Basic Characteristics of Edge Turbulence with Zonal Flow and Global Alfvén Interactions

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Outline

- **Basic Ideas**

  — tokamak geometry, computational domain
  — basic dynamics, scales, computational resolution
  — gyrofluid model GEM3, treatment of various species
  — boundary conditions for “global” computation

- **Basic Results**

  — standard DALF turbulence
  — **nonlinear self sustained drift wave instability**
  — zonal flows and sidebands, global Alfvén transient → P-S current (2D MHD equilibrium)

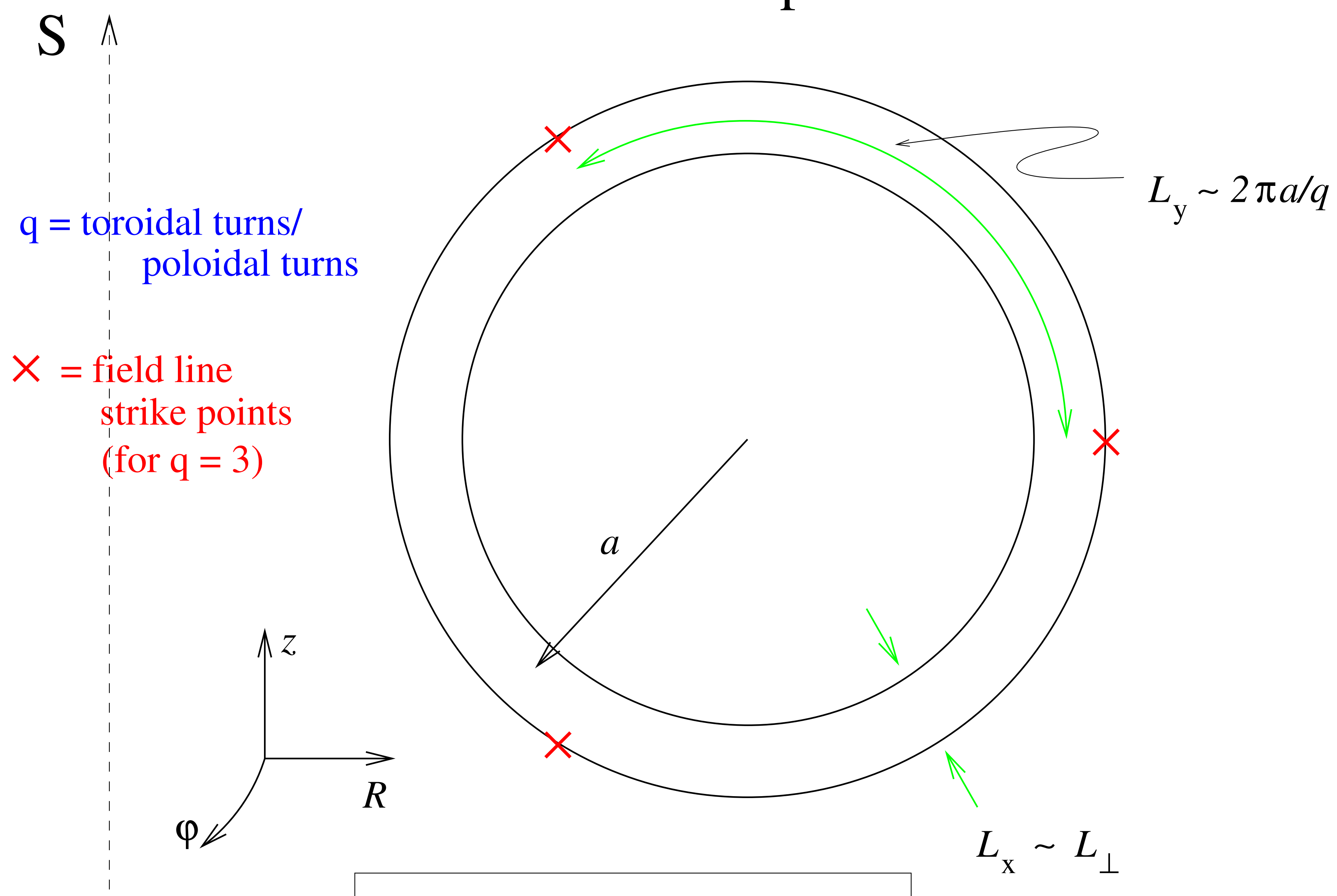
- **Specific Applications**

  — transport of a trace fluid (e.g., tritium)
  — SOL/edge coupling

- **Future Generalisation**

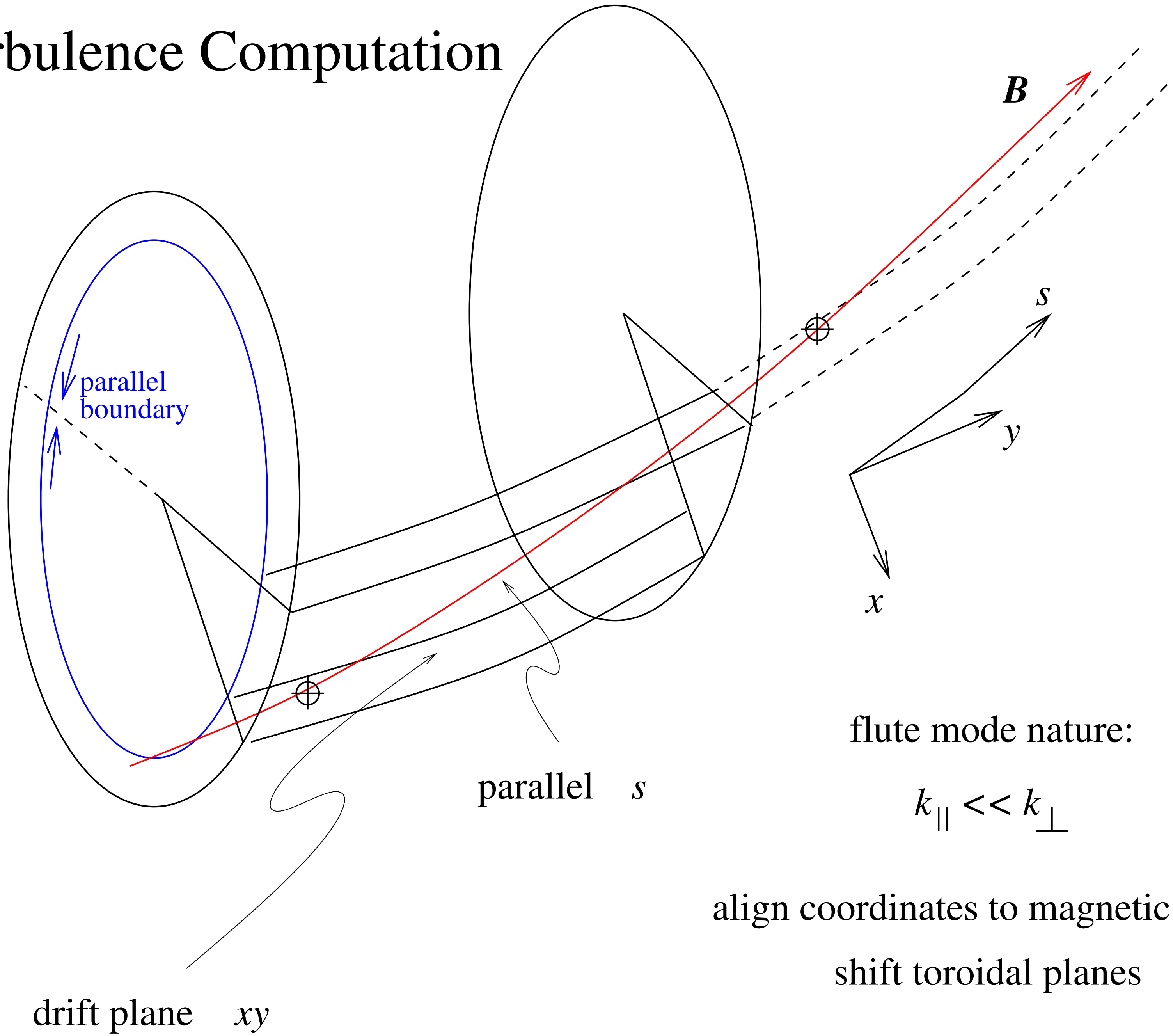
  — edge/core coupling, requires full inhomogeneity
  — hot trace ion adiabaticity (diamagnetic flows)

Computational Domain



large aspect ratio: $L_y >> L_x$

Turbulence Computation



flute mode nature:

$k_{||} << k_{\perp}$

align coordinates to magnetic field

shift toroidal planes

Magnetic Field

- toroidal coordinates (r, θ, ζ)

- **field aligned coordinate representation** (simple version, normalised)

$$x = r - a \qquad\qquad y_k = \frac{a}{q_a}\left[q(\theta - s_k) - \zeta\right] \qquad\qquad s = \theta$$

- pitch parameter $q(r) = B^\zeta / B^\theta$, constants $q_a = q(a)$ and s_k

- **globally consistent boundary conditions**
  - (preserves set of allowed $k_\parallel$ values)
  - **required to capture field line connection** (B Scott, Phys Plasmas 5 [1998] 2334)

- **shifted metric**: chosen so that $g_k^{xy} = 0$ at $s = s_k$, hence ...
  - perp metric is unit diagonal at $s = s_k$
  - $\partial/\partial s$ incurs shifts in y since $y_{k\pm1} \neq y_k$
  - **required to capture slab modes** (B Scott, Phys Plasmas 8 [2001] 447)

- normalised magnetic field strength is unity

- $\nabla_\perp^2$ and $\mathbf{v}_E \cdot \nabla$ and $\mathcal{K}$ involve only perp coordinates (x, y_k)

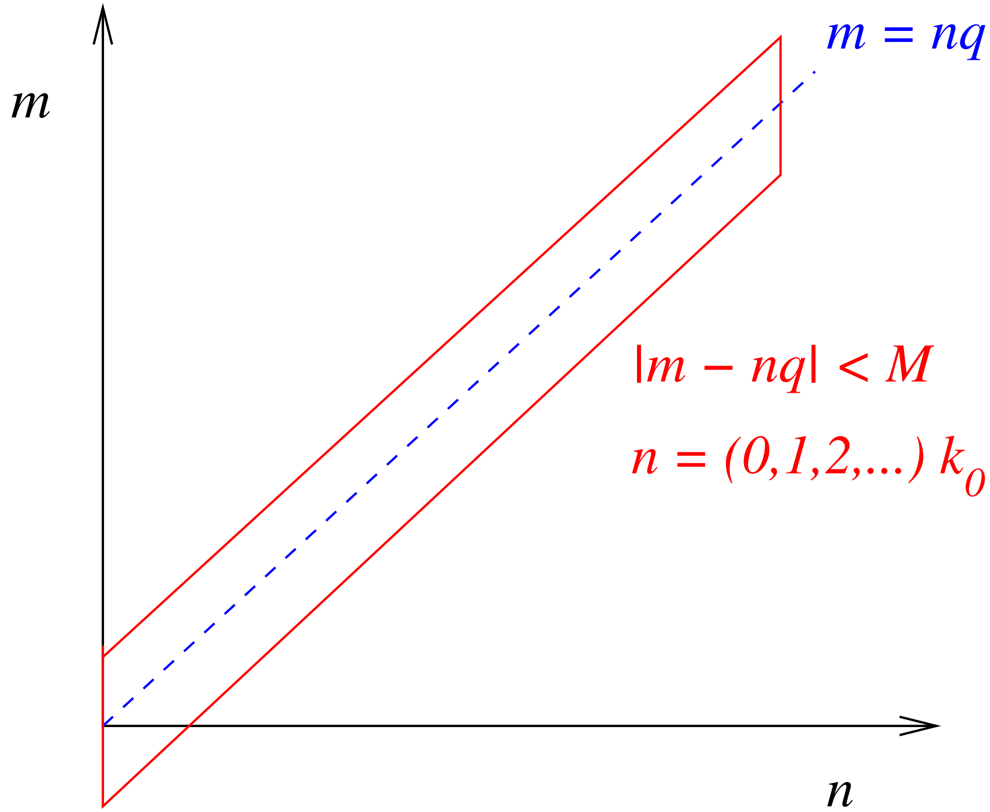
- geometric quantities depend only upon parallel coordinate s

- (all can be generalised to an arbitrary separatrix-free tokamak equilibrium)

coordinates

- aligned to mag field
- reference q sweeps with q(r)
- require far fewer grid nodes than for traditional methods
  scalability!
- resolve scales:
  gradient to ion gyroradius
  require >100 toroidal modes!

mode space



global consistency

$$k_{\|} qR = m - nq$$

parallel grid nodes

$$M << n \ (\text{delta } q)$$

The GEM3 Model

- simplest electromagnetic gyrofluid model   (B Scott, PPCF 45 [2003] A385)
- same ordering as in fluid drift model (often called "gyrokinetic")
- two moments for each species: density, parallel velocity
- version incorporating background gradient into dependent variables
  - profile variation, self consistent transport equilibrium
  - however, still homogeneous equations

- parameters and normalisation convention
- magnetic field representation and coordinates
- moment equations and polarisation

- full generalisation (not shown): six moments (incl $T_\perp$ and $T_\|$ and $q_{\perp\|}$ and $q_{\|\|}$)

Parameters

- **gyro Bohm normalisation**, to T_e and M_D
- drift scale ρ_s, sound speed c_s, following “electron mobility, ion inertia”
- basic profile scale $L_\perp$, hence scale ratio “drift parameter” $\delta = \rho_s/L_\perp \ll 1$
- field line connection length $2\pi qR$ hence scale ratio $qR/L_\perp \gg 1$
- low frequency drift turbulence scales

$$\rho_s = \frac{c}{eB}\sqrt{M_D T_e} \qquad c_s = \sqrt{\frac{T_e}{M_D}} \qquad \text{frequency} \quad \frac{c_s}{L_\perp} \ll \Omega_i$$

- overall edge turbulence parameters (all > 1 but not $\gg 1$)

$$\hat{\beta} = \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 \qquad \hat{\mu} = \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 \qquad C = \frac{0.51\nu_e}{c_s/L_\perp}\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2$$

- background species parameters

$$a_z = \frac{Zn_z}{n_e} \qquad \tau_z = \frac{T_z}{ZT_e} \qquad \rho_z^2 = \frac{M_z T_z}{Z^2 M_D T_e} \qquad \epsilon_z = \frac{M_z}{ZM_D}\left(\frac{qR}{L_\perp}\right)^2$$

Moment Equations for Each Species, Polarisation

- density moment with source

$$\frac{d\widetilde{n}_z}{dt} = -B\nabla_\parallel \frac{\widetilde{u}_{z\parallel}}{B} + \mathcal{K}\left(\widetilde{\phi}_G + \tau_z \widetilde{n}_z\right) + S$$

- velocity moment

$$\hat{\beta}\frac{\partial \widetilde{A}_\parallel}{\partial t} + \epsilon_z \frac{d\widetilde{u}_{z\parallel}}{dt} + C\widetilde{J}_\parallel = -\nabla_\parallel\left(\widetilde{\phi}_G + \tau_z \widetilde{n}_z\right) + \epsilon_z \tau_z \mathcal{K}\left(2\widetilde{u}_{z\parallel}\right)$$

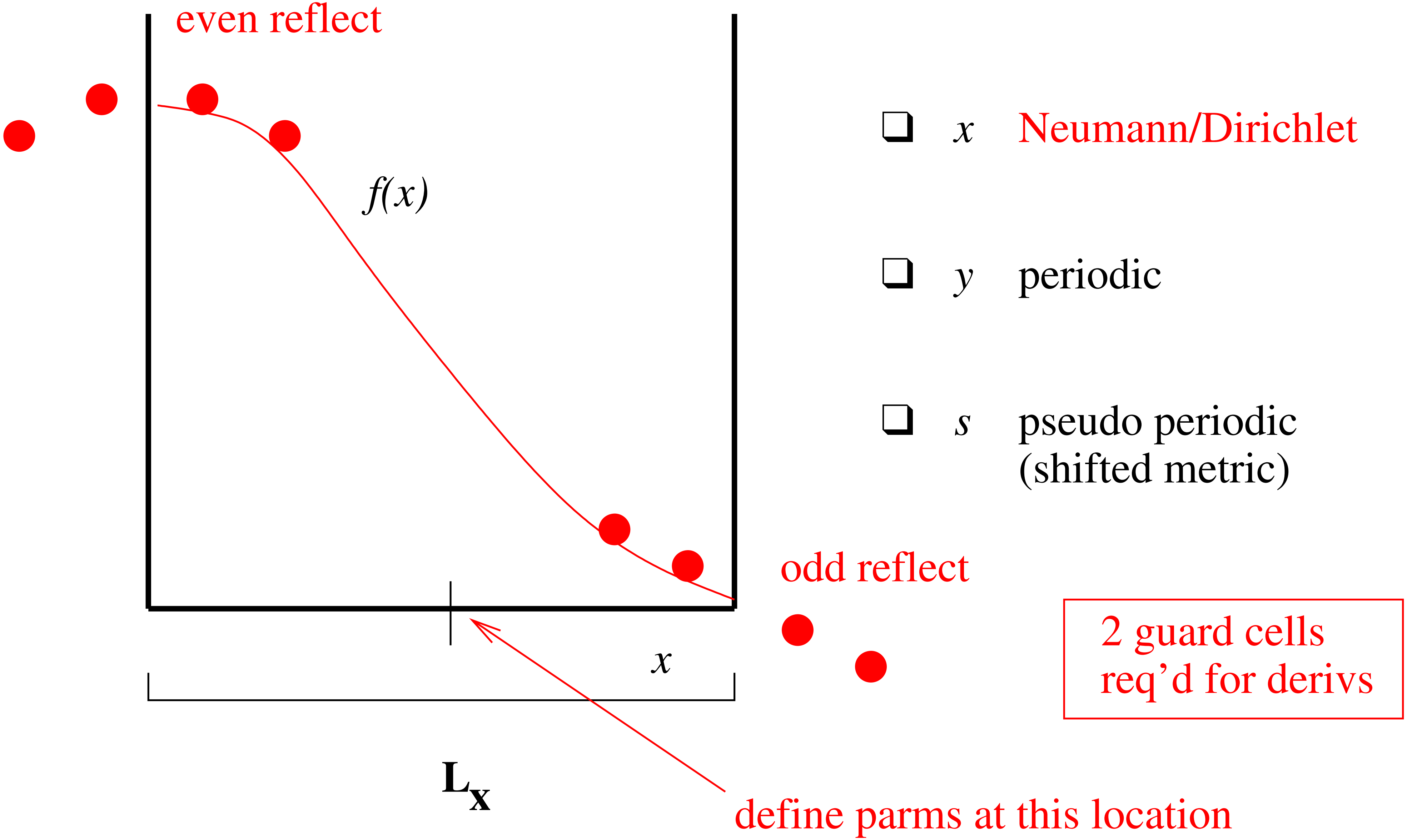
- gyroaveraged potential, species specific

$$\widetilde{\phi}_G = \Gamma_0^{1/2}(\widetilde{\phi}), \qquad \text{with} \qquad \Gamma_0 = \Gamma_0(k_\perp^2 \rho_z^2)$$

- polarisation: Poisson (quasineutrality) and Ampere
  - ◦ species moments → potentials, only place various species communicate

$$\sum_z a_z \left[\Gamma_0^{1/2}(\widetilde{n}_z) + \frac{\Gamma_0 - 1}{\tau_z}\widetilde{\phi}\right] = 0 \qquad\qquad -\nabla_\perp^2 \widetilde{A}_\parallel = \sum_z a_z \widetilde{u}_{z\parallel}$$

Boundary Conditions



- x Neumann/Dirichlet
- y periodic
- s pseudo periodic (shifted metric)

GEM3 Energetics

- very similar to fluid version, things in different places
  - ◦ role of polarisation: ExB energy looks like a charge potential energy

- potential (ExB) energy ($\Gamma_0^{1/2}$ is Hermitian ...)

$$\frac{\partial}{\partial t}\sum_z a_z \, \frac{1-\Gamma_0}{\tau_z}\frac{\widetilde{\phi}^2}{2} = \sum_z a_z \, \widetilde{\phi}_z \, \frac{\partial \widetilde{n}_z}{\partial t}$$

- thermal free energy (local eqs: like entropy)

$$\frac{\partial}{\partial t}\sum_z a_z \tau_z \, \frac{\widetilde{n}_z^2}{2} = \sum_z a_z \tau_z \widetilde{n}_z \, \frac{\partial \widetilde{n}_z}{\partial t}$$

- parallel kinetic energy

$$\frac{\partial}{\partial t}\sum_z a_z \frac{\epsilon_z \widetilde{u}_{z\parallel}^{\,2}}{2} = \sum_z a_z \, \widetilde{u}_{z\parallel} \, \epsilon_z \frac{\partial \widetilde{u}_{z\parallel}}{\partial t}$$

- magnetic energy

$$\frac{\partial}{\partial t}\frac{\hat{\beta}}{2}\left|\nabla_\perp \widetilde{A}_\parallel\right|^2 = \hat{\beta}\widetilde{J}_\parallel \frac{\partial \widetilde{A}_\parallel}{\partial t} = \sum_z a_z \, \widetilde{u}_{z\parallel} \, \hat{\beta}\frac{\partial \widetilde{A}_\parallel}{\partial t}$$

Conservation

- ExB and thermal free energy combine ...

$$\sum_z a_z \left(\widetilde{\phi}_z + \tau_z \widetilde{n}_z\right) \frac{\partial \widetilde{n}_z}{\partial t} = \sum_z a_z \left(\widetilde{\phi}_z + \tau_z \widetilde{n}_z\right) \left[- B\nabla_\parallel \frac{\widetilde{u}_{z\parallel}}{B} + \mathcal{K}\left(\widetilde{\phi}_G + \tau_z \widetilde{n}_z\right) + S\right]$$

- magnetic and parallel kinetic energy combine ...

$$\sum_z a_z \widetilde{u}_{z\parallel} \left(\hat{\beta}\frac{\partial \widetilde{A}_\parallel}{\partial t} + \epsilon_s \frac{\partial \widetilde{u}_\parallel}{\partial t}\right) = \sum_z a_z \widetilde{u}_{z\parallel} \left[- \nabla_\parallel \left(\widetilde{\phi}_G + \tau_z \widetilde{n}_z\right) + \epsilon_z \tau_z \mathcal{K}\left(2\widetilde{u}_{z\parallel}\right)\right] - C\widetilde{J}_\parallel^2$$

  - ○ transfer and conservation is clear (force potential versus parallel flow)

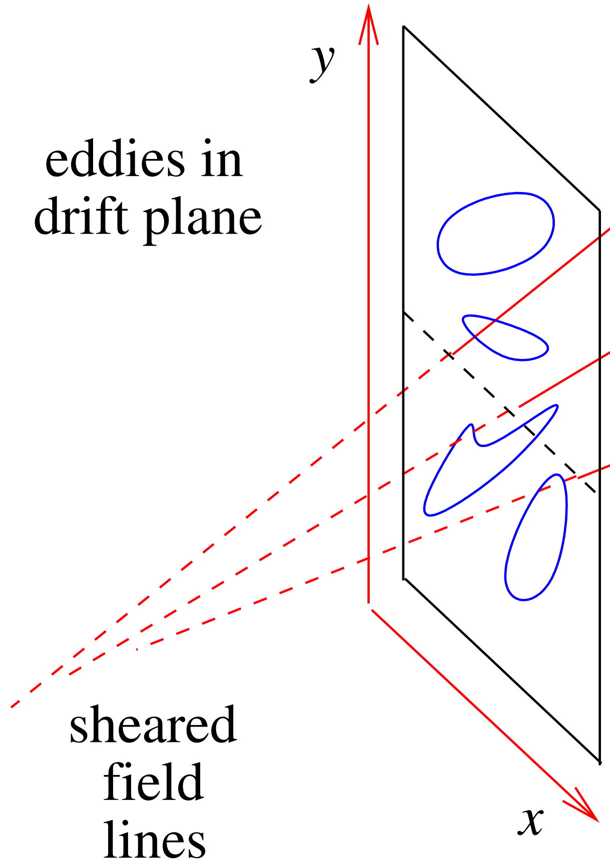
- zonal flow energy

$$\frac{\partial}{\partial t} \sum_z a_z \frac{1-\Gamma_0}{\tau_z} \frac{\left\langle \widetilde{\phi} \right\rangle^2}{2} = \sum_z a_z \left\langle \widetilde{\phi}_z \right\rangle \left\langle \frac{\partial \widetilde{n}_z}{\partial t} \right\rangle$$

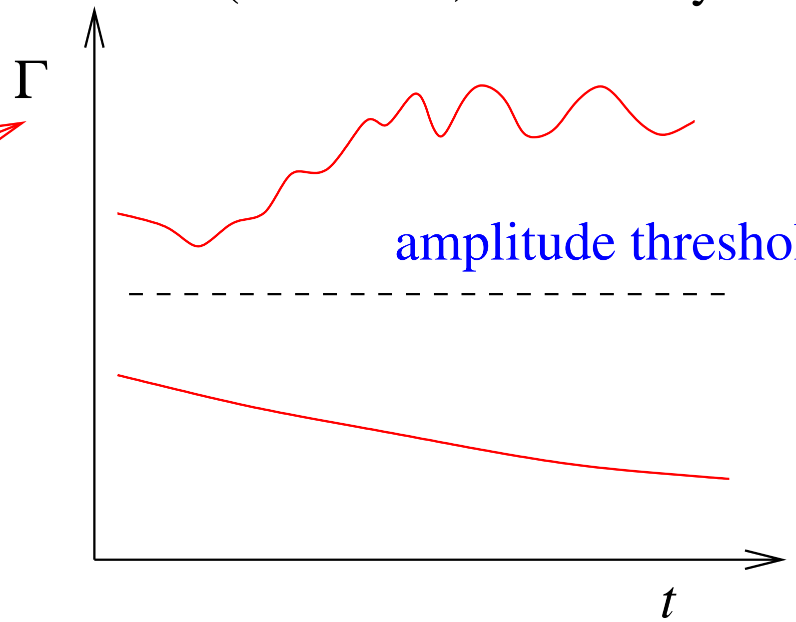
- further manipulations extract Reynolds, Maxwell stresses and geodesic coupling
  - ○ vorticity is the same thing as “gyrofluid charge density”
  - ○ etc, etc (not worked out here)

Nonlinear Instability for Drift Waves

(Scott, Phys Rev Lett 1990; Phys Fl B 1992)



(3D: Scott, New J Phys 2002)



nonlinear vorticity rips open adiabatic response

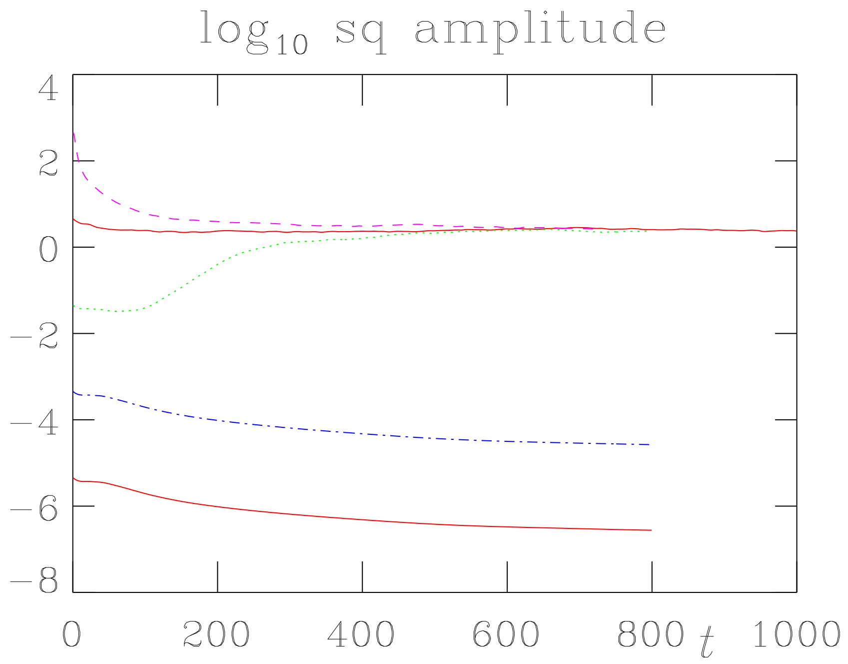
larger accessible space for parallel currents

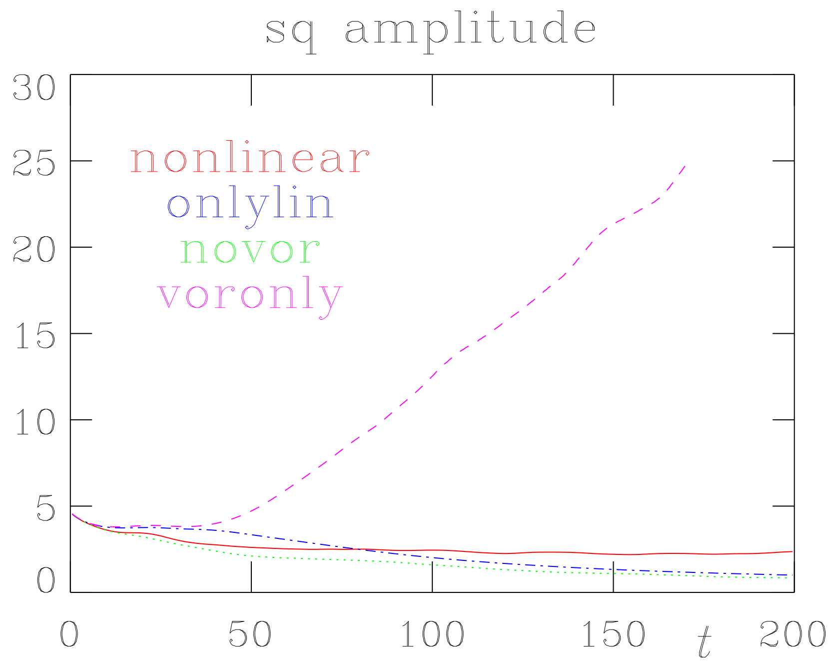
larger phase shifts

--> nonlinear excitation/instability, ‘‘self sustained turbulence’’

self sustained turbulence

C = 10, beta = mu = 0.1, 256 x 256





existence of amplitude threshold

convergence at late times (short correlation time)

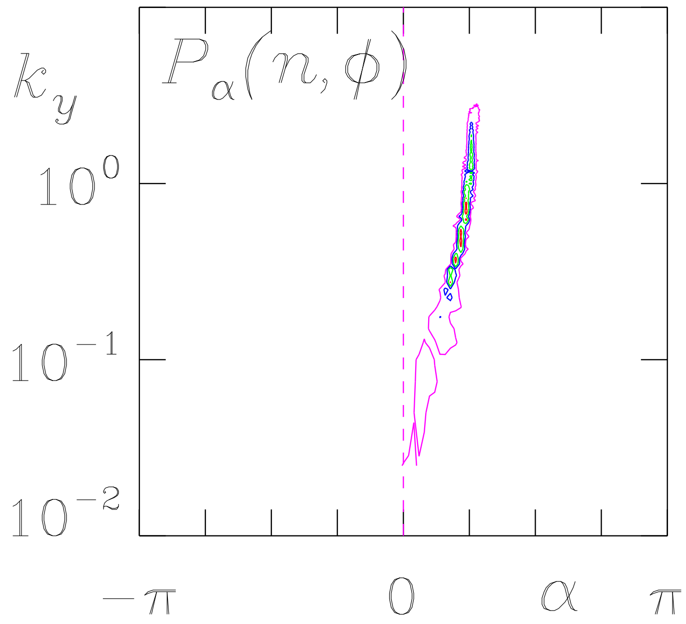
$\mathbf{v}_E \cdot \nabla\widetilde{\Omega}$ is necessary and sufficient for self-sustainment

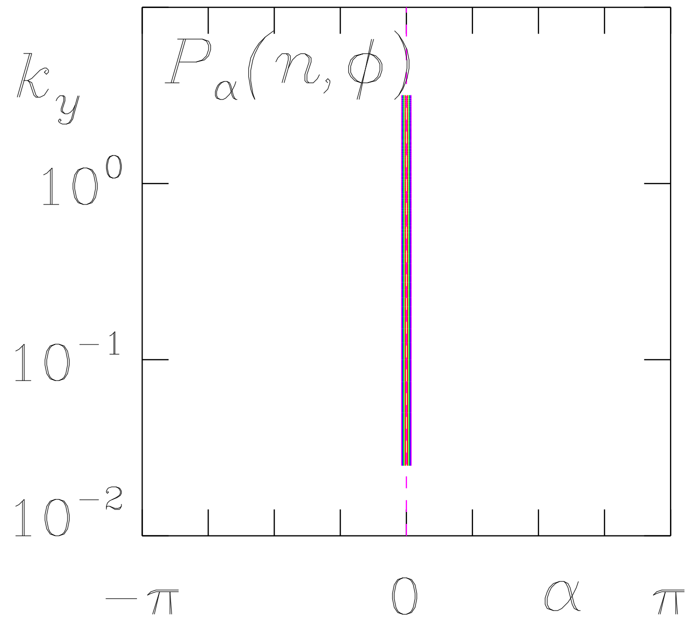
$\mathbf{v}_E \cdot \nabla\widetilde{p}_e$ is necessary for saturation of nonlinear cases

linear drift wave phase shifts

2D Hasegawa Wakatani, D = 0.01, 64 x 256

- linear start, all terms (left), no gradient (right)



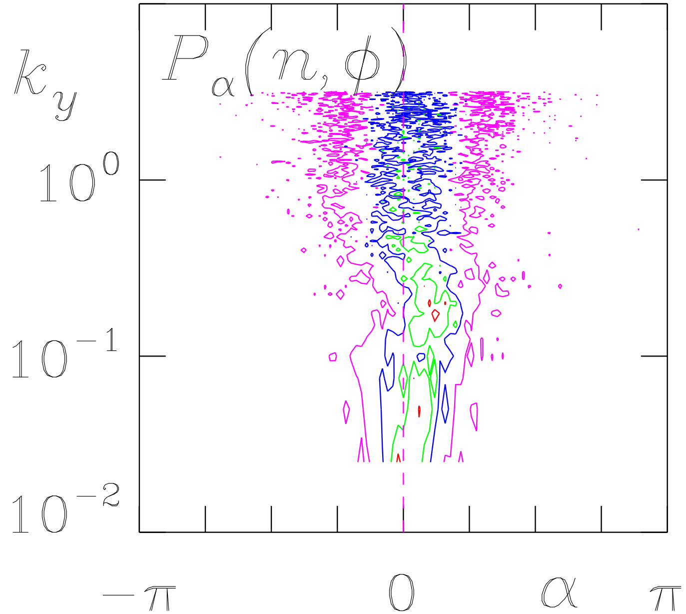


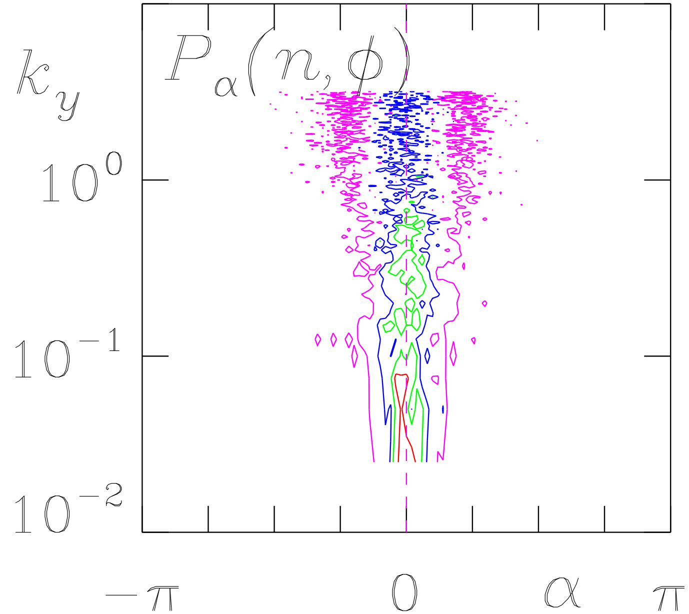
- phase shift caused by gradient, part of dispersion relation
  - scalar function of perpendicular wavenumber

drift wave turbulence phase shift distributions

2D Hasegawa Wakatani, D = 0.01, 64 x 256

- nonlinear start, all terms (left), no gradient (right)





- phase shifts caused by gradient, finds statistical equilibrium
- nonlinearities broaden distribution, gradient forcing selects for positive phase shifts

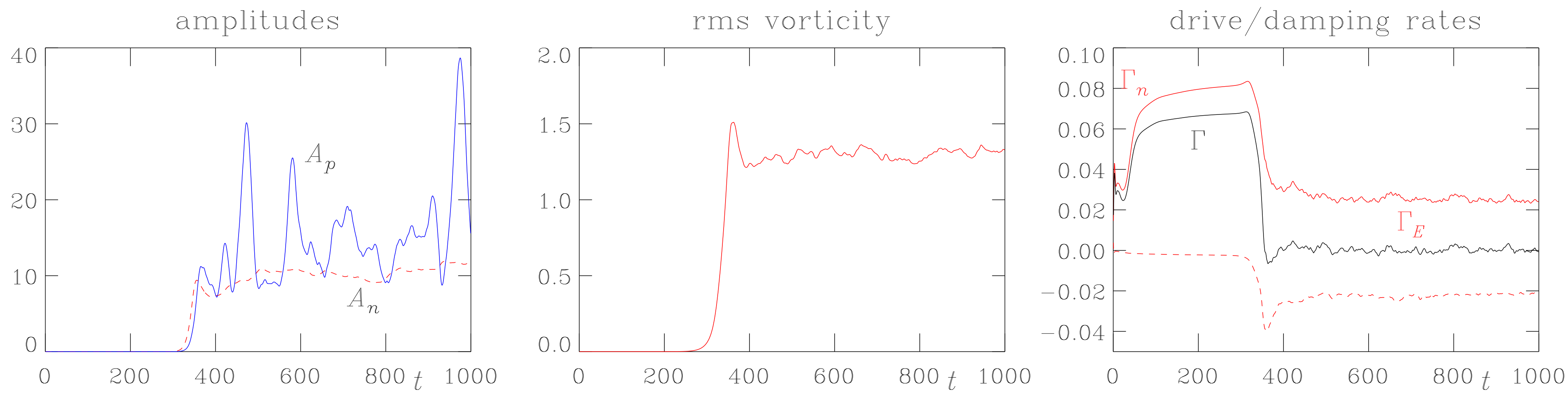
native drift wave vorticity overcomes linear growth rate
mid range of spectrum does not sense action by linear instability

toroidal case — linear start

compare linear and nonlinear mechanisms

- start at small amplitude: $\widetilde{p}_e/p_e \sim 10^{-10}$
- linear eigenmode establishes itself — with clear ballooning mode signature
- transition to turbulence — saturation as $\gamma \to 0$
- well developed turbulence at late times — statistical equilibrium

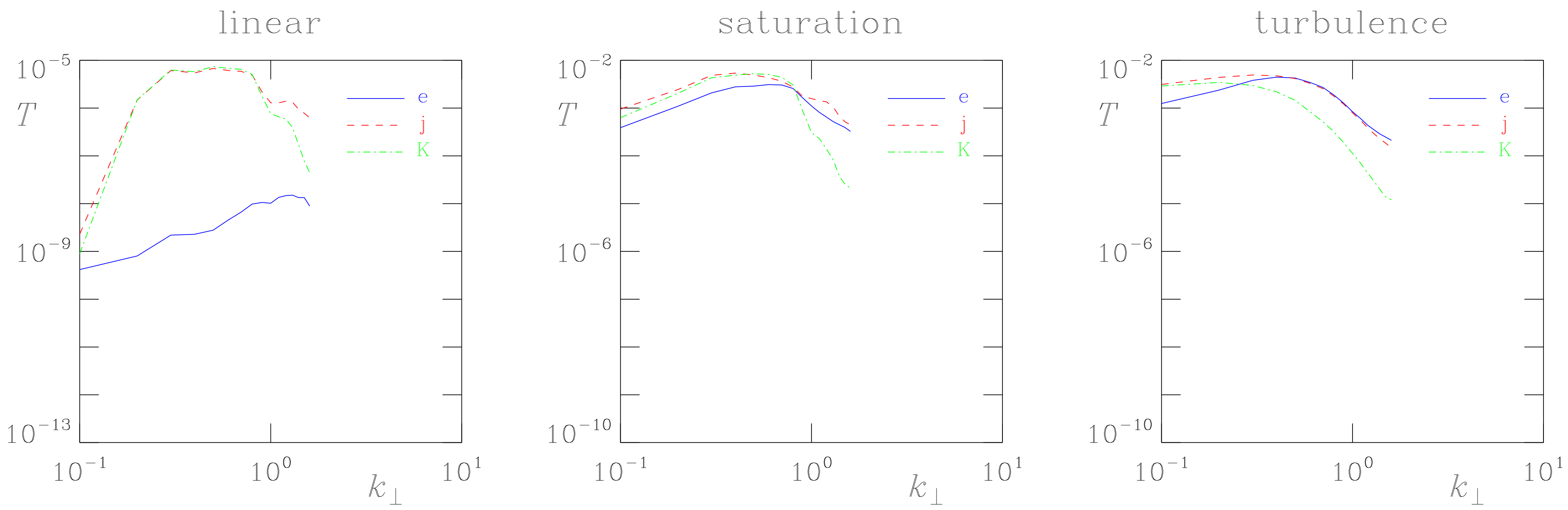
self sustained drift wave turbulence supersedes ballooning instability



RMS transfer spectra — toroidal linear start

measures energy transfer in and out of ExB eddies at given scale

red: parallel currents, green: interchange forcing, blue: nonlinear vorticity scattering



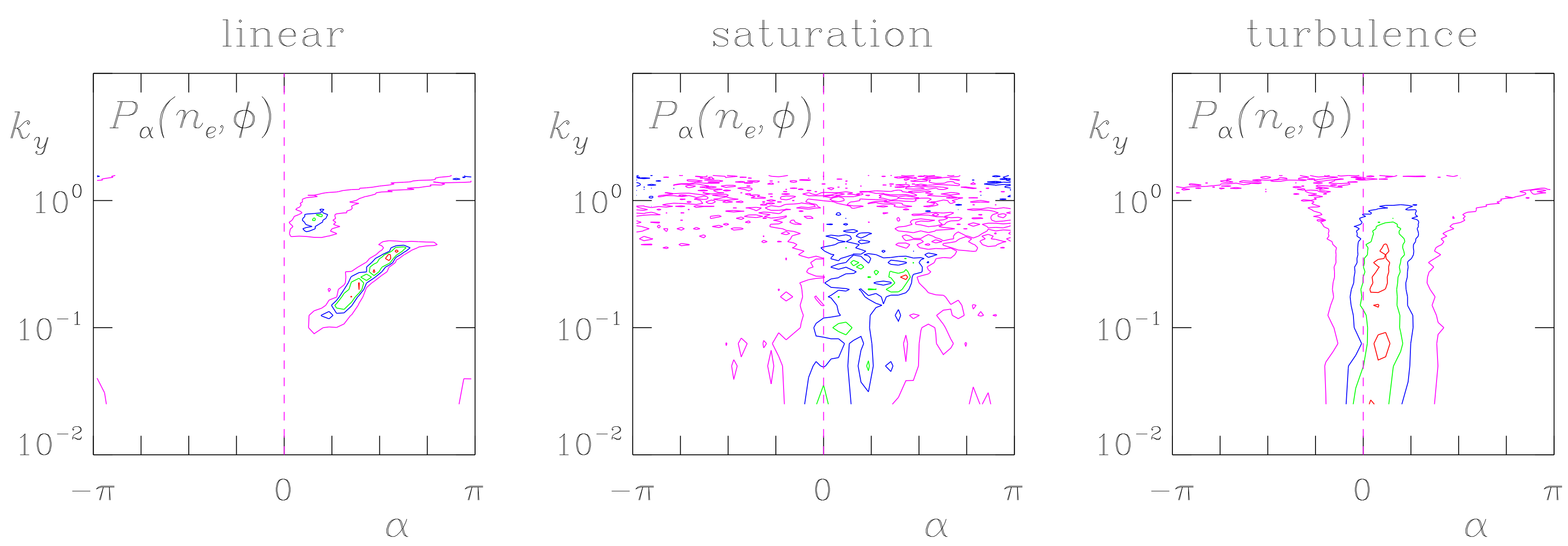
linear modes and fully developed turbulence
have fundamentally different excitation mechanisms

phase shift distributions — toroidal linear start

measures phase shift distributions for each wavelength

for linear modes: part of the dispersion relation

for turbulence: part of the statistical character involving damped or stable, as well as driven transients

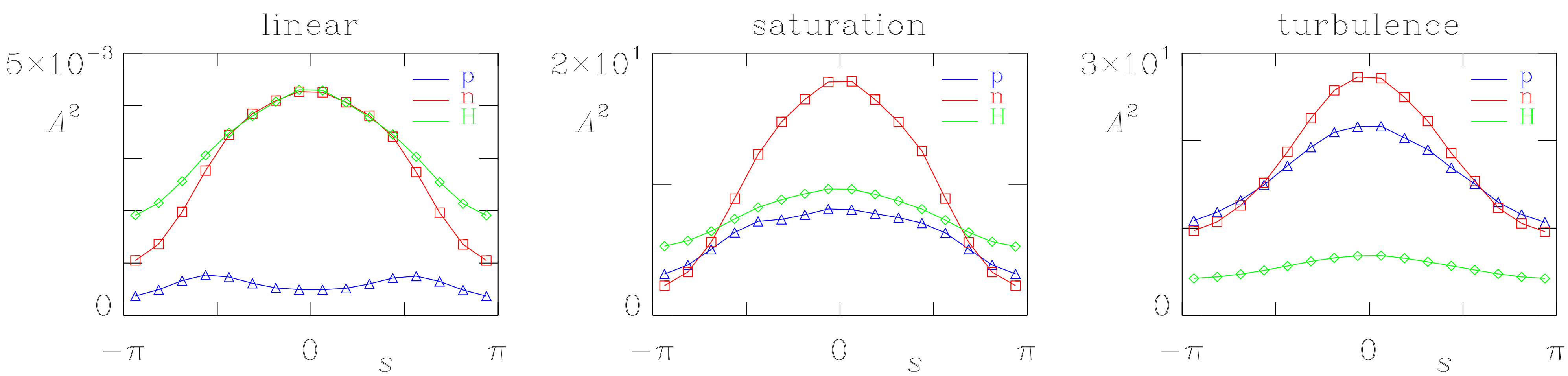


strong linear instability with large phase shift
is no longer felt in the fully developed turbulence

parallel mode structure — toroidal linear start

measures squared amplitudes as function of position along field lines

basic ballooning signal is ballooning in force potential, $\widetilde{h}_e = \widetilde{n}_e - \widetilde{\phi}$ (green line)



linear ballooning mode character is lost in fully developed turbulence

Implications of the Nonlinear Instability

linear results cannot be *a priori* applied to edge turbulence

- no applicability for gamma/k scaling models
- linear mode relevance to be decided on case-by-case basis
- resistive ballooning wiped out by drift waves
- **ideal ballooning** is large scale → still relevant (also peeling), but sets boundaries
- **most L-H models are ruled out** (linear $\hat{\beta}$ or C dependence)
- they *can* be tested by nonlinear code (homogeneous models)
- failing the test, the linear models are inapplicable to experiment

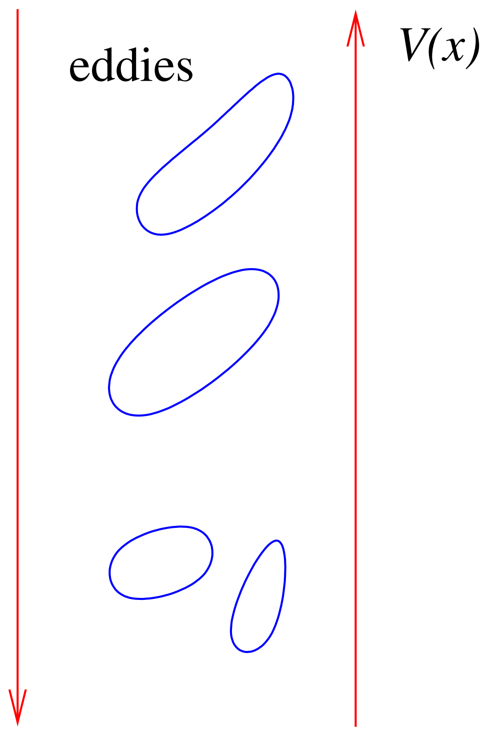
Zonal ExB Flows

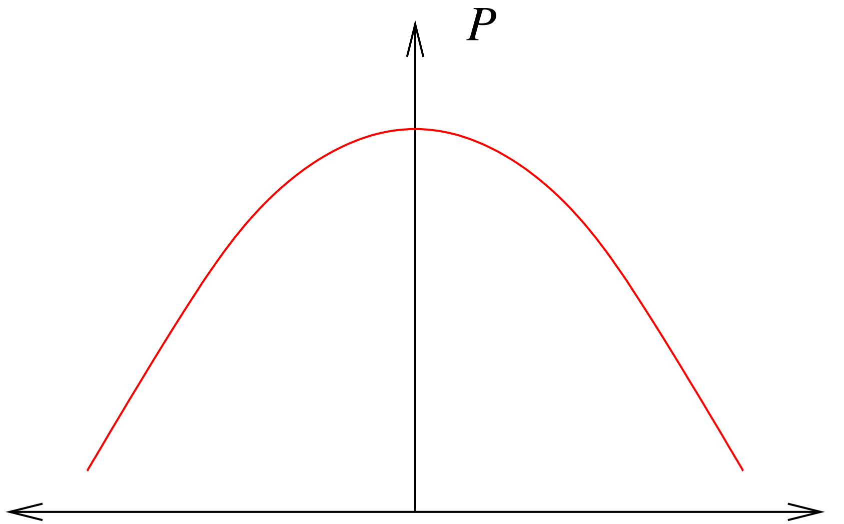
- zonal potential — flux surface averaged $\widetilde{\phi}$
- zonal flow is the ExB flow resulting from a zonal potential
- ExB flow is the advecting agent...
- a sheared ExB flow leads to reduced drive rates, transport
  - ◦ can suppress turbulence altogether if large enough
- shear suppression is conservative transfer from eddies to flow
  - ◦ Reynolds stress
- in toroidal geometry, zonal flow is compressible (quasistatically)
  - ◦ result of geodesic curvature
- "geodesic transfer effect" couples flows back to turbulence

  (B Scott, Phys Letters A 320 [2003] 53)
- entire system reaches statistical equilibrium
  - ◦ flows are not "spun up" indefinitely by turbulence

Reynolds Stress and Zonal Flows

(Diamond and Kim, Phys Fl B 1991)





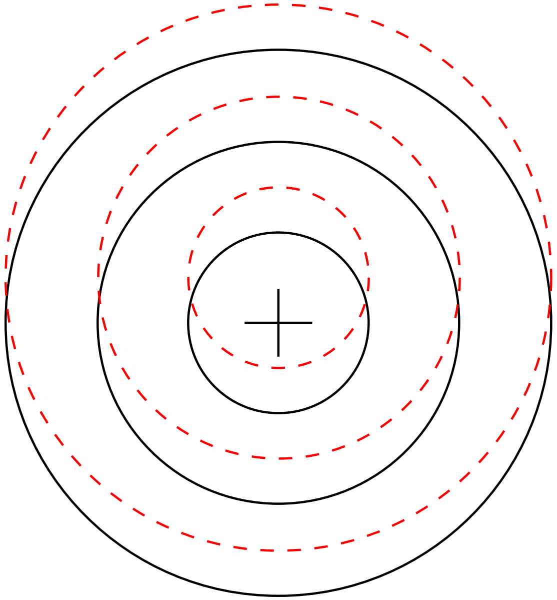
with no flow, zonal vorticity
and eddy Reynolds stress are random

random alignment between zonal vorticity and Reynolds stress

--> self generation/amplification of zonal flow layers

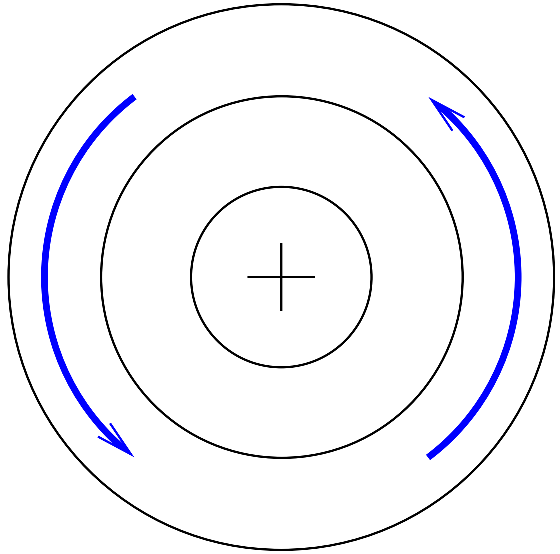
Geodesic Acoustic Oscillation

(Winsor et al, Phys Fl 1968)



pressure sideband

$\langle p \sin\theta \rangle$



compression at top

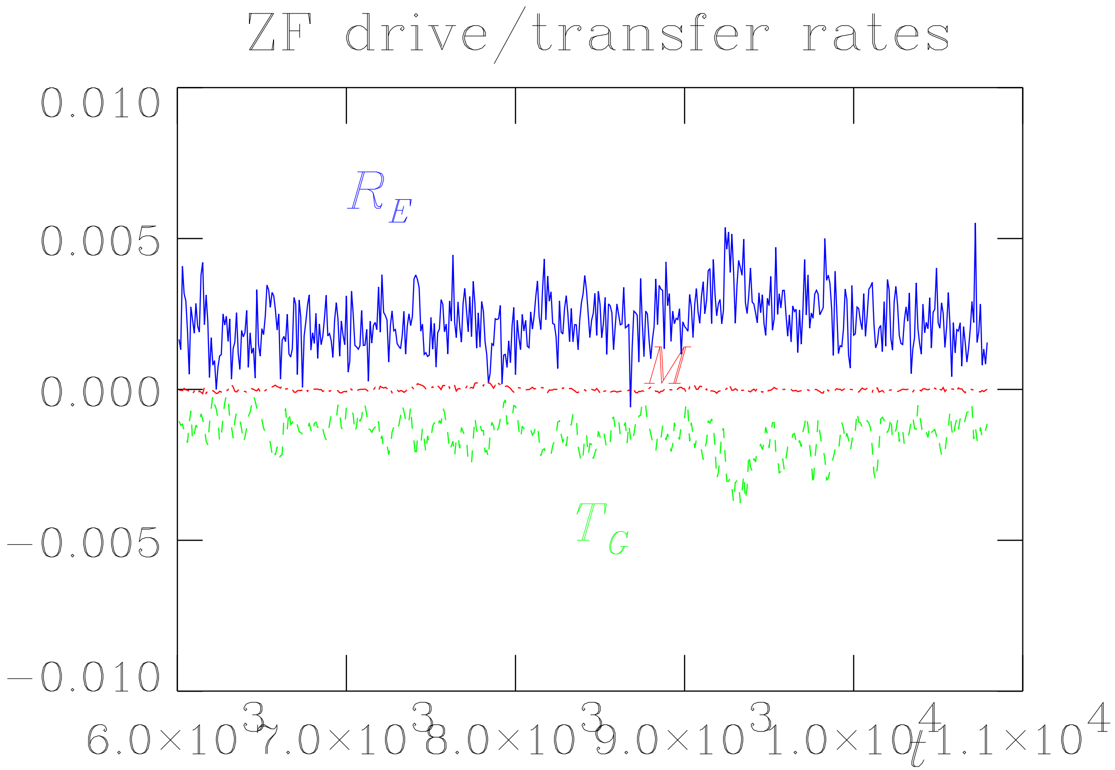
divergence at bottom

zonal flow exchanges conservatively with pressure sideband

--> transfer pathway, equipartition

Identification of Geodesic Transfer Effect

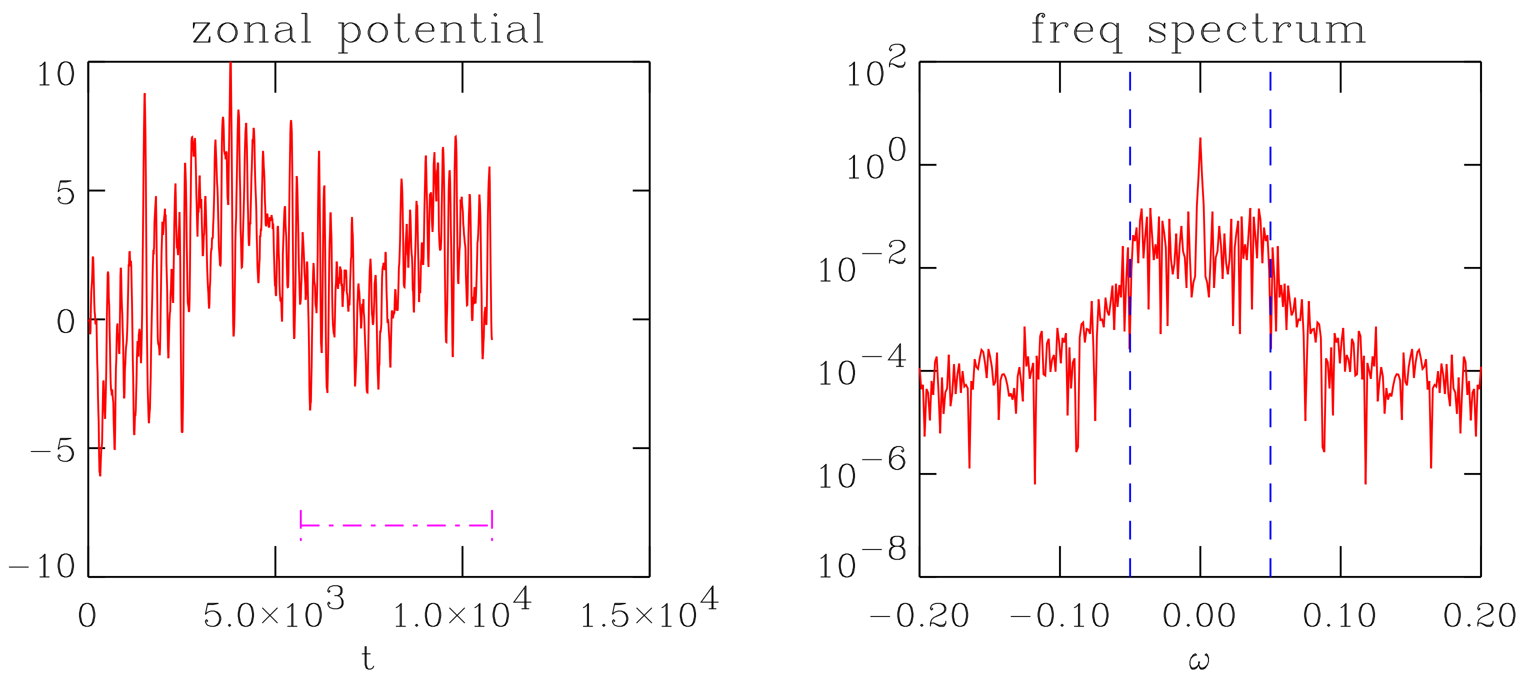
- measure Reynolds stress drive, geodesic transfer, and Maxwell stress in long term saturation



Reynolds stress/ geodesic transfer in clear balance
Maxwell stress is small

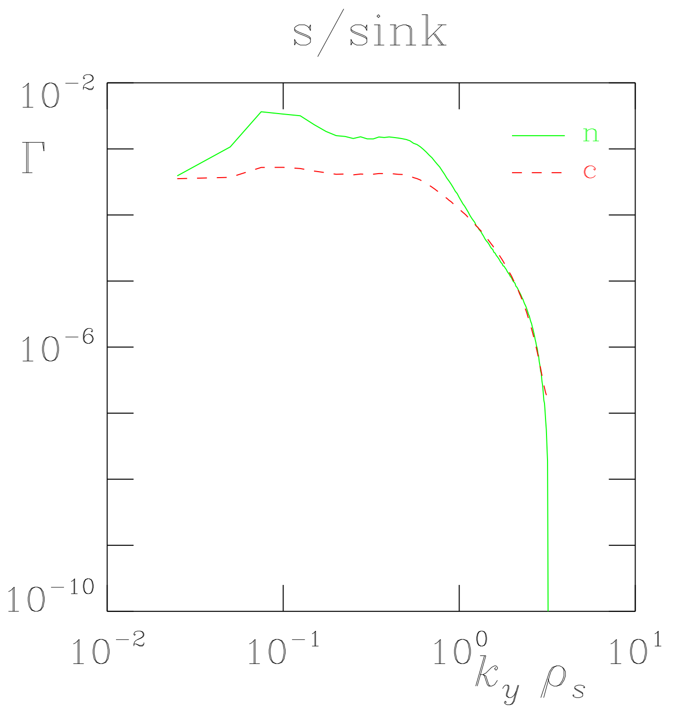
Zonal Potential Frequency Spectrum

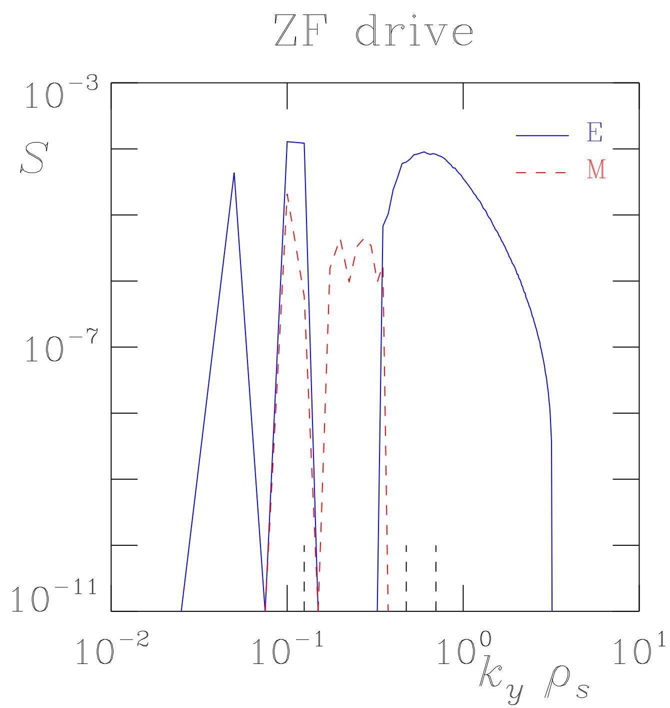
- zonal potential (check also flow, vorticity)
- geodesic acoustic oscillation small but visible



geodesic transfer, not oscillation, is main agent

Transport and Reynolds Stress Spectra



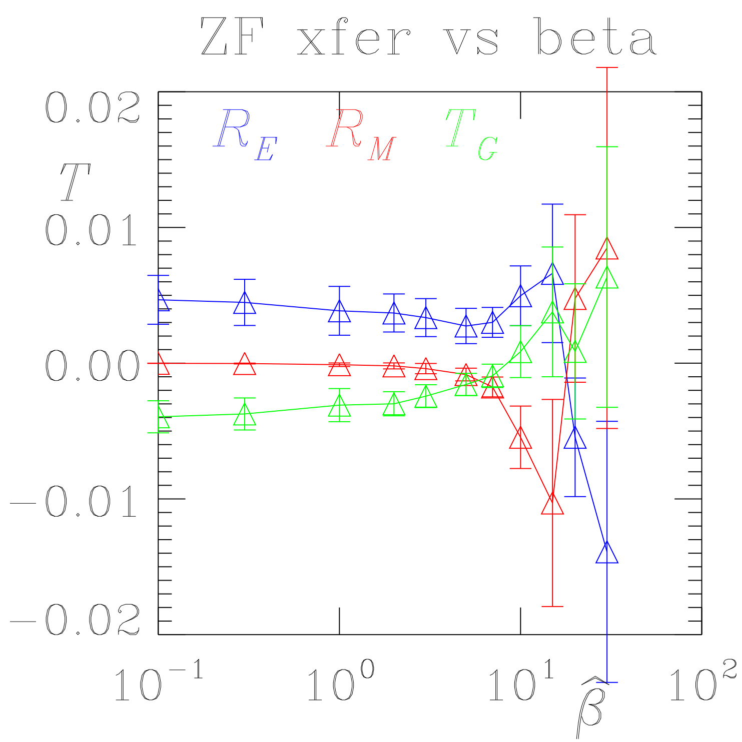


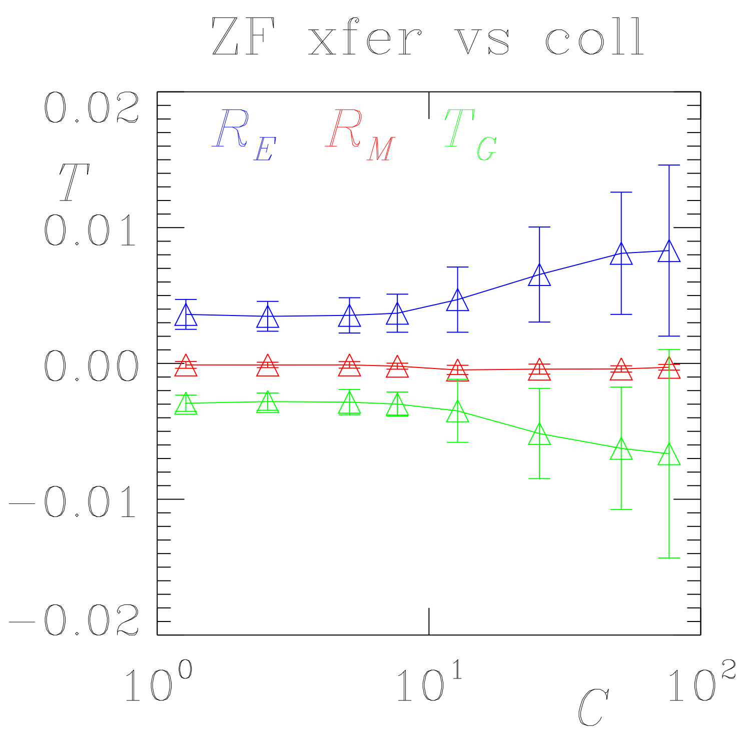
- transport: low (MHD mode) and high (DWT) k_y ranges
- Reynolds stress: 10, 20, and 50 percent lines

most of the transport and nearly all of the Reynolds stress done in the DWT spectral range near ρ_s

Scaling of Geodesic Coupling Effect

- basic case, ASDEX-Upgrade L-mode parameters (80 eV and $4.5 \times 10^{13}\,\mathrm{cm}^{-3}$)
- sweep $\hat{\beta}$ and C around this case





geodesic coupling balances Reynolds stress in operational regime
changes (PS-Alfvén oscillation) enter beyond ideal ballooning boundary

Zonal Flow and Sideband Energetics (simplified)

- zonal flow energy (Ω is vorticity)

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\left\langle v^y\right\rangle^2\right\} = \left\{\langle\Omega\rangle\left[\langle v^x v^y\rangle - \frac{1}{\hat{\beta}}\langle b^x b^y\rangle\right]\right\} - \omega_B\left\{\langle p_e \sin s\rangle\langle v^y\rangle\right\}$$

- the geodesic/Alfvén sideband system (Q^x is radial turbulent flux, $u_\parallel$ gives ion Landau losses)

$$\begin{aligned}\frac{\partial}{\partial t}\left\{\left\langle p_e \sin s\right\rangle^2\right\} = {} & 2\left\{\left\langle\frac{\partial p_e}{\partial x}\sin s\right\rangle\langle Q^x \sin s\rangle\right\} + \omega_B\left\{\langle p_e \sin s\rangle\langle v^y\rangle\right\} \\ & - \omega_B\left\{\langle p_e \sin s\rangle\left\langle\frac{\partial p_e}{\partial x}\right\rangle\right\} - 2\left\{\langle p_e \sin s\rangle\left\langle(J_\parallel - u_\parallel)\cos s\right\rangle\right\}\end{aligned}$$

$$\frac{\partial}{\partial t}\left\{\hat{\beta}^{-1}\left\langle B^y \cos s\right\rangle^2 + \hat{\mu}\left\langle J_\parallel \cos s\right\rangle^2\right\} = 2\left\{\langle(p_e - \phi)\sin s\rangle\left\langle J_\parallel \cos s\right\rangle\right\} - 2C\left\{\left\langle J_\parallel \cos s\right\rangle^2\right\}$$

$$\frac{\partial}{\partial t}\left\{\left\langle v^y \sin s\right\rangle^2\right\} = 2\left\{\langle\Omega \sin s\rangle\langle v^x v^y \sin s\rangle\right\} + \omega_B\left\{\langle\phi \sin s\rangle\left\langle\frac{\partial p_e}{\partial x}\right\rangle\right\} + 2\left\{\langle\phi \sin s\rangle\left\langle J_\parallel \cos s\right\rangle\right\}$$

- the thermal reservoir (background pressure)

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\left\langle p_e\right\rangle^2\right\} = \left\{\left\langle\frac{\partial p_e}{\partial x}\right\rangle\langle Q^x\rangle\right\} - \omega_B\left\{\langle(\phi - p_e)\sin s\rangle\left\langle\frac{\partial p_e}{\partial x}\right\rangle\right\}$$

Zonal Flow and Sideband Dynamics (simplified)

- zonal potential $\langle\phi\rangle$, flow $\langle v^y\rangle = \partial\langle\phi\rangle/\partial x$, and vorticity $\langle\Omega\rangle = \partial\langle v^y\rangle/\partial x$
- zonal vorticity (with nonlinear Reynolds and Maxwell stresses, geodesic curvature)

$$\frac{\partial}{\partial t}\langle\Omega\rangle = -\frac{\partial^2}{\partial x^2}\left[\langle v^x v^y\rangle - \frac{1}{\hat\beta}\langle b^x b^y\rangle\right] - \omega_B \frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

- the geodesic/Alfvén sideband system (Q^x is radial turbulent flux, $u_{||}$ gives ion Landau losses)

$$\frac{\partial}{\partial t}\langle p_e \sin s\rangle = \frac{\partial}{\partial x}\langle Q^x \sin s\rangle + \frac{\omega_B}{2}\langle v^y\rangle - \frac{\omega_B}{2}\left\langle\frac{\partial p_e}{\partial x}\right\rangle - \langle (J_{||} - u_{||})\cos s\rangle$$

$$\frac{\partial}{\partial t}\left\langle\left(\hat\beta A_{||} + \hat\mu J_{||}\right)\cos s\right\rangle = \langle p_e \sin s\rangle - \langle\phi \sin s\rangle - C\left\langle J_{||}\cos s\right\rangle$$

$$\frac{\partial}{\partial t}\langle\Omega \sin s\rangle = -\frac{\partial^2}{\partial x^2}\left\langle\left(v^x v^y - \frac{1}{\hat\beta}b^x b^y\right)\sin s\right\rangle - \frac{\omega_B}{2}\left\langle\frac{\partial p_e}{\partial x}\right\rangle - \langle J_{||}\cos s\rangle$$

PS Alfven

- the thermal reservoir (background pressure)

$$\frac{\partial}{\partial t}\langle p_e\rangle = \langle Q^x\rangle + \omega_B\frac{\partial}{\partial x}\langle\phi \sin s\rangle - \omega_B\frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

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$$\frac{\partial}{\partial t}\langle\Omega\rangle = -\frac{\partial^2}{\partial x^2}\left[\langle v^x v^y\rangle - \frac{1}{\hat\beta}\langle b^x b^y\rangle\right] - \omega_B\frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

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$$\frac{\partial}{\partial t}\langle p_e \sin s\rangle = \frac{\partial}{\partial x}\langle Q^x \sin s\rangle + \frac{\omega_B}{2}\langle v^y\rangle - \frac{\omega_B}{2}\left\langle\frac{\partial p_e}{\partial x}\right\rangle - \left\langle\left(J_{||} - u_{||}\right)\cos s\right\rangle$$

PS adiabatic

$$\frac{\partial}{\partial t}\left\langle\left(\hat\beta A_{||} + \hat\mu J_{||}\right)\cos s\right\rangle = \langle p_e \sin s\rangle - \langle\phi \sin s\rangle - C\left\langle J_{||}\cos s\right\rangle$$

$$\frac{\partial}{\partial t}\langle\Omega \sin s\rangle = -\frac{\partial^2}{\partial x^2}\left\langle\left(v^x v^y - \frac{1}{\hat\beta}b^x b^y\right)\sin s\right\rangle - \frac{\omega_B}{2}\left\langle\frac{\partial p_e}{\partial x}\right\rangle - \left\langle J_{||}\cos s\right\rangle$$

PS Alfven

- the thermal reservoir (background pressure)

$$\frac{\partial}{\partial t}\langle p_e\rangle = \langle Q^x\rangle + \omega_B\frac{\partial}{\partial x}\langle\phi \sin s\rangle - \omega_B\frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

Zonal Flow and Sideband Dynamics (simplified)

- zonal potential $\langle\phi\rangle$, flow $\langle v^y\rangle = \partial\langle\phi\rangle/\partial x$, and vorticity $\langle\Omega\rangle = \partial\langle v^y\rangle/\partial x$
- zonal vorticity (with nonlinear Reynolds and Maxwell stresses, geodesic curvature)

$$\frac{\partial}{\partial t}\langle\Omega\rangle = -\frac{\partial^2}{\partial x^2}\left[\langle v^x v^y\rangle - \frac{1}{\hat\beta}\langle b^x b^y\rangle\right] - \omega_B\frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

geodesic acoustic

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DW TURB

geodesic acoustic

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$$\frac{\partial}{\partial t}\langle p_e \sin s\rangle = \frac{\partial}{\partial x}\langle Q^x \sin s\rangle + \frac{\omega_B}{2}\langle v^y\rangle - \frac{\omega_B}{2}\left\langle\frac{\partial p_e}{\partial x}\right\rangle - \left\langle (J_\parallel - u_\parallel)\cos s\right\rangle$$

PS adiabatic

$$\frac{\partial}{\partial t}\left\langle\left(\hat\beta A_\parallel + \hat\mu J_\parallel\right)\cos s\right\rangle = \langle p_e \sin s\rangle - \langle\phi \sin s\rangle - C\left\langle J_\parallel \cos s\right\rangle$$

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turb sink

PS Alfven

- the thermal reservoir (background pressure)

$$\frac{\partial}{\partial t}\langle p_e\rangle = \langle Q^x\rangle + \omega_B \frac{\partial}{\partial x}\langle\phi \sin s\rangle - \omega_B \frac{\partial}{\partial x}\langle p_e \sin s\rangle$$

sign of
PS Alf xfer
is reversed!

Summary and Outlook

- **Basic Results**

  — standard DALF turbulence

  — **nonlinear self sustained drift wave instability**

  — zonal flows and sidebands, global Alfvén transient $\rightarrow$ P-S current (2D eq)

  — zonal flows system reaches equilibrium on transport time scales

  — run to transport equilibrium very feasible for edge turbulence

- **Things GEM can do Now**

  — basic description of edge turbulence, **warm ions**

  — resolve ρ_i: note $k_\perp \rho_i$ must go past unity to resolve DWT with $T_e = T_i$

  — self consistent profiles with sources, turbulence

  — SOL/edge coupling, self consistent shear layer

  — experiment with sources, distribution thereof

- **Future Generalisation**

  — inhomogeneous equations (under development)

  — extension of gyrokinetic GENE code to edge turbulence

  — how much of this is needed to get pedestal (already know homogenous models don't work)