Correspondence between the gyrofluid and low frequency Braginskii equations

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Outline

- **Braginskii Fluid Equations**
  — moments over kinetic particle motion
  — diamagnetic flows, stress tensor, heat fluxes
  — dissipation: parallel viscosity and heat fluxes

- **Gyrofluid Equations**
  — moments over kinetic gyrocenter motion
  — FLR nonlinearities, curvature and grad-B drifts
  — dissipation: thermal anisotropy and heat fluxes
  — correspondence in large scales limit

- **How Braginskii treated Thermal Anisotropy**
  — isotropic pressure plus (small) trace-free stress tensor

- **How the Gyrofluid Model captures Viscosity**
  — thermal anisotropy versus flow divergences
  — correspondence in strongly collisional limit

The Drift Approximation

- fluid: solve equation of motion with inertia as correction
  - assumes gyrofrequency is arbitrarily fast

$$\mathbf{u}_\perp = \frac{c}{B^2}\mathbf{B}\times\nabla\phi + \frac{1}{ne}\frac{c}{B^2}\mathbf{B}\times\nabla p + \frac{1}{ne}\frac{c}{B^2}\mathbf{B}\times\left(nM\frac{d\mathbf{u}_\perp}{dt} + \nabla\cdot\mathbf{\Pi}\right)$$

- gyrokinetic: low-frequency limit in Lagrangian
  - treats gyrocenters directly

$$L_p = \left(\frac{e}{c}\mathbf{A} + Mv_\parallel\mathbf{b}\right)\cdot\dot{\mathbf{R}} + \mu\frac{Mc}{e}\dot{\theta} - H$$

ultimately these are the same approximation
equations should agree in the common limit

Pressure Anisotropy — Definitions

- diagonal pressure, perp and parallel different

$$\mathbf{P} = p_\parallel \mathbf{bb} + p_\perp \left(\mathbf{g} - \mathbf{bb}\right) \qquad p_\parallel = \int d\mathcal{W}\, M w_\parallel^2\, f \qquad p_\perp = \int d\mathcal{W}\, M \frac{w_\perp^2}{2}\, f$$

- expression within Braginskii — pressure plus traceless tensor

$$\mathbf{P} = p\,\mathbf{g} + \mathbf{\Pi} \qquad g^{ij}\Pi_{ij} = 0$$

- isotropisation — extra potential appearing in reduced Braginskii model

$$p = \frac{2}{3}p_\perp + \frac{1}{3}p_\parallel \qquad \delta p = p_\parallel - p_\perp \qquad G \equiv \frac{1}{4}\Pi_{\parallel\parallel} = \frac{1}{6}\delta p$$

- appearances in the gyrofluid model — same as in reduced Braginskii model

$$\frac{p_\parallel + p_\perp}{2} = p + \frac{1}{6}\delta p \equiv p + G \qquad p_\parallel = p + \frac{2}{3}\delta p \equiv p + 4G$$

Braginskii's Expansion

- order collision and gyro frequencies large in Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{e}{M}\mathbf{E}\cdot\frac{\partial f}{\partial \mathbf{v}} = C(f) - \frac{e}{Mc}\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f}{\partial \mathbf{v}}$$

- left side pieces given by drifting Maxwellian

$$f^{(0)} = F^M = \frac{n}{(4\pi^2 T/M)^{3/2}} \exp\left[-\frac{m}{2T}\left(\mathbf{v}-\mathbf{u}\right)^2\right]$$

- expansion of velocity gradient terms — symmetrisation

$$\mathbf{v}\cdot\nabla f = \cdots + F^M \mathbf{v}\mathbf{v}:\mathbf{W} \qquad \mathbf{W} = \nabla\mathbf{u} + (\nabla\mathbf{u})^T - \frac{2}{3}\mathbf{g}(\nabla\cdot\mathbf{u})$$

- matching correction on RHS to match (no gyro term piece for diagonal elements)

$$\delta f = \Phi_{ij}\left(v^i v^j - \frac{v^2}{3}g^{ij}\right) \qquad \mathbf{\Pi} = \int d\mathcal{W}M\left(\mathbf{v}\mathbf{v} - \frac{v^2}{3}\mathbf{g}\right)\delta f$$

Gyroviscosity

- Braginskii's Stress Tensor — off-diagonal elements
- take the off-diagonal $\mathbf{vv}$ moment, neglect dissipation, use $\mathbf{\Omega} = e\mathbf{B}/Mc$

$$p\mathbf{W} = \mathbf{\Omega}\times\mathbf{\Pi}_* - \mathbf{\Pi}_*\times\mathbf{\Omega}$$

hence

$$\mathbf{\Pi}_* = \frac{1}{4}\frac{p}{\Omega^2}\left(\mathbf{\Omega}\times\mathbf{W} - \mathbf{W}\times\mathbf{\Omega}\right)$$

- principal effect – cancels diamagnetic advection

$$\nabla\cdot\mathbf{\Pi}_* = -nM\mathbf{u}_*\cdot\nabla\mathbf{u} + (\mathrm{FLR})$$

where

$$\mathbf{u}_* = \frac{1}{ne}\frac{c}{B^2}\mathbf{B}\times\nabla p$$

is the diamagnetic velocity

Diamagnetic Cancellation

- in all the equations: effect is to cancel diamagnetic advection

$$\mathbf{u}_* \cdot \nabla n + n \nabla \cdot \mathbf{u}_* = \frac{1}{e} \nabla \cdot \frac{c}{B^2} \mathbf{B} \times \nabla p$$

$$n M \mathbf{u}_* \cdot \nabla \mathbf{u} + \nabla \cdot \mathbf{\Pi}_* = (\mathrm{FLR})$$

$$\frac{3}{2} n \mathbf{u}_* \cdot \nabla T + p \nabla \cdot \mathbf{u}_* + \nabla \cdot \mathbf{q}_\wedge = \frac{5/2}{e} \nabla \cdot \frac{c}{B^2} \mathbf{B} \times \nabla (pT)$$

- in each case the cancellation is complete up to curvature terms and FLR corrections

- with this information the low frequency drift equations emerge easily

Fluid Drift Equations

- low frequency, small amplitude drift ordering, gyro-Bohm normalisation

$$\frac{\partial n}{\partial t} + [\phi, n] - \nabla \cdot \left(\frac{\partial}{\partial t} + [\phi,] \right) \nabla_\perp (\phi + p) + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$

$$\frac{3}{2}\frac{\partial T}{\partial t} + \frac{3}{2}[\phi, T] - \nabla \cdot \left(\frac{\partial}{\partial t} + [\phi,] \right) \nabla_\perp \left(\phi + p + \frac{5}{2}T \right) + B\nabla_\parallel \frac{u_\parallel + q_\parallel}{B} = \mathcal{K} \left(\phi + p + \frac{5}{2}T \right)$$

- polarisation terms: lowest order forms in inertial corrections
  - velocity: force potential $W = \phi + p$
  - heat flux: thermal potential $(5/2)T$
- nonlinear polarisation terms, e.g.,

$$\frac{dn}{dt} + \nabla \cdot \frac{d}{dt}\nabla_\perp W = \frac{d}{dt}(n - \nabla_\perp^2 W) + [\nabla_\perp \phi, \nabla_\perp W]$$

where $d/dt = \partial/\partial t + [\phi,]$ is ExB advection

Gyrofluid Equations

- low frequency, small amplitude drift ordering, gyro-Bohm normalisation

$$\frac{\partial n}{\partial t} + [\phi_G, n] + [\Omega_G, T_\perp] + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}\left(\phi_G + \frac{\Omega_G + p_\parallel + p_\perp}{2}\right)$$

$$\frac{\partial T_\perp}{\partial t} + [\phi_G, T_\perp] + [\Omega_G, (n + 2T_\perp)] + B\nabla_\parallel \frac{q_{\perp\parallel}}{B} = \mathcal{K}\left(\frac{\phi_G + p_\perp + 4\Omega_G + 3T_\perp}{2}\right)$$

$$\frac{1}{2}\frac{\partial T_\parallel}{\partial t} + \frac{1}{2}[\phi_G, T_\parallel] + B\nabla_\parallel \frac{u_\parallel + q_{\parallel\parallel}}{B} = \mathcal{K}\left(\frac{\phi_G + p_\parallel + 2T_\parallel}{2}\right)$$

- Padé approximants for gyroaveraged potentials

$$\phi_G = \left(1 - \frac{\rho_i^2}{2}\nabla_\perp^2\right)^{-1}\phi \qquad \Omega_G = T\frac{\partial}{\partial T}\left(1 - \frac{\rho_i^2}{2}\nabla_\perp^2\right)^{-1}\phi$$

Gyroviscosity versus FLR (polarisation) nonlinearities

- gyrofluid model uses gyrocenter moment variables
  - representation depends on underlying gyrokinetic Lie transform

- lowest order moment over the Lie transform is the quasineutrality constraint
  - relates gyrocenter densities to space densities

- fluid model uses the particle moment variables
  - quasineutrality constraint equates the densities

purpose is to show equivalence at low-$k_\perp$ between these forms
ultimately, the two representations treat the same dynamics

Gyrocenter Representation

- let $f(\mathbf{R}, v_\|, \mu)$ be the gyrocenter distribution function and $F(\mathbf{x}, \mathbf{v})$ the particle one
- the transform from gyrocenter to space coordinates is ... in the delta-f limit

$$F = J_0 f + e\frac{F^M}{T}(J_0^2 - 1)\phi$$

- moments over this give the fluid variables in terms of the gyrofluid ones (normalised)

$$n_f = \Gamma_1 n_g + \Gamma_2 T_{g\perp} + \frac{e}{T}(\Gamma_0 - 1)\phi$$

$$T_{f\perp} = \Gamma_1 T_{g\perp} + \Gamma_2(n_g + 2T_{g\perp}) + \frac{e}{T}(\Gamma_0 - 1)\phi$$

$$T_{f\|} = \Gamma_1 T_{g\|}$$

- definitions: J_0 is $J_0(k_\perp \rho_i)$ and $\Gamma_0(b) = I_0(b)e^{-b}$, where $b = k_\perp^2 \rho_i^2$
- closure: $\Gamma_1(b) = \Gamma_0(b)^{1/2}$ and $\Gamma_2(b) = b(\partial/\partial b)\Gamma_1(b)$

Longwave Limit

- neglecting corrections of $O(b^2)$

$$\Gamma_0 = 1 - b \qquad \Gamma_1 = 1 - b/2 \qquad \Gamma_2 = -b/2$$

- note the factor of T in the potential:

$$\frac{b}{T} \rightarrow b_s \qquad \text{hence} \qquad b \rightarrow -\rho_i^2 \nabla_\perp^2 \qquad b_s \rightarrow -\rho_s^2 \nabla_\perp^2$$

- these normalise in terms of τ_i, the normalised temperature

$$-b_s \phi \rightarrow \nabla_\perp^2 \phi \qquad -b(n + T_\perp) \rightarrow \nabla_\perp^2 p_\perp \qquad \text{with} \qquad p_\perp = \tau_i (n + T_\perp)$$

- gyroaveraged potentials (note signs)

$$\phi_G \rightarrow \phi - \frac{b}{2}\phi \rightarrow \phi + \frac{\rho_i^2}{2} \nabla_\perp^2 \phi \qquad \Omega_G \rightarrow -\frac{b}{2}\phi \rightarrow \frac{\rho_i^2}{2} \nabla_\perp^2 \phi$$

Representations in Longwave Limit

- fluid variables are

$$n_f = n_g - \frac{b}{2}(n_g + T_{g\perp}) - b_s\phi$$

$$T_{f\perp} = T_{g\perp} - \frac{b}{2}(n_g + 3T_{g\perp}) - b_s\phi$$

$$T_{f\parallel} = T_{g\parallel} - \frac{b}{2}T_{g\parallel}$$

- in the $O(b)$ terms the representations are interchangeable

$$n_g = n_f + \frac{b}{2}(n_f + T_{f\perp}) + b_s\phi$$

$$T_{g\perp} = T_{f\perp} + \frac{b}{2}(n_f + 3T_{f\perp}) + b_s\phi$$

$$T_{g\parallel} = T_{f\parallel} + \frac{b}{2}T_{f\parallel}$$

Continuity Equation

- gyrofluid density equation

$$\frac{\partial n}{\partial t} + [\phi_G, n] + [\Omega_G, T_\perp] + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}\left(\phi_G + \frac{\Omega_G + p_\parallel + p_\perp}{2}\right)$$

- use the representation relation for n,

$$n_g = n_f - \frac{\tau_i}{2}\nabla_\perp^2(n_f + T_{f\perp}) - \nabla_\perp^2\phi \qquad = \quad n_f - \nabla_\perp^2\left(\phi + \frac{p}{2}\right)$$

- use the definitions of ϕ_G and Ω_G

$$\phi_G = \phi + \frac{\tau_i}{2}\nabla_\perp^2\phi \qquad\qquad \Omega_G = \frac{\tau_i}{2}\nabla_\perp^2\phi$$

- note under $\nabla_\perp^2$ the fluid and gyrofluid variables are the same
- keep $O(b)$ terms only in the inertia (reason: quasineutrality)

Continuity Equation, cont'd

- gyrofluid density equation

$$\frac{\partial n}{\partial t} + [\phi_G, n] + [\Omega_G, T_\perp] + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}\left(\phi_G + \frac{\Omega_G + p_\parallel + p_\perp}{2}\right)$$

- put in fluid variables, FLR potentials, keep $O(b)$ in polarisation only, drop anisotropy

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial t}\nabla_\perp^2\left(\phi + \frac{p}{2}\right) + [\phi, n] + \left[\nabla_\perp^2\phi, \frac{p}{2}\right] - \left[\phi, \nabla_\perp^2\left(\phi + \frac{p}{2}\right)\right] + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$

- collect nonlinear terms with $\nabla_\perp^2$

$$\frac{\partial n}{\partial t} + [\phi, n] - \frac{\partial}{\partial t}\nabla_\perp^2\left(\phi + \frac{p}{2}\right) + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$

$$+ \left[\nabla_\perp^2\phi, \frac{p}{2}\right] - \left[\phi, \nabla_\perp^2\left(\phi + \frac{p}{2}\right)\right]$$

Operations with Brackets

- chain rule manipulations

$$[\nabla_\perp^2\phi, p] = \nabla\cdot[\nabla_\perp\phi, p] - [\nabla_\perp\phi, \nabla_\perp p]$$

$$\nabla\cdot[\nabla_\perp\phi, p] = \nabla_\perp^2[\phi, p] - \nabla\cdot[\phi, \nabla_\perp p]$$

$$[\phi, \nabla_\perp^2 p] = \nabla\cdot[\phi, \nabla_\perp p] - [\nabla_\perp\phi, \nabla_\perp p]$$

- polarisation manipulations, note symmetry in $\nabla_\perp^2\phi$ piece

$$[\nabla_\perp^2\phi, p] - [\phi, \nabla_\perp^2(\phi+p)]$$

$$= -\nabla\cdot[\phi, \nabla_\perp\phi] + \nabla\cdot[\nabla_\perp\phi, p] - [\nabla_\perp\phi, \nabla_\perp p] - \nabla\cdot[\phi, \nabla_\perp p] + [\nabla_\perp\phi, \nabla_\perp p]$$

$$= -\nabla\cdot[\phi, \nabla_\perp\phi] + \nabla_\perp^2[\phi, p] - \nabla\cdot[\phi, \nabla_\perp p] - \nabla\cdot[\phi, \nabla_\perp p]$$

- hence

$$\left[\nabla_\perp^2\phi, \frac{p}{2}\right] - \left[\phi, \nabla_\perp^2\left(\phi + \frac{p}{2}\right)\right] = -\nabla\cdot[\phi, \nabla_\perp(\phi+p)] + \nabla_\perp^2\left[\phi, \frac{p}{2}\right]$$

Continuity Equation, final

- we had

$$\frac{\partial n}{\partial t} + [\phi, n] - \frac{\partial}{\partial t}\nabla_\perp^2\left(\phi + \frac{p}{2}\right) + B\nabla_\parallel\frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$
$$+\left[\nabla_\perp^2\phi, \frac{p}{2}\right] - \left[\phi, \nabla_\perp^2\left(\phi + \frac{p}{2}\right)\right]$$

which is now

$$\frac{\partial n}{\partial t} + [\phi, n] - \frac{\partial}{\partial t}\nabla_\perp^2\left(\phi + \frac{p}{2}\right) - \nabla\cdot[\phi, \nabla_\perp(\phi + p)] + \nabla_\perp^2\left[\phi, \frac{p}{2}\right] + B\nabla_\parallel\frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$

- advection under $\nabla_\perp^2$: no further $O(b)$ terms

$$\nabla_\perp^2\left[\phi, \frac{p}{2}\right] = -\frac{\partial}{\partial t}\nabla_\perp^2\frac{p}{2} + O(b\nabla_\parallel) + O(b\mathcal{K})$$

- final form agrees with original fluid version (with $d/dt = \partial/\partial t + [\phi,]$)

$$\frac{dn}{dt} - \nabla\cdot\frac{d}{dt}\nabla_\perp(\phi + p) + B\nabla_\parallel\frac{u_\parallel}{B} = \mathcal{K}(\phi + p)$$

Temperature Equations

- same manipulations as with continuity equations give

$$\frac{dT_\perp}{dt} - \nabla\cdot\frac{d}{dt}\nabla_\perp(\phi + p + 2T_\perp) + B\nabla_\parallel\frac{q_{\perp\parallel}}{B} = \mathcal{K}\left(\frac{\phi + p_\perp + 3T_\perp}{2}\right)$$

$$\frac{1}{2}\frac{dT_\parallel}{dt} - \nabla\cdot\frac{d}{dt}\nabla_\perp\frac{T_\parallel}{2} + B\nabla_\parallel\frac{u_\parallel + q_{\parallel\parallel}}{B} = \mathcal{K}\left(\frac{\phi + p_\parallel + 2T_\parallel}{2}\right)$$

- add them, use $q_\parallel = q_{\parallel\parallel} + q_{\perp\parallel}$ and $T_\parallel = T_\perp = T$

$$\frac{3}{2}\frac{dT}{dt} - \nabla\cdot\frac{d}{dt}\nabla_\perp\left(\phi + p + \frac{5}{2}T\right) + B\nabla_\parallel\frac{u_\parallel + q_\parallel}{B} = \mathcal{K}\left(\phi + p + \frac{5}{2}T\right)$$

- the $5/2$ under $\nabla_\perp^2$ is the polarisation heat flux

agreement with the Braginskii forms is found
if the latter keeps heat flux polarisation

Intermission

- we have shown the gyrofluid FLR forms reproduce gyroviscosity to $O(b)$
- we now turn to parallel viscosity, the anisotropy correction
  - nonlinear equations for $\delta p = p_\| - p_\perp$ in limit $\nu \gg d/dt$
- later do the parallel heat flux
  - nonlinear equations for $q_\| = q_{\|\,\|} + q_{\perp\,\|}$ in limit $\nu \gg d/dt$

these three pieces form the gyrofluid/fluid correspondence

Pressure Anisotropy — Definitions

- diagonal pressure, perp and parallel different

$$\mathbf{P} = p_\parallel \mathbf{bb} + p_\perp (\mathbf{g} - \mathbf{bb}) \qquad p_\parallel = \int d\mathcal{W}\, M w_\parallel^2\, f \qquad p_\perp = \int d\mathcal{W}\, M \frac{w_\perp^2}{2}\, f$$

- expression within Braginskii — pressure plus traceless tensor

$$\mathbf{P} = p\,\mathbf{g} + \mathbf{\Pi} \qquad g^{ij}\Pi_{ij} = 0$$

- isotropisation — extra potential appearing in reduced Braginskii model

$$p = \frac{2}{3}p_\perp + \frac{1}{3}p_\parallel \qquad \delta p = p_\parallel - p_\perp \qquad G \equiv \frac{1}{4}\Pi_{\parallel\parallel} = \frac{1}{6}\delta p$$

- appearances in the gyrofluid model — same as in reduced Braginskii model

$$\frac{p_\parallel + p_\perp}{2} = p + \frac{1}{6}\delta p \equiv p + G \qquad p_\parallel = p + \frac{2}{3}\delta p \equiv p + 4G$$

Parallel Viscosity

- Braginskii's Stress Tensor — diagonal elements
- result of expansion (z is parallel, xy are perpendicular)

$$\Pi_{xx} = \Pi_{yy} = -\frac{\eta_0}{2}\left(W_{xx} + W_{yy}\right) \qquad\qquad \Pi_{zz} = -\eta_0\, W_{zz}$$

- bulk compression viscosity coefficient (for ions)

$$\eta_0 = 0.96\frac{p_i}{\nu_i}$$

- form of stress tensor in terms of divergences

$$\Pi_{\|\|} = -\eta_0\left[2\nabla_\| u_\| - \frac{2}{3}(\nabla\cdot\mathbf{u})\right] = -\frac{2}{3}\eta_0\left(2\nabla_\| u_\| - \nabla\cdot\mathbf{u}_\perp\right)$$

- expression in terms of anisotropy

$$\delta p = \frac{3}{2}\Pi_{\|\|} \sim p\frac{(\nabla\cdot\mathbf{u})}{\nu}$$

Corrections for Toroidal Magnetic Field

- Braginskii's component expansion is only valid in a slab, due to finite $\nabla\cdot\mathbf{bb}$ otherwise
- correct using CGL model with adiabatic invariants (add dissipation)

$$n(dT_\parallel/dt) + (\nu/3)\delta p = -2p_\parallel \mathbf{bb}\!:\!\nabla\mathbf{u}$$

$$n(dT_\perp/dt) - (\nu/3)\delta p = -p_\perp \left(\nabla\cdot\mathbf{u} - \mathbf{bb}\!:\!\nabla\mathbf{u}\right)$$

- ordering: $\delta p \ll p_\parallel \sim p_\perp \to p$

$$\nu\delta p = -p\left(3\mathbf{bb}\!:\!\nabla\mathbf{u} - \nabla\cdot\mathbf{u}\right) = -p\left(2\nabla_\parallel u_\parallel - \nabla\cdot\mathbf{u}_\perp - 3\mathbf{u}\nabla\!:\!\mathbf{bb}\right)$$

- also use $\mathbf{u}\cdot(\mathbf{b}\cdot\nabla\mathbf{b}) \approx -(1/2)\nabla\cdot\mathbf{u}_\perp$ **(note signs, factors change)**

$$\nu\delta p = -\frac{p}{2}\left(4\nabla_\parallel u_\parallel + \nabla\cdot\mathbf{u}_\perp\right)$$

- we'll show this is also the gyrofluid result in the appropriate limit

Physical Effect — Magnetic Pumping

- parallel/perp divergences change parallel/perp temperatures (pressures)
  - changes add to pressures, through stress tensor pieces

- perp compression in vorticity equation

$$p \to p + G \qquad\qquad \mathcal{K}(p+G) = -\nabla \cdot \frac{c}{B^2}\mathbf{B}\times\nabla(p+G)$$

- parallel compression in parallel velocity equation

$$p \to p + 4G \qquad\qquad \nabla_\parallel(p+4G) = \mathbf{b}\cdot\nabla(p+4G)$$

- in reduced Braginskii equations (normalised) — dissipation forces G towards zero
  - numerical coefficients ensure positive definite damping

$$\frac{\partial}{\partial t}\nabla_\perp^2(\phi+p) = \cdots - \mathcal{K}(p+G) \qquad\qquad \frac{\partial u_\parallel}{\partial t} = \cdots - \nabla_\parallel(p+4G)$$

Pressure Anisotropy in the Gyrofluid Model

- moment hierarchy on the gyrokinetic equation
  - $w_\parallel^2$ moment gives $p_\parallel$ equation
  - $w_\perp^2/2$ moment gives $p_\perp$ equation
  - collisional dissipation $\nu(p_\parallel - p_\perp)/3$ gives relaxation
  - nonlinear ExB advective derivative $\mathbf{u}_E \cdot \nabla$ gives forcing
  - dissipation assumed in collisional theory to balance flow divergences

- required limits for Braginskii model
  - strongly collisional, $\nu \gg (\partial/\partial t)$, also $\nu \gg V_t/R$
  - small anisotropy, $\delta p \ll p$, where $\delta p = p_\parallel - p_\perp$
  - slow heat fluxes, $\mathbf{q} \ll p\mathbf{u}$

these conditions are not usually met

Appearance in the Gyrofluid Equations

- curvature terms, density equation (gyro-Bohm units)

$$\frac{\partial n_i}{\partial t} = \cdots + \mathcal{K}\left(\phi_G + \frac{p_\| + p_\perp + \Omega_G}{2}\right)$$

  becomes, for small anisotropy

$$\frac{\partial n_i}{\partial t} = \cdots + \mathcal{K}\left(\phi_G + p + \frac{\Omega_G}{2} + \frac{\delta p}{6}\right)$$

- this enters both temperatures as well

- parallel gradient term in parallel velocity

$$\frac{\partial u_\parallel}{\partial t} = \cdots - \nabla_\parallel \left(p_\parallel + \phi_G\right)$$

- in combination, note the factor of four:

$$\frac{\partial n_i}{\partial t} = \cdots + \mathcal{K}\left(\phi_G + p + \frac{\Omega_G}{2} + \frac{\delta p}{6}\right)$$

$$\frac{\partial u_\parallel}{\partial t} = \cdots - \nabla_\parallel \left(p + \frac{2}{3}\delta p + \phi_G\right)$$

- expressing $G = \delta p/6$, this is the same as the Braginskii/CGL forms
  - recall the ion gyrocenter density tracks the total fluid vorticity
  - fluid vorticity equation formed from the fluid density difference
  - polarisation current from gyroviscosity, diamagnetic current from $\mathcal{K}$ terms

Anisotropy Dissipation in the Temperature Equations

- direct damping of the temperature difference, coefficient $\eta = 0.96$ (for electrons, 0.73)

$$\frac{\partial T_\perp}{\partial t} + [\phi_G, T_\perp] + [\Omega_G, (n + 2T_\perp)] + B\nabla_\parallel \frac{q_{\perp\parallel}}{B} = \mathcal{K}\left(\frac{\phi_G + p_\perp + 4\Omega_G + 3T_\perp}{2}\right) + \frac{\nu}{3\eta}\delta T$$

$$\frac{1}{2}\frac{\partial T_\parallel}{\partial t} + \frac{1}{2}[\phi_G, T_\parallel] + B\nabla_\parallel \frac{u_\parallel + q_{\parallel\parallel}}{B} = \mathcal{K}\left(\frac{\phi_G + p_\parallel + 2T_\parallel}{2}\right) - \frac{\nu}{3\eta}\delta T$$

- note that $\delta p = \delta T = T_\parallel - T_\perp$ in normalised units

- find the equation for δp to obtain the gyrofluid's parallel viscosity

$$\left(\frac{\partial}{\partial t} + [\phi_G,] + \frac{\nu}{\eta}\right)\delta p - [\Omega_G, (n + 2T_\perp)] + B\nabla_\parallel \frac{2u_\parallel + 2q_{\parallel\parallel} - q_{\perp\parallel}}{B}$$

$$= \mathcal{K}\left(\frac{\phi_G - \Omega_G}{2} + \frac{2p_\parallel - p_\perp}{2} + 2T_\parallel - 3\frac{\Omega_G + T_\perp}{2}\right)$$

Parallel Viscosity from the Temperature Equations

- the formed equation for the anisotropy (again)

$$\left(\frac{\partial}{\partial t} + [\phi_G,] + \frac{\nu}{\eta}\right)\delta p - [\Omega_G,(n+2T_\perp)] + B\nabla_\parallel \frac{2u_\parallel + 2q_{\parallel\parallel} - q_{\perp\parallel}}{B}$$

$$= \mathcal{K}\left(\frac{\phi_G - \Omega_G}{2} + \frac{2p_\parallel - p_\perp}{2} + 2T_\parallel - 3\frac{\Omega_G + T_\perp}{2}\right)$$

- neglect FLR effects, also anisotropy except on left hand side

$$\left(\frac{d}{dt} + \frac{\nu}{\eta}\right)\delta p = \frac{1}{2}\mathcal{K}(\phi + p + T) - B\nabla_\parallel \frac{2u_\parallel + 2q_{\parallel\parallel} - q_{\perp\parallel}}{B}$$

- Braginskii limit: neglect heat fluxes, assume $\nu \gg d/dt$, use $\Pi_{\parallel\parallel} = (2/3)\delta p = 4G$

$$\Pi_{\parallel\parallel} = \frac{\eta}{3\nu}\left[\mathcal{K}(\phi + p) - 4B\nabla_\parallel \frac{u_\parallel}{B}\right]$$

- this is the same form as in the CGL/Braginskii fluid model

Parallel Heat Flux Equations

- as with δp, these are dynamical variables ($v_\parallel^2 v_\parallel/2$ and $\mu B v_\parallel$ moments)
- with Landau damping a_L and collisional dissipation ν
  - thermal conduction coefficient κ
  - thermal force coefficient $\alpha = 0.71$ present for electrons only

$$\frac{\partial q_{\parallel\,\parallel}}{\partial t} + [\phi_G, q_{\parallel\,\parallel}] = -\frac{3}{2}\nabla_\parallel T_\parallel + \mathcal{K}\left(\frac{3u_\parallel + 8q_{\parallel\,\parallel}}{2}\right) - a_L q_{\parallel\,\parallel} - \frac{3}{2}\frac{\nu}{\kappa}\left(q_{\parallel\,\parallel} + 0.6\alpha J_\parallel\right) - 1.28\nu\left(q_{\parallel\,\parallel} - \frac{3}{2}q_{\perp\,\parallel}\right)$$

$$\frac{\partial q_{\perp\,\parallel}}{\partial t} + [\phi_G, q_{\perp\,\parallel}] + [\Omega_G, (u_\parallel + 2q_{\perp\,\parallel})] = -\nabla_\parallel(\Omega_G + T_\perp) + \mathcal{K}\left(\frac{u_\parallel + 6q_{\perp\,\parallel}}{2}\right) - a_L q_{\perp\,\parallel} - \frac{\nu}{\kappa}\left(q_{\perp\,\parallel} + 0.4\alpha J_\parallel\right) + 1.28\nu\left(q_{\parallel\,\parallel} - \frac{3}{2}q_{\perp\,\parallel}\right)$$

- form the equation for the total $q_\parallel = q_{\parallel\,\parallel} + q_{\perp\,\parallel}$, neglect FLR effects

Parallel Heat Flux in Collisional Limit

- form the equation for the total $q_\parallel = q_{\parallel\,\parallel} + q_{\perp\,\parallel}$, neglect FLR effects

$$\frac{dq_\parallel}{dt} + a_L q_\parallel - \mathcal{K}(2u_\parallel + 4q_{\parallel\,\parallel} + 3q_{\perp\,\parallel}) = -\nabla_\parallel \left(\frac{3}{2} T_\parallel + T_\perp \right) \frac{5/2}{\kappa} \nu (q_\parallel + \alpha J_\parallel)$$

- Braginskii limit: terms on the left side are small
- neglecting anisotropy, the Braginskii form is recovered

$$q_\parallel + \alpha J_\parallel = -\frac{\kappa}{\nu} \frac{T_0}{M} \nabla_\parallel T$$

- putting the physical units back in (T and M, especially), recover

$$q_\parallel + \alpha \frac{T}{e} J_\parallel = -\kappa \, n \frac{V_t^2}{\nu} \nabla_\parallel T$$

Main Points

- starting with the gyrofluid model with dissipation effects:
- the gyrofluid FLR forms reproduce gyroviscosity to $O(k_\perp^2 \rho_i^2)$
- the gyrofluid anisotropy reproduces the parallel viscosity, magnetic pumping
- the gyrofluid heat flux equations reduce to the Braginskii forms
- NB: the dissipation coefficients and the thermal force model were so chosen
- however, the gyroviscosity result emerges naturally
- the form of the divergences in the parallel viscosity also emerge naturally

the gyrofluid GEM model with dissipation is a complete superset
of the low frequency Braginskii equations