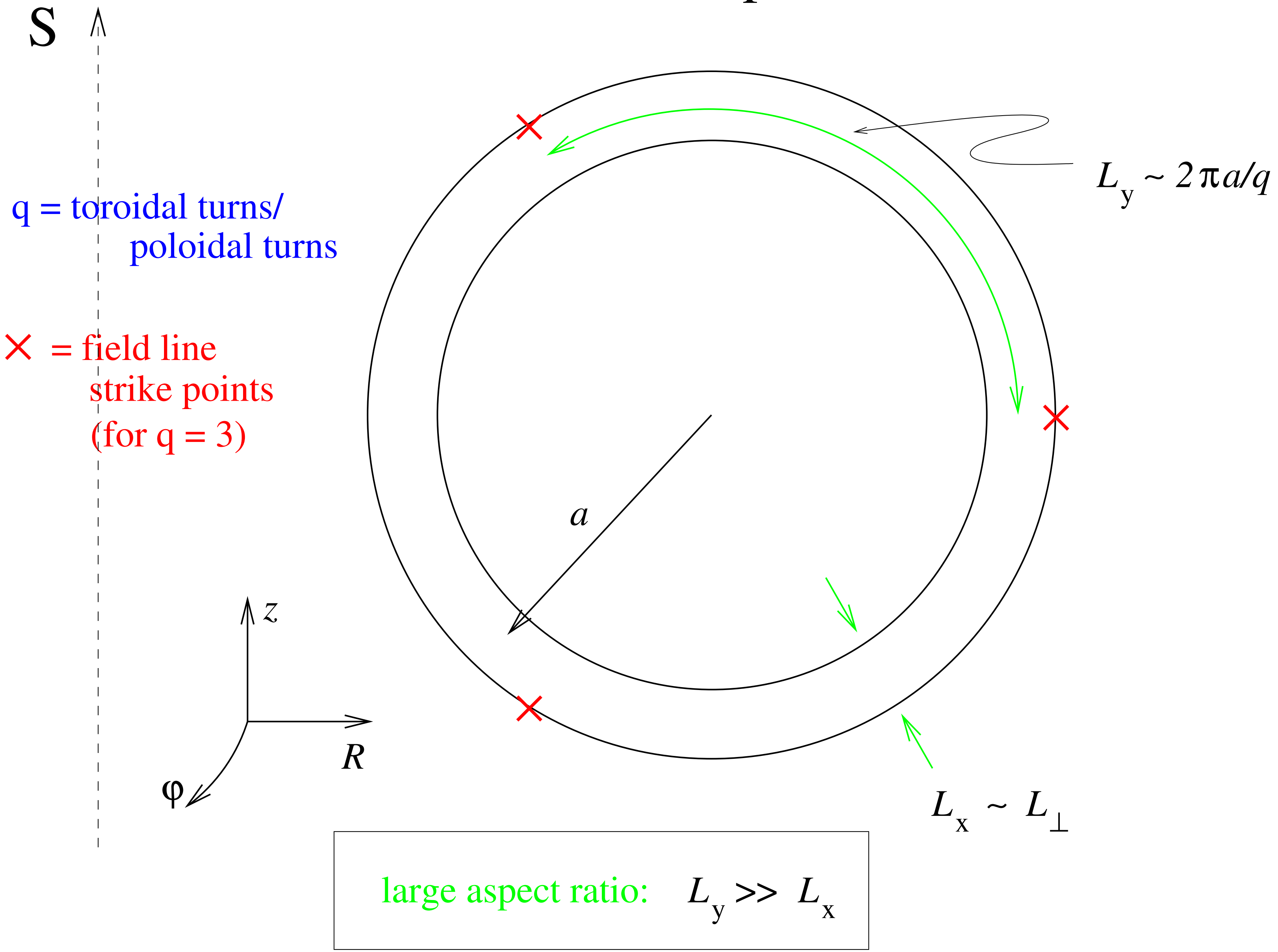
- **Future Generalisation**

  — size scaling

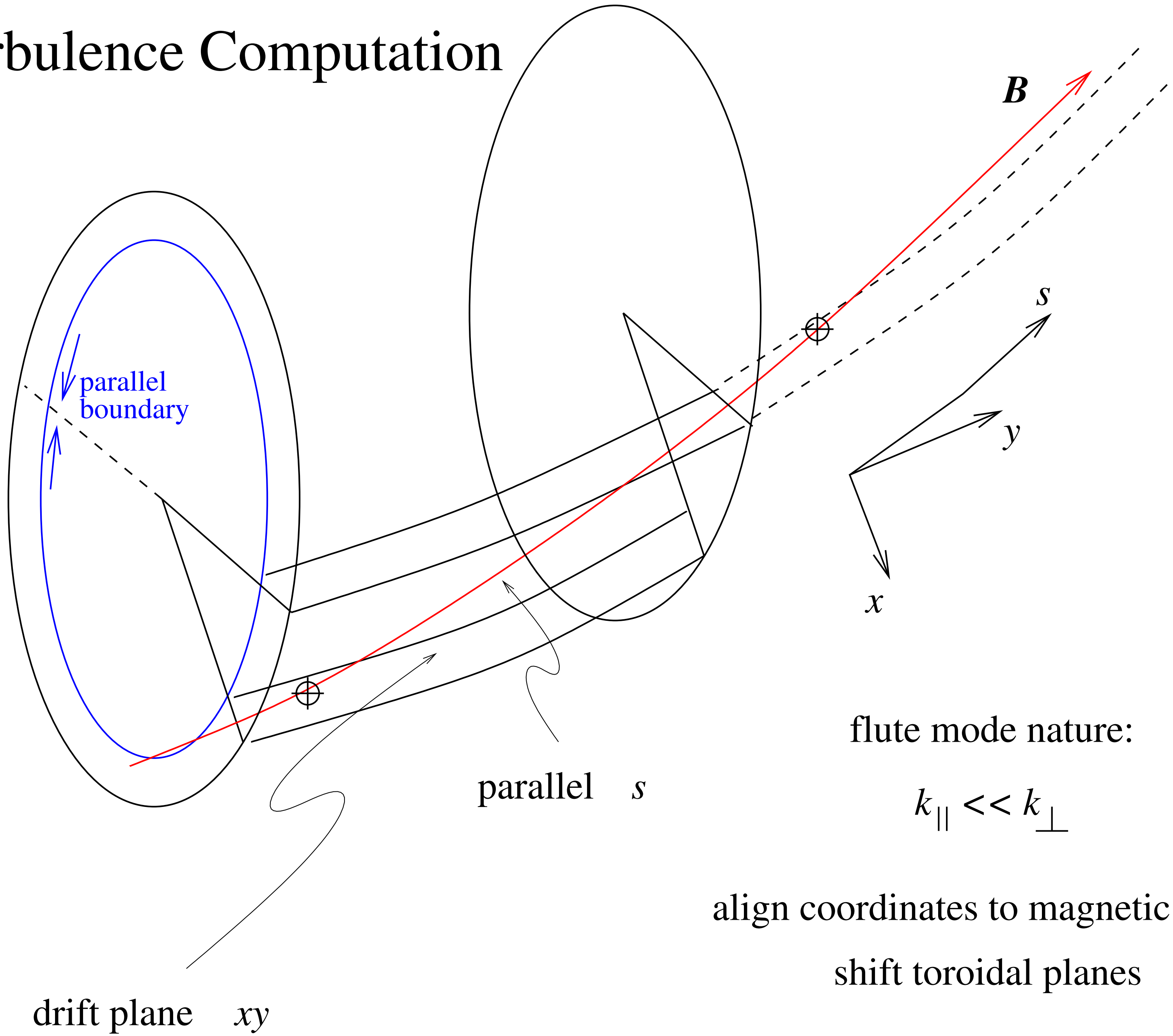
  — shear reversal

  — why gyrofluid is still relevant

Computational Domain



Turbulence Computation



flute mode nature:

$k_{||} << k_{\perp}$

align coordinates to magnetic field

shift toroidal planes

Tokamak Core Conditions

- fast passing electrons

$$\frac{V_e}{qR} \gg \frac{c_s}{L_T} \qquad \text{hence expect} \quad \omega \ll k_\parallel V_e$$

- closed flux surfaces

$$k_\parallel qR = m - nq \qquad \text{hence} \quad k_\parallel \neq 0 \quad \text{and} \quad k_\parallel qR = O(1)$$

- "ignore" beta: passing electrons are adiabatic (Hasegawa/Mima)

$$\nabla_\parallel \widetilde{p}_e = n_e e \nabla_\parallel \widetilde{\phi} \qquad \text{and} \qquad \nabla_\parallel \widetilde{T}_e = 0$$

- zonal flow consideration (Dorland/Hammett)

$$\left\langle \widetilde{\phi} \right\rangle \neq 0 \qquad \text{but} \quad \langle \widetilde{n}_e \widetilde{v}_E^x \rangle = 0 \qquad \text{hence} \quad \frac{\partial}{\partial t} \langle \widetilde{n}_e \rangle = 0$$

- incorporate into polarisation, include unchanging profile piece

$$\widetilde{n}_e = \langle \widetilde{n}_e \rangle + \widetilde{\phi} - \left\langle \widetilde{\phi} \right\rangle$$

The GEM Model

- electrostatic version
  - ∘ (not the full electromagnetic gyrofluid model)
  - ∘ no “epsilon” effects (exc. curvature)

- same ordering as in fluid drift model (often called “gyrokinetic”)

- six moments for each species: density, parallel velocity, perp/parallel T’s and and $q_{\|}$’s

- version incorporating background gradient into dependent variables
  - ∘ profile variation, self consistent transport equilibrium
  - ∘ however, still homogeneous equations

- dependent variables appear with tilde symbol, others are constant parameters

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- parameters and normalisation convention

- magnetic field representation and coordinates, boundary conditions for global model

- what are “gyrofluid” and “gyroaveraging”

- moment equations and polarisation, and energetics

- run campaign based upon Cyclone project

Parameters

- drift scale ρ_s, sound speed c_s, following “electron mobility, ion inertia”
- basic profile scale $L_\perp$, hence scale ratio “drift parameter” $\delta = \rho_s / L_\perp \ll 1$
- field line connection length $2\pi qR$ hence scale ratio $qR/L_\perp \gg 1$
- low frequency drift turbulence scales

$$\rho_s = \frac{c}{eB}\sqrt{M_D T_e} \qquad c_s = \sqrt{\frac{T_e}{M_D}} \qquad \text{frequency} \quad \frac{c_s}{L_\perp} \ll \Omega_i$$

- overall edge turbulence parameters (all SMALL for “core adiabatic”)

$$\hat\beta = \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 \qquad \hat\mu = \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 \qquad C = \frac{0.51\nu_e}{c_s/L_\perp}\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2$$

- ion dynamics parameters

$$\tau_i = \frac{T_i}{T_e} \to 1 \qquad \rho_i^2 = \tau_i \times \rho_s^2 \qquad \hat\epsilon = \left(\frac{qR}{L_\perp}\right)^2 \gg 1$$

Normalisation Convention

- fold scale ratios δ and $qR/L_\perp$ into parameters, normalisation
- perp and parallel space and time derivatives

$$\rho_s \nabla_\perp \sim \text{unity} \qquad qR\nabla_\parallel \sim \text{unity} \qquad \frac{\partial}{\partial t} \sim \frac{c_s}{L_\perp}$$

- dependent variables: e.g., relative disturbance $\widetilde{n}_e/n_e \sim \delta$ normalised to unity

$$\widetilde{n}_i \leftarrow \delta^{-1}\frac{\widetilde{n}_i}{n_i} \qquad \widetilde{u}_\parallel \leftarrow \delta^{-1}\frac{\widetilde{u}_\parallel}{c_s}(L_\perp/qR) \qquad \widetilde{\phi} \leftarrow \delta^{-1}\frac{e\widetilde{\phi}}{T_e}$$

- ensures scale ratios normalised away
- note profiles (zonal state variables) appear "big" due to $\delta \ll 1$

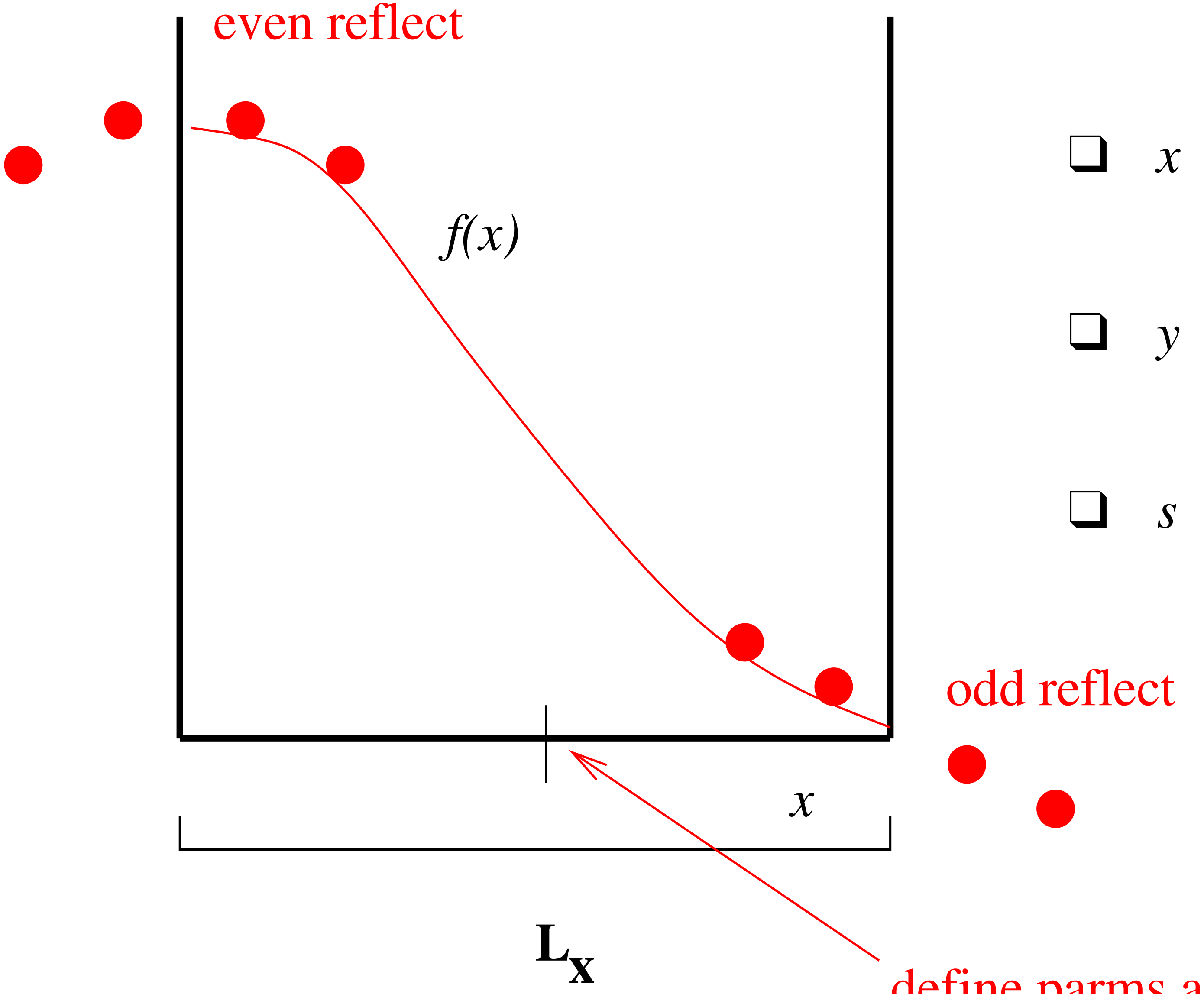
Magnetic Field

- toroidal coordinates (r, θ, ζ), neglect $1/r$ effects
- fluxtube coordinate representation (simple version, normalised)

$$x = r - a \qquad\qquad y_k = \frac{a}{q_a}\left[q(\theta - s_k) - \zeta\right] \qquad\qquad s = qR\,\theta$$

- pitch parameter $q = B^\zeta / B^\theta$
- constants $q_a = q(a)$ and s_k
- shifted metric: chosen so that $g_k^{xy} = 0$ at $s = s_k$, hence $\partial/\partial s$ incurs shifts in y since $y_{k\pm1} \neq y_k$
- perp metric is unit diagonal at $s = s_k$
- normalised magnetic field strength is unity
- $\nabla_\perp^2$ and $\mathbf{v}_E \cdot \nabla$ and $\mathcal{K}$ involve only perp coordinates (x, y_k)
- geometric quantities depend only upon parallel coordinate s
- (all can be generalised to an arbitrary separatrix-free tokamak equilibrium)

Boundary Conditions



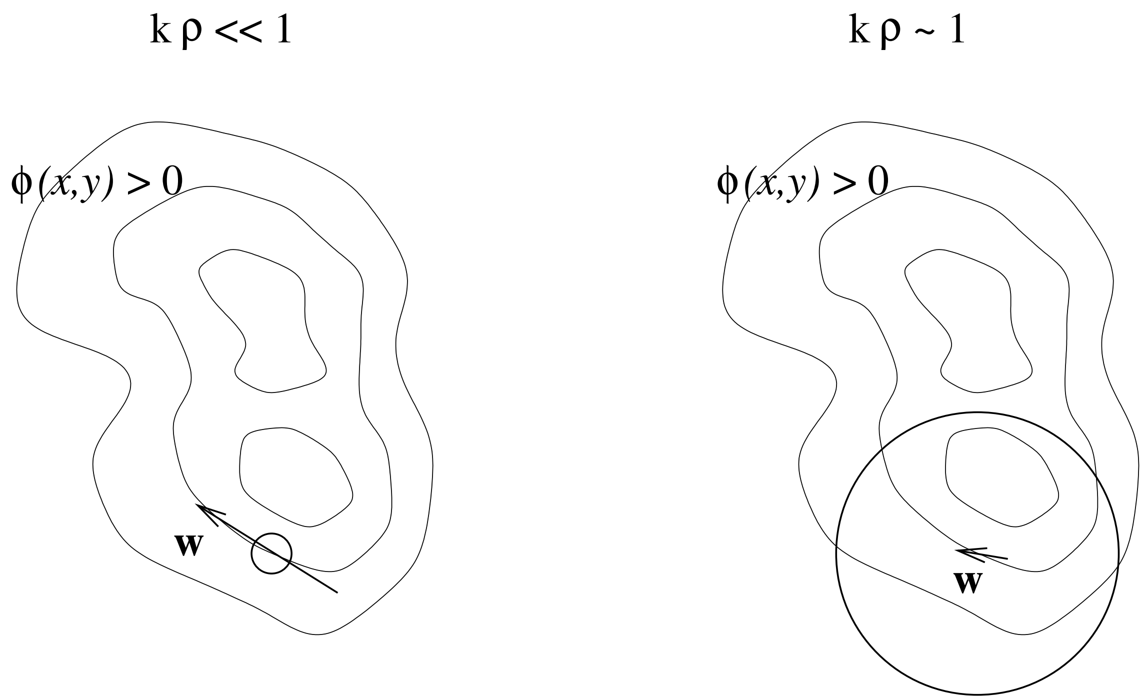
- x Neumann/Dirichlet
- y periodic
- s pseudo periodic (shifted metric)

2 guard cells req'd for derivs

The Gyrofluid Model

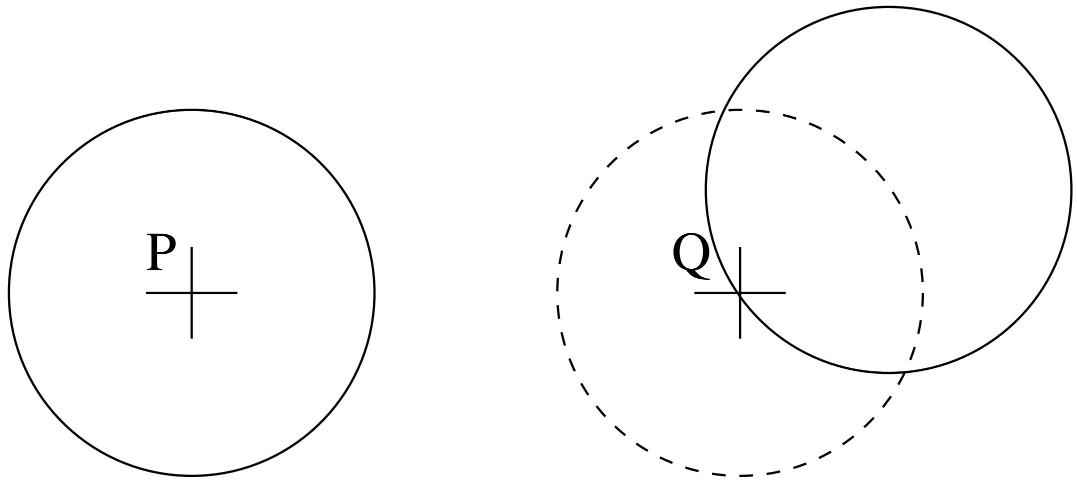
- low frequencies $\omega \ll \Omega_c = eB/mc$ for each species

treat particles as rings of charge in spatially varying fields



- gyrofluid: moment equations substituted for the gyrokinetic equation

Gyrokinetic/Gyrofluid Polarisation



gyrocenter at P senses potential at points on the ring

gyrocenters on dashed ring contribute charge to Q

mathematics identical in both procedures (Hermitian operators)

polarisation density <–– part of f dependent upon gyrophase angle

Gyroaveraging

- main point: expansion of model due to temperature variations
- gyroaverage from kinetic theory

$$\widetilde{\phi}_J(\mathbf{k}, v_\perp) = J_0(k_\perp v_\perp/\Omega)\,\widetilde{\phi}(\mathbf{k}) \qquad \text{follows from} \quad J_0(x) = \oint \frac{d\theta}{2\pi}\, e^{ix\cos\theta}$$

- symmetric role with gyrocenter density

$$\widetilde{n}_i(\text{space}) = \int dW\, FT^{-1}[J_0(k_\perp v_\perp/\Omega)\, f(\mathbf{k}, v_\perp)]$$

- fluid gyrocenter density

$$\frac{\widetilde{n}_i}{n_i}(\text{space}) = \Gamma_1\left(\frac{\widetilde{n}_i}{n_i}\right) + \Gamma_2\left(\frac{\widetilde{T}_i}{T_i}\right)$$

- gyroaveraging operators

$$\Gamma_1 = \Gamma_0^{1/2}(k_\perp^2\rho_i^2) \qquad \Gamma_2 = \frac{\partial}{\partial \log T_i}\Gamma_1$$

- use same operators also on the potential

$$\widetilde{\phi}_G = \Gamma_1(\widetilde{\phi}) \qquad \widetilde{\Omega}_G = \Gamma_2(\widetilde{\phi})$$

Electrostatic Moment Equations for Ions

$$\frac{d\widetilde{n}_i}{dt} + \mathbf{w}_E \cdot \nabla \widetilde{T}_{i\perp} = -B\nabla_\parallel \frac{\widetilde{u}_\parallel}{B} + \mathcal{K}\left(\widetilde{\phi}_G + \frac{\tau_i \widetilde{p}_{i\parallel} + \tau_i \widetilde{p}_{i\perp} + \widetilde{\Omega}_G}{2}\right)$$

$$\hat{\epsilon}\frac{d\widetilde{u}_\parallel}{dt} + \hat{\epsilon}\mathbf{w}_E \cdot \nabla \widetilde{q}_{i\perp} = -\nabla_\parallel \left(\widetilde{\phi}_G + \tau_i \widetilde{p}_{i\parallel}\right) + \frac{\hat{\epsilon}}{2}\tau_i \mathcal{K}\left(4\widetilde{u}_\parallel + 2\widetilde{q}_{i\parallel} + \widetilde{q}_{i\perp}\right)$$

$$\frac{1}{2}\frac{d\widetilde{T}_{i\parallel}}{dt} = -B\nabla_\parallel \frac{\widetilde{u}_\parallel + \widetilde{q}_{i\parallel}}{B} + \mathcal{K}\left(\frac{\widetilde{\phi}_G + \tau_i \widetilde{p}_{i\parallel}}{2} + \tau_i \widetilde{T}_{i\parallel}\right)$$

$$\frac{d\widetilde{T}_{i\perp}}{dt} + \mathbf{w}_E \cdot \nabla \left(\widetilde{n}_i + 2\widetilde{T}_{i\perp}\right) = -B\nabla_\parallel \frac{\widetilde{q}_{i\perp}}{B} + \mathcal{K}\left(\frac{\widetilde{\phi}_G + \tau_i \widetilde{p}_{i\perp} + \widetilde{\Omega}_G}{2} + 3\tau_i \frac{\widetilde{T}_{i\perp} + \widetilde{\Omega}_G}{2}\right)$$

$$\hat{\epsilon}\frac{d\widetilde{q}_{i\parallel}}{dt} + a_{LD}\left(\widetilde{q}_{i\parallel}\right) = -\frac{3}{2}\nabla_\parallel \left(\tau_i \widetilde{T}_{i\parallel}\right) + \frac{\hat{\epsilon}}{2}\tau_i \mathcal{K}\left(3\widetilde{u}_\parallel + 8\widetilde{q}_{i\parallel}\right)$$

$$\hat{\epsilon}\frac{d\widetilde{q}_{i\perp}}{dt} + \hat{\epsilon}\mathbf{w}_E \cdot \nabla \left(\widetilde{u}_\parallel + 2\widetilde{q}_{i\perp}\right) + a_{LD}\left(\widetilde{q}_{i\perp}\right) = -\nabla_\parallel \left(\tau_i \widetilde{T}_{i\perp} + \widetilde{\Omega}_G\right) + \frac{\hat{\epsilon}}{2}\tau_i \mathcal{K}\left(\widetilde{u}_\parallel + 6\widetilde{q}_{i\perp}\right)$$

- gyroaveraged and FLR ExB velocities

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla \qquad \mathbf{u}_E = -\widehat{\mathbf{F}} \cdot \nabla \widetilde{\phi}_G \qquad \mathbf{w}_E = -\widehat{\mathbf{F}} \cdot \nabla \widetilde{\Omega}_G$$

Polarisation

- species moments $\rightarrow$ potentials, only place various species communicate
- gyrofluid Poisson equation (quasineutrality)

$$\Gamma_1(\widetilde{n}_i) + \Gamma_2(\widetilde{T}_{i\perp}) + \frac{\Gamma_0 - 1}{\tau_i}\widetilde{\phi} = \widetilde{n}_e$$

- adiabatic electrons

$$\widetilde{n}_e = \langle \widetilde{n}_e \rangle + \widetilde{\phi} - \left\langle \widetilde{\phi} \right\rangle$$

- note zonal mode has $k_\parallel = 0$ everywhere, hence not adiabatic
- FLR evaluation: finite-difference in x, Padé approximations

$$\Gamma_0^{1/2} = \left(1 - \frac{\rho_i^2}{2}\nabla_\perp^2\right)^{-1} \qquad \Gamma_0 = \left(1 - \rho_i^2\nabla_\perp^2\right)^{-1}$$

GEM Energetics

- very similar to fluid version, things in different places
- role of polarisation: ExB energy looks like a charge potential energy
  - multiply by $\widetilde{\phi}$, use Hermitian property of Γ's

$$\widetilde{\phi}\ \Gamma_1(\widetilde{n}_i) + \widetilde{\phi}\ \Gamma_2(\widetilde{T}_{i\perp}) + \frac{\Gamma_0 - 1}{\tau_i}\widetilde{\phi}^2 = \widetilde{n}_e\widetilde{\phi}$$

$$\widetilde{\phi}_G\widetilde{n}_i + \widetilde{\Omega}_G\widetilde{T}_{i\perp} + \frac{\Gamma_0 - 1}{\tau_i}\widetilde{\phi}^2 = \widetilde{n}_e\widetilde{\phi}$$

- potential (ExB) energy, use time independence of Γ's

$$\frac{\partial}{\partial t}\frac{1-\Gamma_0}{\tau_i}\frac{\widetilde{\phi}^2}{2} = \widetilde{\phi}_G\,\frac{\partial\widetilde{n}_i}{\partial t} + \widetilde{\Omega}_G\,\frac{\partial\widetilde{T}_{i\perp}}{\partial t} - \widetilde{\phi}\frac{\partial\widetilde{n}_e}{\partial t}$$

- thermal free energy (local eqs: like entropy), and parallel kinetic energy

$$\frac{\partial}{\partial t}\left[\tau_i\frac{\widetilde{n}_i^2 + (1/2)\widetilde{T}_{i\parallel}^2 + \widetilde{T}_{i\perp}^2}{2} + \frac{\widetilde{n}_e^2}{2}\right] \qquad \frac{\partial}{\partial t}\left[\hat{\epsilon}\frac{\widetilde{u}_\parallel^2 + (2/3)\widetilde{q}_{i\parallel}^2 + \widetilde{q}_{i\perp}^2}{2}\right]$$

Conservation

- main point: $\widetilde{n}_i$ and $\widetilde{\phi}_G$, and $\widetilde{T}_{i\perp}$ and $\widetilde{\Omega}_G$, go together
- ExB and thermal free energy combine

$$\left(\widetilde{\phi}_G + \tau_i \widetilde{n}_i\right) \frac{\partial \widetilde{n}_i}{\partial t} + \left(\widetilde{\Omega}_G + \tau_i \widetilde{T}_{i\perp}\right) \frac{\partial \widetilde{T}_{i\perp}}{\partial t} = \cdots$$

- Γ_1 appears through $\widetilde{\phi}_G$ (monents) and $\widetilde{n}_i$ (polarisation)
- Γ_2 appears through $\widetilde{\Omega}_G$ (monents) and $\widetilde{T}_{i\perp}$ (polarisation)
- those pairs of quantities appear together under derivatives (except ExB advection)

Landau Closure

- dissipative terms in heat flux eqs. calibrated to get kinetic response function

$$a_{LD}\left(\widetilde{q}_{i\parallel}\right) \to \left\langle k_\parallel V_i \widetilde{q}_{i\parallel}\right\rangle \to \hat{\epsilon}^{1/2}\left(1 - 0.125\nabla_\parallel^2\right)\widetilde{q}_{i\parallel}$$

- remove the closure terms for $k_y = 0$ modes

$$a_{LD}\left(\widetilde{q}_{i\parallel}\right) \to \hat{\epsilon}^{1/2}\left(1 - 0.125\nabla_\parallel^2\right)\left(\widetilde{q}_{i\parallel} - \int \frac{dy}{L_y}\widetilde{q}_{i\parallel}\right)$$

- basic issue: closure accurate for waves, but not for equilibrium
- Rosenbluth/Hinton: trapped ions → undamped residual flows
- caveat: neoclassical ordering, residual comes from initial zonal flow layer
  - ◦ applicability to turbulent forcing below diamagnetic flow level uncertain
- study possible effects: switch LD in and out of $k_y = 0$ mode

Cyclone Base Case

- hot core parameters – “hot ion H-mode” from D-III-D, eval. at $r = a/2$

$$T_i = T_e = 2\,\mathrm{keV} \qquad n_e = 4.5 \times 10^{13}\,\mathrm{cm}^{-3} \qquad B = 1.91\,\mathrm{T}$$

$$R = 169.6\,\mathrm{cm} \qquad a = 62.5\,\mathrm{cm} \qquad L_T = R/6.9$$

$$M_i = M_D = 3670\,m_e$$

$$L_T/L_n = 1/3.114 \qquad q = 1.4 \qquad \hat{s} = 0.78$$

- physical global domain size (negl. elongation)

$$a = 192\rho_s \qquad 2\pi r/q = 350\rho_s$$

Global Model

- adiabatic electron global model – $\hat{\beta}$ and $\hat{\mu}$ and C do not enter
- electrons: background n_e and T_e, with $v_\parallel = u_\parallel$

$$\widetilde{n}_e = n_e + \widetilde{\phi} - \left\langle \widetilde{\phi} \right\rangle$$

- ions: full six moment model, normalised parameters

$$\hat{\epsilon} = 90.9 \qquad \omega_B = 0.29 \qquad \hat{s} = 0.78 \qquad \omega_n = 0.321$$

- domain size, depends on rho-star value used; nominal model is

$$K = 0.025 \qquad L_x = L_y = 2\pi/K \qquad -\pi < s < \pi$$

- "global conditions" with constant parameters; for all variables f,

$$\frac{\partial f}{\partial x} = 0 \quad \text{at} \quad x = -\frac{L_x}{2} \qquad\qquad f = 0 \quad \text{at} \quad x = \frac{L_x}{2}$$

Initialisation – Run Campaign

- initial profiles

$$T_e = T_i = \frac{L_x}{2}\left(1 - \sin\frac{\pi x}{L_x}\right) \qquad\qquad n_e = n_i = T_e/3.114$$

- note *average* gradient is equal to unity

- turbulence – random bath of "ballooning blobs"
  - random phase distribution in drift plane
  - exponential parallel envelope
  - parallel structure aligned to magnetic field

- no sources — profiles allowed to decay

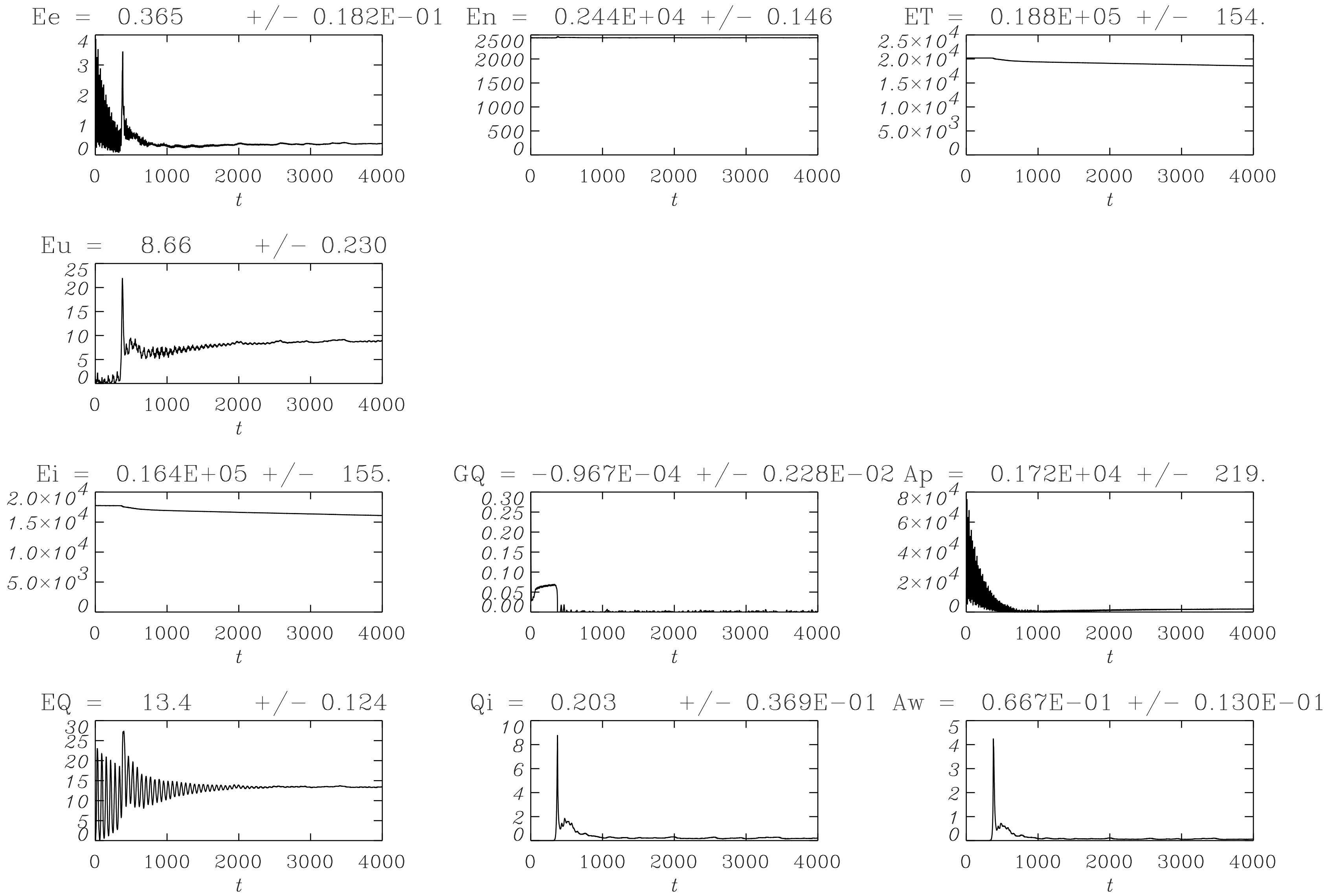
- nominal case shows minimal decay, with

$$\tau_E \gg \text{run time}$$

- parameter sweep: temperature gradient $\omega_i = \omega_t$, nominally 1.0

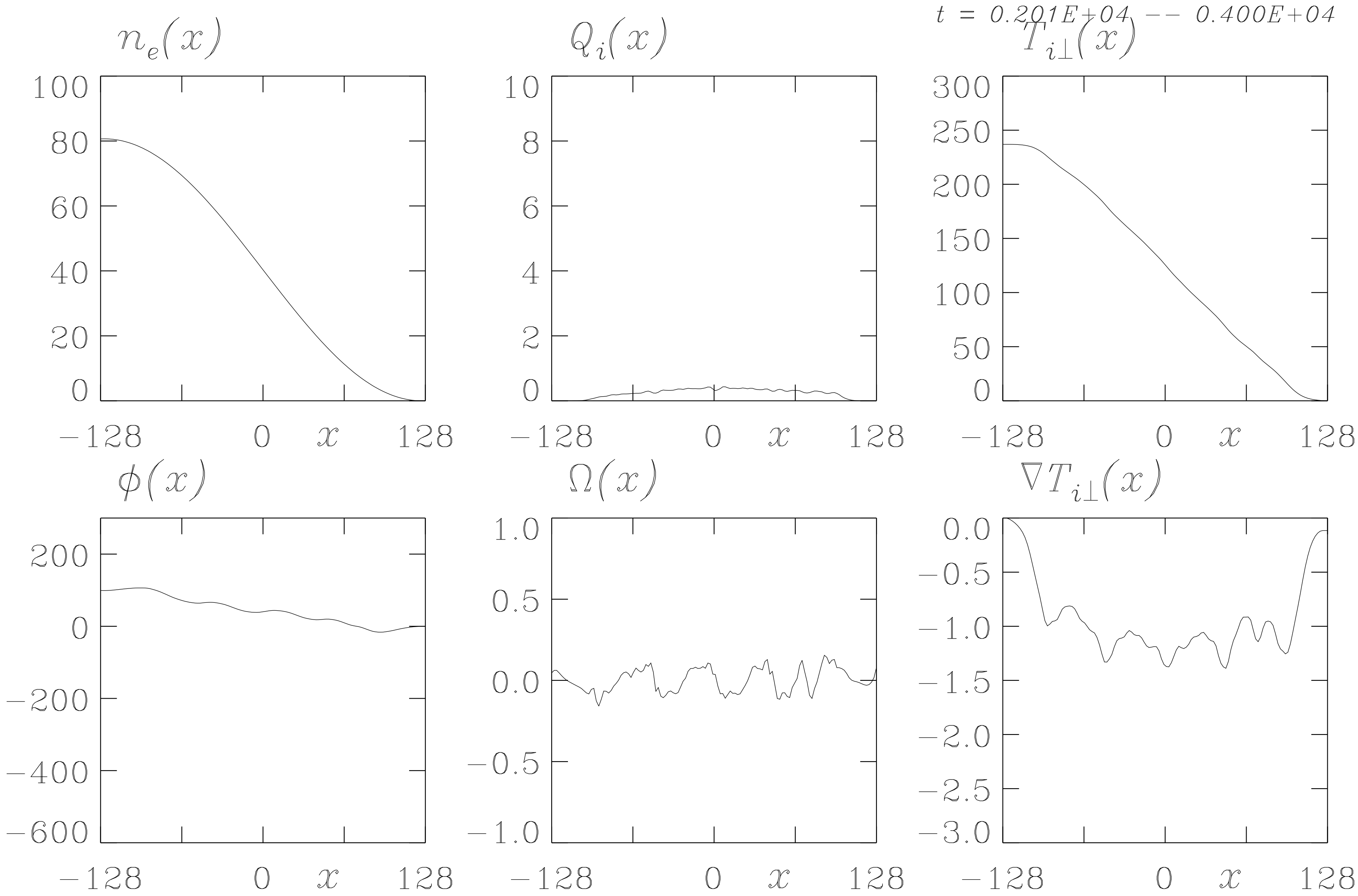
$$\eta_i = 3.114 \times \omega_i \qquad R/L_T = 6.9 \times \omega_i \qquad T_{i0} = T_{e0} = \omega_i \times \text{nominal}$$

Base Case Run with LD on Waves Only



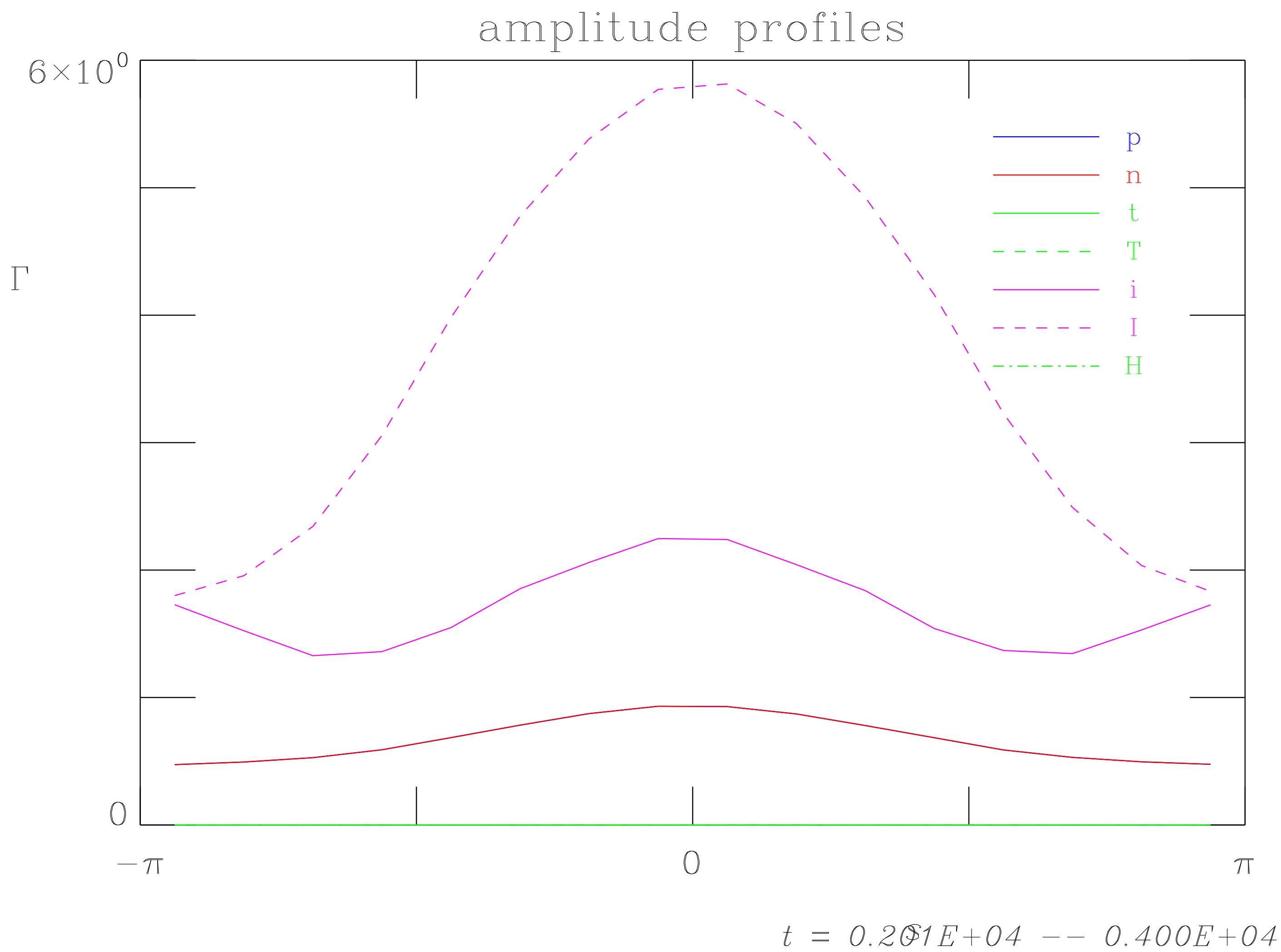
zonal profiles

Landau damping on waves only



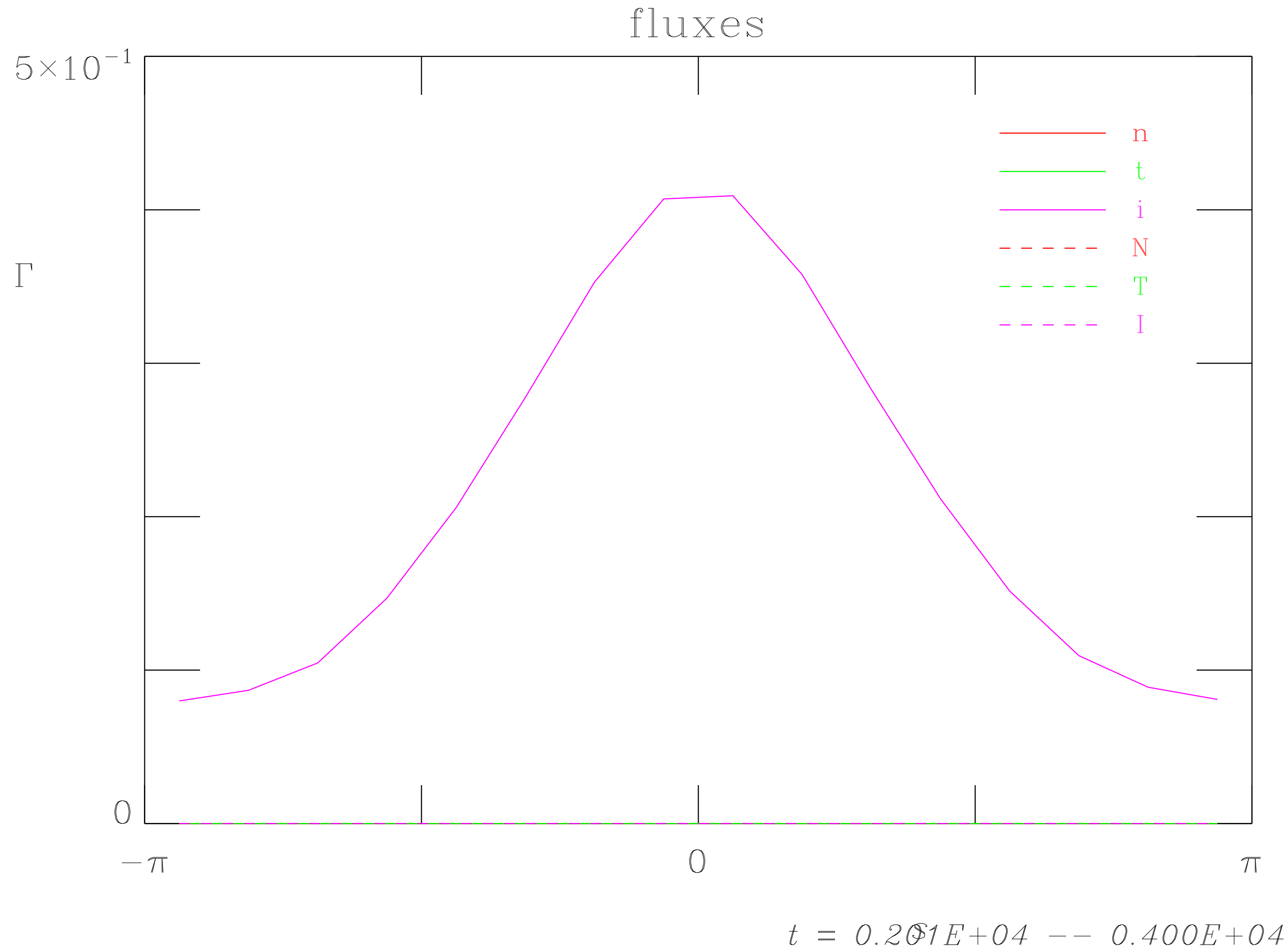
parallel mode structure

Landau damping on waves only



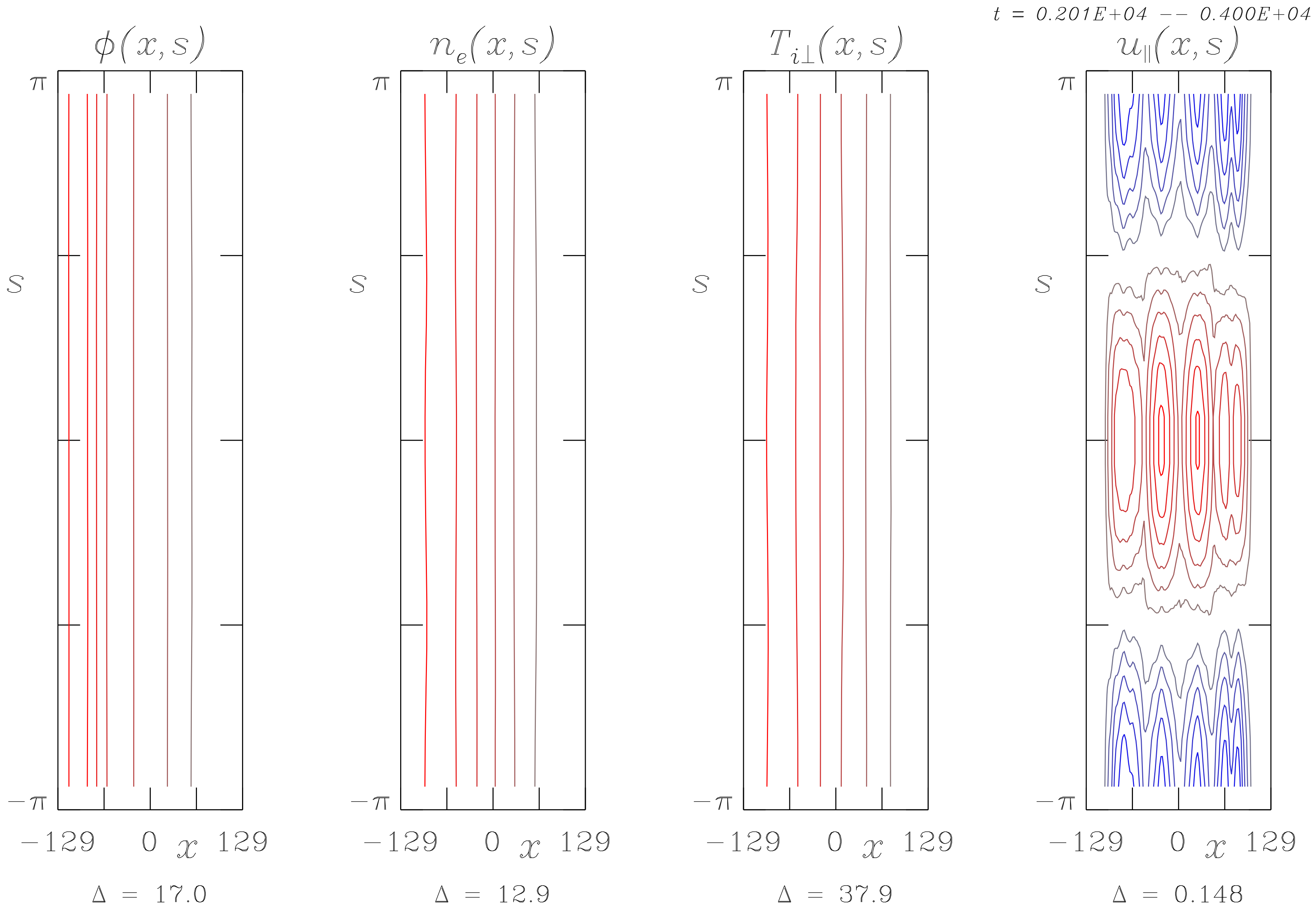
transport ballooning

Landau damping on waves only

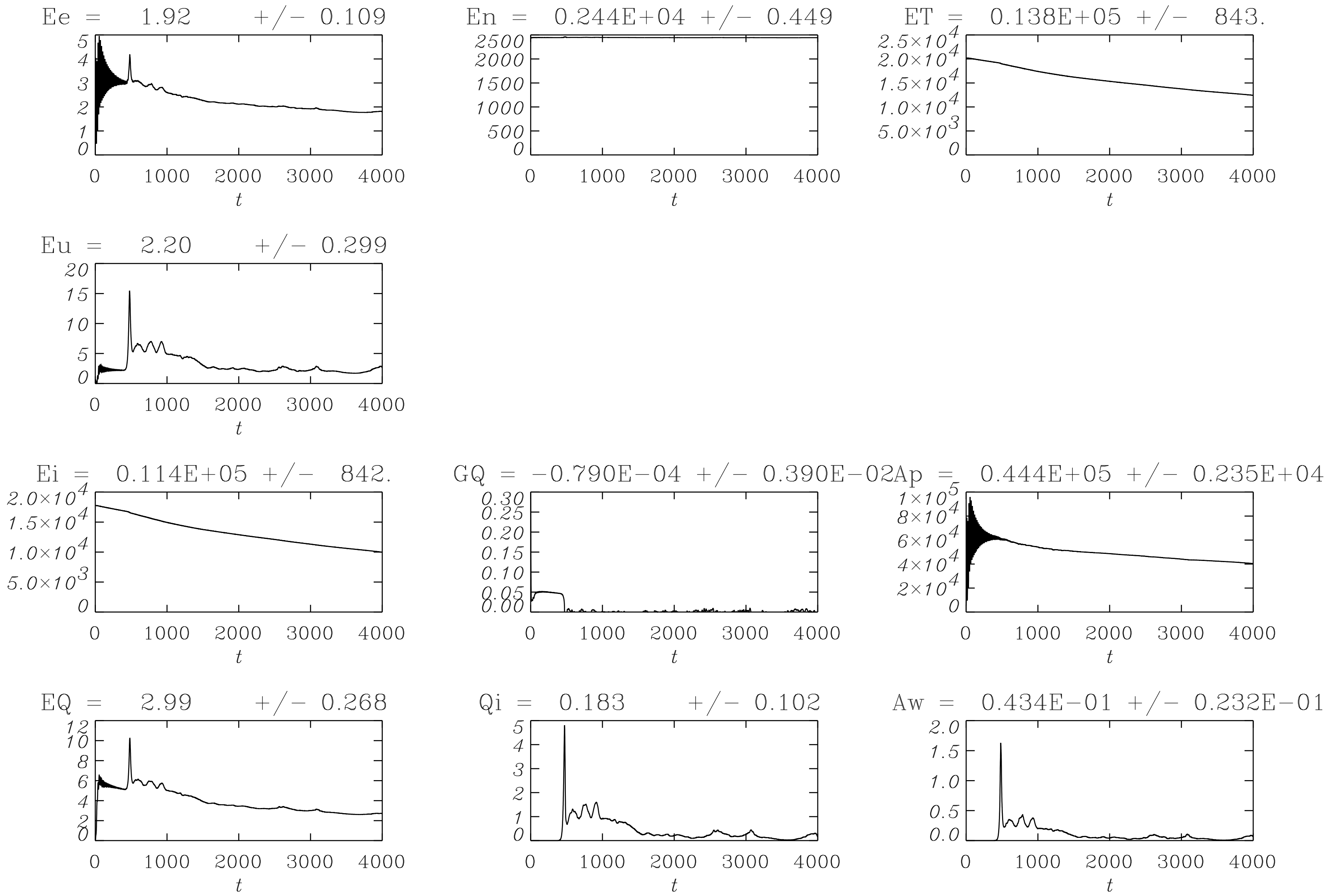


poloidal plane morphology

Landau damping on waves only

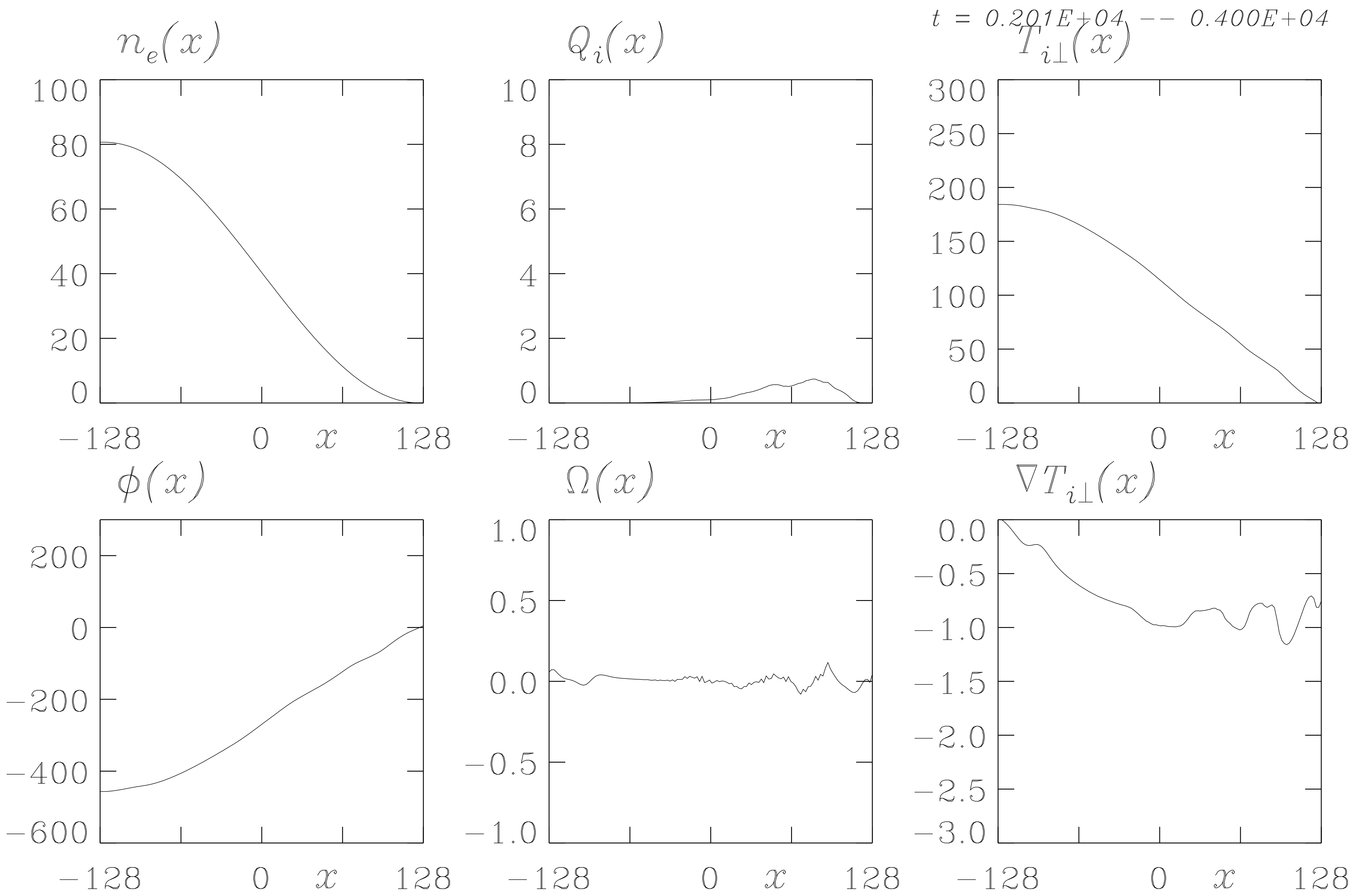


Base Case Run with LD on All Modes



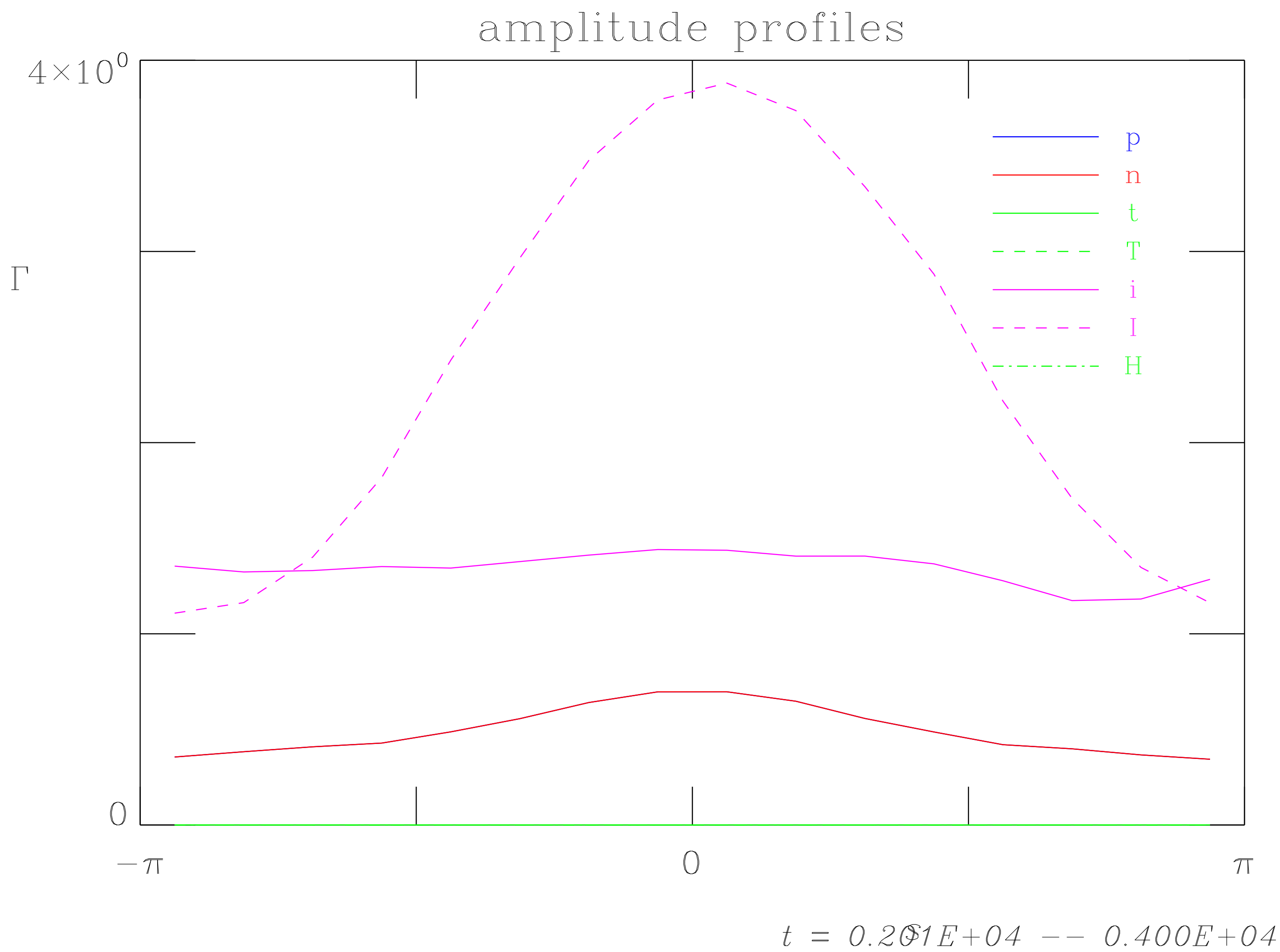
zonal profiles

Landau damping on all modes



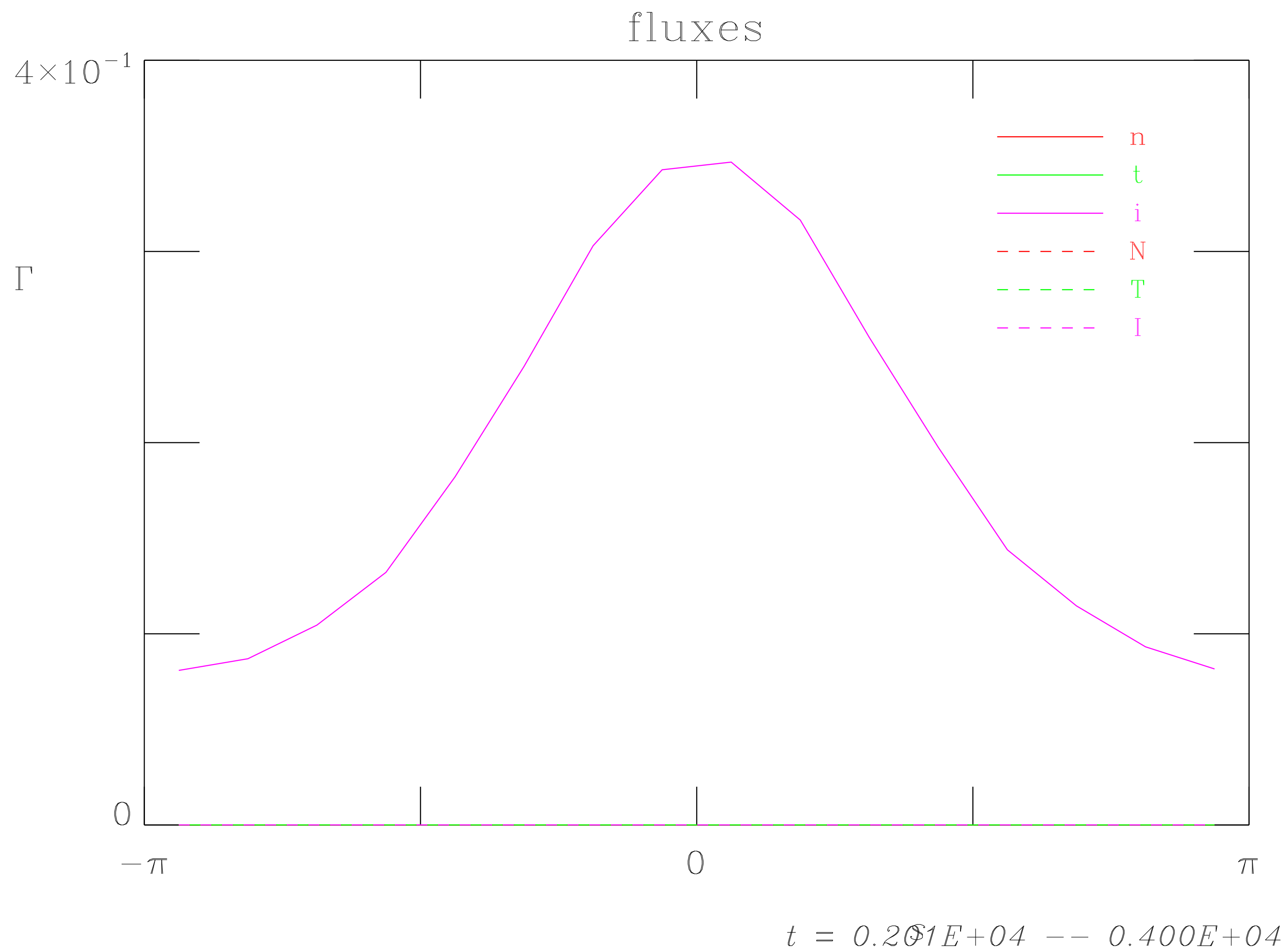
parallel mode structure

Landau damping on all modes



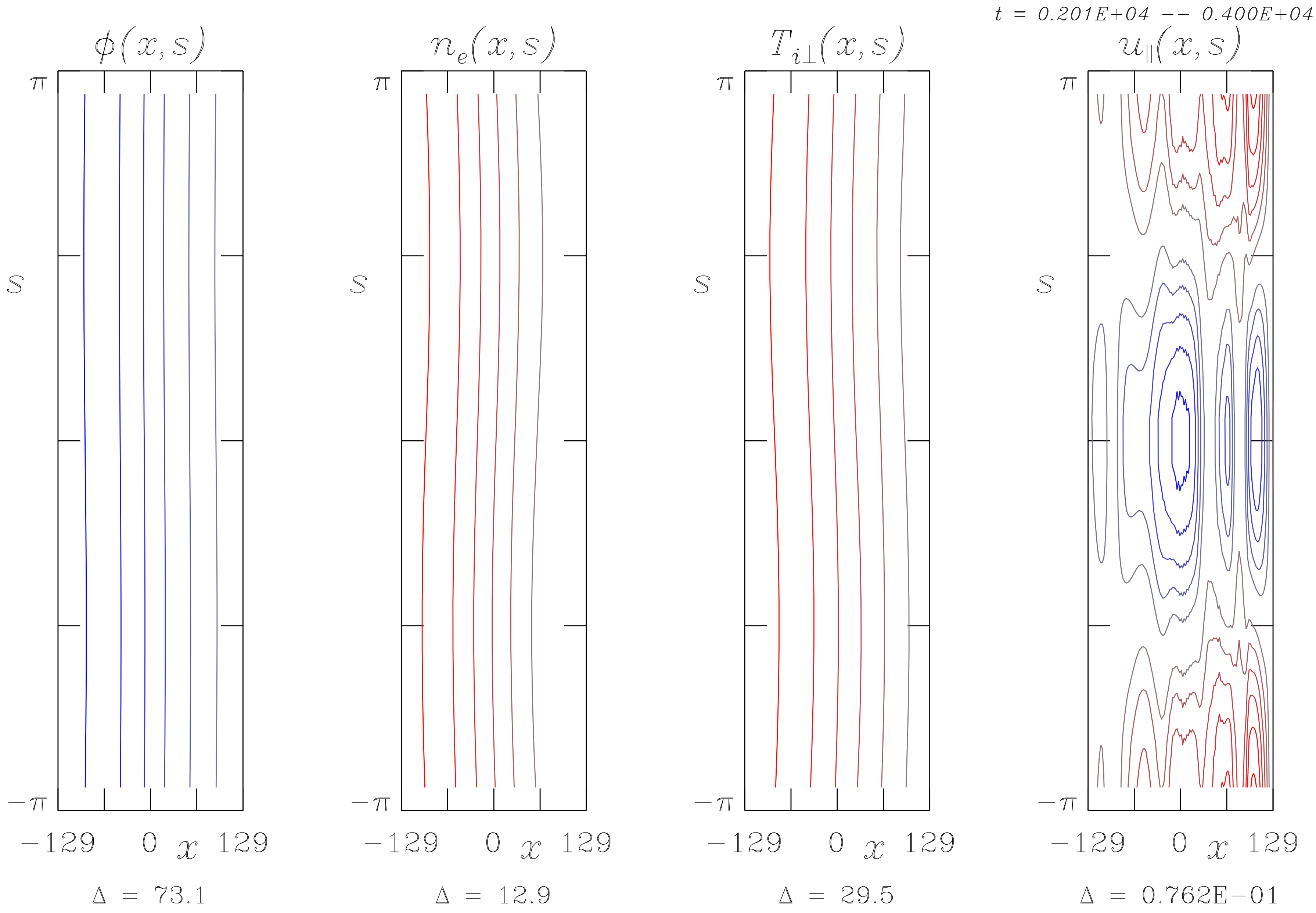
transport ballooning

Landau damping on all modes



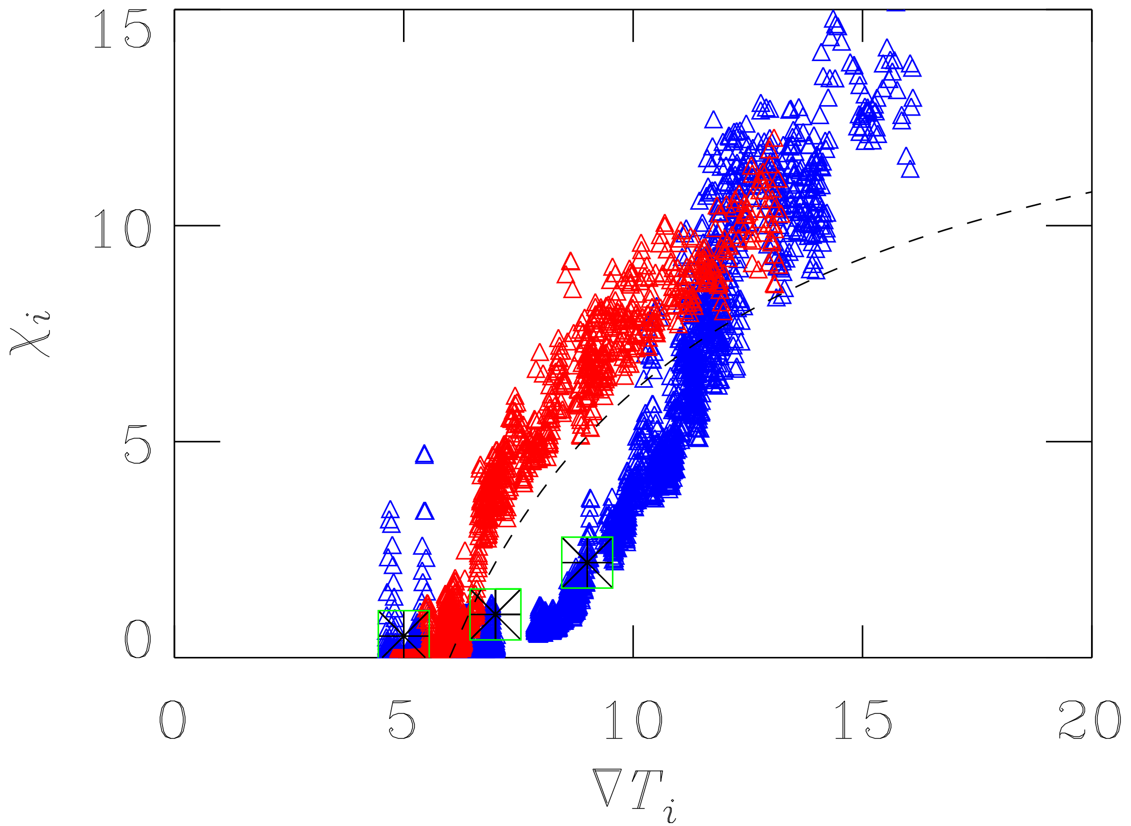
poloidal plane morphology

Landau damping on all modes



Transport Result

- LD on all modes (red) waves only (blue) Dimits (dashed) Sydora (green)



- uncertain conclusion: agreement in principle with either
  - Sydora probably did not keep toroidal ExB compression

Summary and Outlook

- **Basic Results**

  — standard ITG turbulence

  — large changes (better?) if LD does not act on $k_y = 0$ mode

  — nonlinear upshift of threshold, similar to GK results

  — decaying cases most relevant to realistic core situation

- **Issues for Discussion**

  — thresholds, linear and nonlinear

  — transport curve more "gyro-Bohm" away from threshold

  — box size problems for older studies (only ca. $100\rho_s$)

  — particle code problems, maybe finally solved only by GENE

- **Future Generalisation**

  — size scaling

  — shear reversal

  — why gyrofluid is still relevant