Gyrokinetic Energetics

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Outline

- **What Gyrokinetics Means, Gyrokinetic Equation**

  — electrostatic, fully nonlinear

  — Lagrangian, Hamiltonian structure

  — variational principle → polarisation

- **Energy conservation**

  — particle versus field Hamiltonian

  — drift energy versus thermal energy

  — polarisation equation and energy theorem

  — phase space conservation

- **Flow Energy**

  — flows: field energy

  — zonal flows: contribution of zonal component to field energy

  — energy exchange mechanisms

- **Delta-F Codes GENE and FEFC**

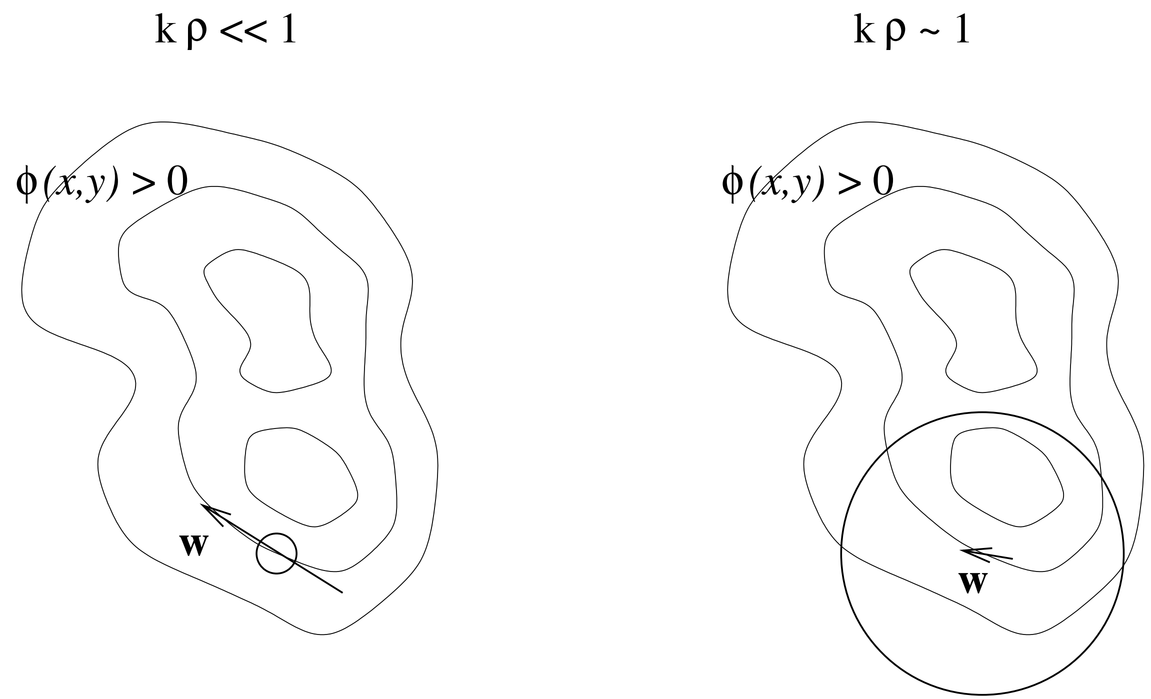
  — delta-f approximations

  — free energy conservation

The Meaning of Gyrokinetics

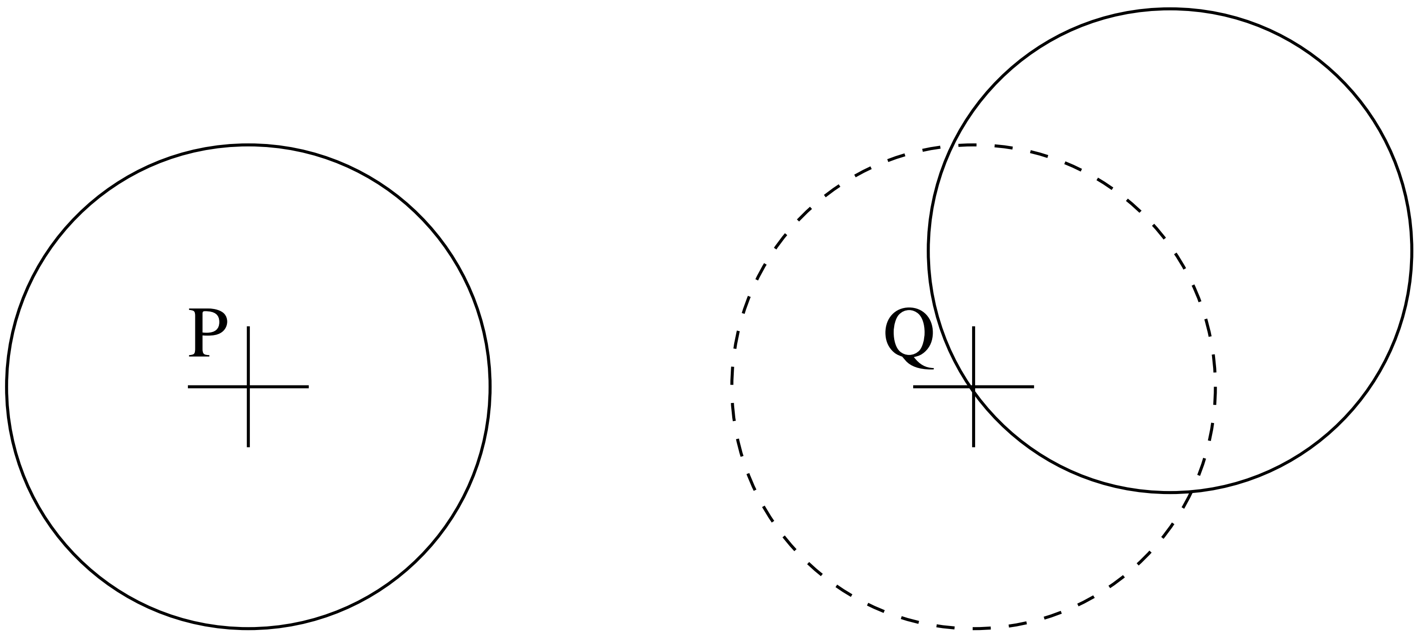
- low frequencies $\omega \ll \Omega_c = eB/mc$ for each species

treat particles as rings of charge in spatially varying fields



- reduced response: "gyroaveraging"
- reaction to fields, polarisation density "gyroscreening"

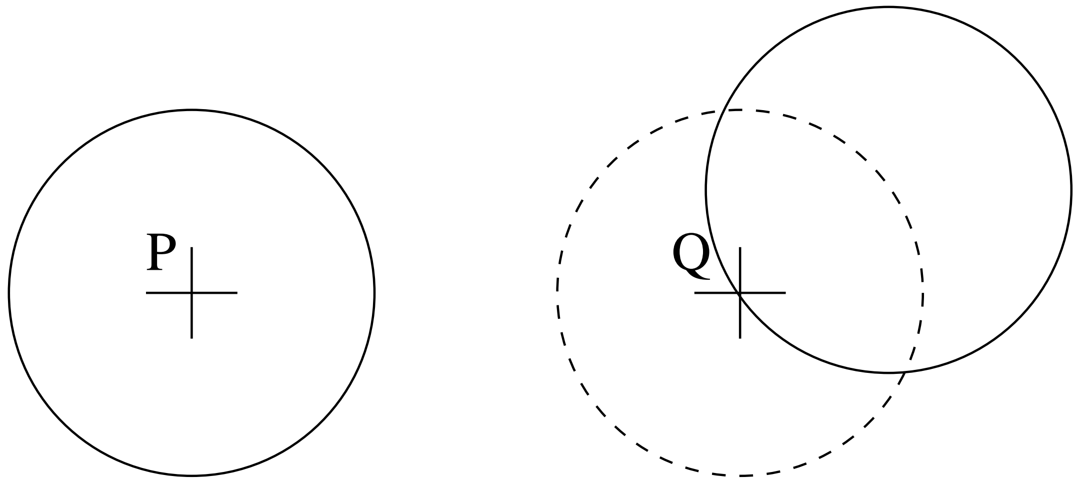
Gyroaveraging Operations



gyrocenter at P contributes charge to points on the ring

gyrocenters contributing charge to Q lie on dashed ring

Gyrokinetic/Gyrofluid Polarisation



gyrocenter at P senses potential at points on the ring

gyrocenters on dashed ring contribute charge to Q

mathematics identical in both procedures (Hermitian operators)

polarisation density <–– part of f dependent upon gyrophase angle

The Gyrokinetic Model

- electrostatic, full-f version to show structure
- particle Lagrangian

$$L_p = \left(\frac{e}{c}\mathbf{A} + mv_\parallel\mathbf{b}\right)\cdot\dot{\mathbf{R}} + \mu\frac{B}{\Omega}\dot{\theta} - \left(m\frac{v_\parallel^2}{2} + \mu B - e\psi\right)$$

- Hamiltonian

$$H = m\frac{v_\parallel^2}{2} + \mu B - e\psi$$

- gyroaveraged and (low-$k_\perp$) screened potential

$$e\psi = eJ_0\phi - m\frac{v_E^2}{2}$$

- modified field

$$\mathbf{A}^* = \mathbf{A} + \frac{mc}{e}v_\parallel\mathbf{b} \qquad \mathbf{B}^* = \nabla\times\mathbf{A}^* \qquad B^* = \mathbf{B}^*\cdot\mathbf{b} \qquad \mathbf{b}^* = \frac{\mathbf{B}^*}{B^*}$$

The Gyrokinetic Equation

- particle Lagrangian

$$L_p = \frac{e}{c}\mathbf{A}^* \cdot \dot{\mathbf{R}} + \mu \frac{B}{\Omega}\dot{\theta} - H$$

- equation of motion for distribution function

$$\frac{\partial f}{\partial t} = \frac{1}{m}\left(\frac{\partial f}{\partial v_\parallel}\mathbf{b}^* \cdot \nabla H - \frac{\partial H}{\partial v_\parallel}\mathbf{b}^* \cdot \nabla f\right) + \frac{c}{eBB^*}\mathbf{F}:(\nabla H \nabla f)$$

- tensor representation for magnetic field

$$\mathbf{F} = \epsilon \cdot \mathbf{B} \qquad \text{hence} \qquad \mathbf{v}_E = \frac{c}{B^2}\mathbf{B}\times\nabla\phi = -\frac{c}{B^2}\mathbf{F}\cdot\nabla\phi$$

- Hamiltonian structure

$$\frac{\partial f}{\partial t} = [H, f]$$

where ...

… where we have a spatial coordinate system $\{x, y, s\}$ so that

$$[H, f] = \frac{b^{*x}}{m}\left(\frac{\partial f}{\partial v_\parallel}\frac{\partial H}{\partial x} - \frac{\partial H}{\partial v_\parallel}\frac{\partial f}{\partial x}\right) + \frac{c\,F^{xy}}{eBB^*}\left(\frac{\partial f}{\partial x}\frac{\partial H}{\partial y} - \frac{\partial H}{\partial x}\frac{\partial f}{\partial y}\right)$$

$$+\frac{b^{*y}}{m}\left(\frac{\partial f}{\partial v_\parallel}\frac{\partial H}{\partial y} - \frac{\partial H}{\partial v_\parallel}\frac{\partial f}{\partial y}\right) + \frac{c\,F^{xs}}{eBB^*}\left(\frac{\partial f}{\partial x}\frac{\partial H}{\partial s} - \frac{\partial H}{\partial x}\frac{\partial f}{\partial s}\right)$$

$$+\frac{b^{*s}}{m}\left(\frac{\partial f}{\partial v_\parallel}\frac{\partial H}{\partial s} - \frac{\partial H}{\partial v_\parallel}\frac{\partial f}{\partial s}\right) + \frac{c\,F^{ys}}{eBB^*}\left(\frac{\partial f}{\partial y}\frac{\partial H}{\partial s} - \frac{\partial H}{\partial y}\frac{\partial f}{\partial s}\right)$$

- notes:
  - ○ curvature drift is in the b^* terms
  - ○ grad-B drift is in the μB part of the drift terms
  - ○ (generalised) ExB drift is in the $e\psi$ part of the drift terms
  - ○ parallel dynamics is in the b terms

- note especially:
  - ○ parallel dynamics and trapping go together, in both $e\psi$ and μB
  - ○ (really subtle) phase space conservation:

$$\frac{e}{m}\frac{\partial}{\partial v_\parallel}\left(\mathbf{b}^* \cdot \nabla\psi\right) \qquad \text{versus} \qquad \nabla \cdot \frac{c}{BB^*}\mathbf{F} \cdot \nabla\psi$$

○ that is, curvature drift part of trapping versus ExB advection!

Polarisation: Reaction of the Field

- find from Hamilton's principle, variation of ϕ

$$\sum_{\text{species}} \left[\int d\mathcal{W}\, eJ_0 f + \nabla \cdot \frac{nmc^2}{B^2} \nabla_\perp \phi \right] = 0 \qquad \text{where} \qquad n = \int d\mathcal{W}\, f$$

- a little slower ...
  - position gyrocenters in space (G is positioning kernel):

$$\int d\mathcal{V}\mathcal{L} = \int d\mathcal{V} \sum_p L_p\, G(\mathbf{R}, \mathbf{x})$$

  - examine part due to the potential ϕ:

$$\int d\mathcal{V}\mathcal{L} = \int d\mathcal{V} \int d\mathcal{W}\, f\, (\cdots - e\psi) = \cdots - \int d\mathcal{V} \int d\mathcal{W}\, f \left(eJ_0\phi - \frac{mc^2}{2B^2} \left| \nabla_\perp \phi \right|^2 \right)$$

  - vary with respect to ϕ (J_0 is Hermitian, do one $\nabla_\perp$ by parts, incorporate f into n):

$$\delta \left(\int d\mathcal{V}\mathcal{L} \right) = - \int d\mathcal{V}\, \delta\phi \left(\int d\mathcal{W}\, eJ_0 f + \nabla \cdot \frac{nmc^2}{B^2} \nabla_\perp \phi \right)$$

Energy Conservation

- main point:

  particle and field energy work together

- in this case, field energy is actually ExB motion of particles

- an important auxiliary relation using polarisation:

$$\sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, eJ_0\phi\, f = \sum_{\text{species}} \int d\mathcal{V}\, nmv_E^2$$

- proof: multiply polarisation by ϕ, integrate over volume

$$\int d\mathcal{V}\,\phi \sum_{\text{species}} \left[\int d\mathcal{W}\, eJ_0 f + \nabla\cdot\frac{nmc^2}{B^2}\nabla_\perp\phi\right] = 0$$

$$\int d\mathcal{V} \sum_{\text{species}} \left[\int d\mathcal{W}\,\phi\, eJ_0 f - \nabla\phi\cdot\frac{nmc^2}{B^2}\nabla_\perp\phi\right] = 0$$

$$\int d\mathcal{V} \sum_{\text{species}} \int d\mathcal{W}\, f\, eJ_0\phi = \int d\mathcal{V} \sum_{\text{species}} \frac{nmc^2}{B^2}\left|\nabla_\perp\phi\right|^2$$

Energy Conservation (2)

- "charge density" is really ExB energy
  - start with

$$\sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, eJ_0\phi\, f = \sum_{\text{species}} \int d\mathcal{V}\, nmv_E^2$$

  - subtract the ExB energy from both sides

$$\sum_{\text{species}} \int d\mathcal{V} \left(\int d\mathcal{W}\, eJ_0\phi\, f - nm\frac{v_E^2}{2} \right) = \sum_{\text{species}} \int d\mathcal{V}\, nm\frac{v_E^2}{2}$$

  - combine left side into effective potential

$$\sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, e\psi\, f = \sum_{\text{species}} \int d\mathcal{V}\, nm\frac{v_E^2}{2}$$

Energy Conservation (3)

- entire energy is therefore thermal plus field energy

$$\mathcal{U} = \sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, H\, f = \sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, (H_0 + e\psi)\, f$$

  where

$$H_0 = m\frac{v_\parallel^2}{2} + \mu B$$

  and then

$$\sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, H\, f = \sum_{\text{species}} \int d\mathcal{V} \left(\int d\mathcal{W}\, H_0\, f + nm\frac{v_E^2}{2} \right)$$

- part of energy due to H_0 is thermal energy

- part of energy due to $e\psi$ is ExB energy (“drift energy” or “field energy”)

Energy Conservation (4)

- time derivative works the same way (more manipulations):

$$\frac{\partial}{\partial t}\sum_{\text{species}}\int d\mathcal{V}\,nm\frac{v_E^2}{2} = \sum_{\text{species}}\int d\mathcal{V}\int d\mathcal{W}\,e\psi\frac{\partial f}{\partial t}$$

- so that the evolution of the total energy is given by ...

$$\frac{\partial \mathcal{U}}{\partial t} = \sum_{\text{species}}\int d\mathcal{V}\int d\mathcal{W}\,H_0\frac{\partial f}{\partial t} + \frac{\partial}{\partial t}\sum_{\text{species}}\int d\mathcal{V}\,nm\frac{v_E^2}{2}$$

$$= \sum_{\text{species}}\int d\mathcal{V}\int d\mathcal{W}\,(H_0 + e\psi)\,\frac{\partial f}{\partial t}$$

$$= \sum_{\text{species}}\int d\mathcal{V}\int d\mathcal{W}\,H\frac{\partial f}{\partial t}$$

$$= \sum_{\text{species}}\int d\mathcal{V}\int d\mathcal{W}\,H[H,f] = 0$$

... zero!   One more little detail however ...

Phase Space Conservation

... the assertion that the integral over $H[H,f]$ vanishes requires the Liouville theorem

- express the gyrokinetic equation in advection form

$$\frac{\partial f}{\partial t} + V^\alpha \nabla_\alpha f = 0 \qquad \text{where} \qquad V^\alpha = \left\{ -\frac{c}{eBB^*}\mathbf{F}\cdot\nabla H + \frac{\mathbf{b}^*}{m}\frac{\partial H}{\partial v_\parallel}, -\frac{1}{m}\mathbf{b}^*\cdot\nabla H \right\}$$

- it is important to note that not $\nabla_\alpha V^\alpha$ but $\nabla_\alpha (B^* V^\alpha)$ is zero
  - proof (use $\nabla\times\mathbf{b} = -\nabla\cdot(\mathbf{F}/B)$ and $\nabla\cdot\mathbf{B}^* = 0$):

$$\begin{aligned}
\nabla_\alpha (B^* V^\alpha) &= -\nabla\cdot\left(\frac{c}{e}\frac{\mathbf{F}}{B}\cdot\nabla H\right) - \frac{1}{m}\frac{\partial \mathbf{B}^*}{\partial v_\parallel}\cdot\nabla H \\
&= -\nabla\cdot\left(\frac{c}{e}\frac{\mathbf{F}}{B}\cdot\nabla H\right) - \frac{c}{e}(\nabla\times\mathbf{b})\cdot\nabla H \\
&= -\nabla\cdot\left(\frac{c}{e}\frac{\mathbf{F}}{B}\cdot\nabla H\right) + \frac{c}{e}\left(\nabla\cdot\frac{\mathbf{F}}{B}\right)\cdot\nabla H \\
&= 0
\end{aligned}$$

- the phase space element is $\int d\mathcal{W} = 2\pi \int dv_\parallel \, d\mu \, B^*$

Phase Space Conservation (2)

- thus we have

$$H\frac{\partial f}{\partial t} = H[H,f] = \left[\frac{H^2}{2}, f\right] = -V_2^\alpha \nabla_\alpha f$$

  where V_2 is V defined with $H \to H^2/2$

- number and energy conservation follow the same way:

$$\int d\mathcal{V} \int d\mathcal{W}\,\frac{\partial f}{\partial t} = \int d\mathcal{V} \int d\mathcal{W}\,[H,f] = -2\pi \int d\mathcal{V} \int dv_\parallel \int d\mu\,(B^* V^\alpha)\nabla_\alpha f =$$

$$= -2\pi \int d\mathcal{V} \int dv_\parallel \int d\mu\,\nabla_\alpha (f B^* V^\alpha) = 0$$

  that is, zero.

- find energy conservation using $H \to H^2/2$ and $V \to V_2$ (trivial)

Energy Transfer

- using total energy conservation it is simple to get the transfer:

$$\frac{\partial \mathcal{U}_k}{\partial t} = -\frac{\partial \mathcal{U}_e}{\partial t}$$

$$\sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, H_0 \frac{\partial f}{\partial t} = - \sum_{\text{species}} \int d\mathcal{V} \int d\mathcal{W}\, e\psi \frac{\partial f}{\partial t}$$

- the energy theorem for zonal flows follows
  - define the zonal average $\langle \phi \rangle$ according to coordinate system
  - field aligned, unit Jacobian Hamada coordinates x, y, s: $\quad \langle \phi \rangle = \int dy \int ds\, \phi$

$$\int dx \int d\mathcal{W}\, \langle \psi \rangle \frac{\partial}{\partial t} \langle f \rangle = \text{“Reynolds stress”} - \text{“geodesic transfer”}$$

- gyrokinetic codes should be using relations like these as diagnostics!

The Delta-F Model

- using usual local approx, take $\delta f/F^M \sim \delta \ll 1$
  - linearise the trapping term
  - divergence free advection, divergences kept in coupling terms

$$\frac{\partial f}{\partial t} + \mathbf{v}_d \cdot \nabla f + \frac{e}{m} E_\parallel \frac{\partial F^M}{\partial v_\parallel} = 0$$

  - drift velocity: ExB, parallel, curvature/grad-B

- 1st order gyroaveraging

$$\widehat{f} = \sum_{\mathbf{k}_\perp} J_0(k_\perp v_\perp/\Omega)\, f_{\mathbf{k}_\perp}\, e^{i\mathbf{k}_\perp\cdot\mathbf{x}} \qquad E_\parallel = -\left(\frac{1}{c}\frac{\partial \widehat{A}_\parallel}{\partial t} + \mathbf{b}\cdot\nabla\widehat{\phi}\right)$$

- 2nd order → gyroscreening → field energy term

$$\mathcal{U}_E = \int d\mathcal{V} \sum_{\text{species}} \left[\frac{e^2}{T}\,(1-\Gamma_0)\,\frac{\phi^2}{2}\right]$$

where

$$\Gamma_0 = \int d\mathcal{W}\, J_0^2\, F^M \qquad F^M = \text{Maxwellian}(n,T)$$

The Delta-F Equation

- split off nonlinear terms into a bracket form (with gyroaveraged potentials)

$$\mathbf{v}_E \cdot \nabla f = \frac{c}{B}\mathbf{b}\times\nabla\phi \rightarrow [\widehat{\phi}, f] \qquad \mathbf{b}\cdot\nabla f = \frac{\partial f}{\partial s} - \frac{1}{c}[\widehat{A}_\|, f]$$

- generalised nonlinear potential — in general, nonadiabatic potentials

$$\psi = \widehat{\phi} - \frac{v_\|}{c}\widehat{A}_\| \qquad g = f + e\frac{v_\|}{c}\widehat{A}_\|\frac{F^M}{T} \qquad h = f + e\widehat{\phi}\frac{F^M}{T}$$

- divergences kept in curvature terms acting on $\widehat{\phi}$

$$\mathcal{K}(f) = -\frac{mv_\|^2 + \mu B_0}{e}[\log B^2, f] \qquad \rightarrow \qquad \mathcal{K}(h)$$

- entire equation works with nonadiabatic potentials

Gyrokinetic Eq: $$\frac{\partial g}{\partial t} + [\psi, h] + v_\|\frac{\partial h}{\partial s} = \mathcal{K}(h)$$

- it remains to show this equation plus its field equations have an exact energy theorem

Delta-F Polarisation

- field energy works the same way as in full-f model, with linearised, second order field forms

$$\mathcal{U}_E = \int d\mathcal{V} \sum_{\text{species}} \left[\frac{e^2}{T} (1 - \Gamma_0) \frac{\phi^2}{2} \right] \qquad \mathcal{U}_M = \int d\mathcal{V} \frac{1}{8\pi} \left| \nabla_\perp A_\parallel \right|^2$$

- consider first order gyroaveraged Hamiltonian, find field potential Euler-Lagrange equations

$$\sum_{\text{species}} \left[\frac{e^2}{T} (1 - \Gamma_0) \right] \phi = \sum_{\text{species}} \int d\mathcal{W}\, e \widehat{f} \qquad \nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} \sum_{\text{species}} \int d\mathcal{W}\, e v_\parallel \widehat{f}$$

- in the Ampere's law, use $\widehat{g}$, and in ExB polarisation note symmetry, replace $\widehat{f}$ with $\widehat{g}$

Field Eqs: $$\sigma_E^2 \phi = \sum_{\text{species}} \int d\mathcal{W}\, e \,\widehat{g} \qquad \sigma_A^2 A_\parallel = \frac{4\pi}{c} \sum_{\text{species}} \int d\mathcal{W}\, e v_\parallel \,\widehat{g}$$

with screening functions (used in $\mathbf{k}_\perp$-space)

$$\sigma_E^2 = \sum_{\text{species}} \int d\mathcal{W}\, e^2 \left(1 - J_0^2\right) \frac{F^M}{T} \qquad \sigma_A^2 = k_\perp^2 + \sum_{\text{species}} \int d\mathcal{W} \frac{4\pi e^2}{c^2} \left(v_\parallel^2 J_0^2 \right) \frac{F^M}{T}$$

Delta-F Energy

- "free energy" most closely related to entropy
- use field energies, add model for delta-f entropy

$$\mathcal{U}_E = \int d\mathcal{V} \sum_{\text{species}} \left[\frac{e^2}{T} (1 - \Gamma_0) \frac{\phi^2}{2} \right] \qquad \mathcal{U}_M = \int d\mathcal{V} \, \frac{1}{8\pi} \left| \nabla_\perp A_\parallel \right|^2 \qquad \mathcal{U}_F = \int d\Lambda \, \frac{T}{F^M} \frac{f^2}{2}$$

where $d\Lambda = \sum_{\text{species}} d\mathcal{V} \, d\mathcal{W}$

- use polarisation to re-form field pieces

$$\mathcal{U}_E = \frac{1}{2} \int d\Lambda \, e\widehat{\phi} \, f \qquad \mathcal{U}_M = \frac{1}{2} \int d\Lambda \, e\frac{v_\parallel}{c} \widehat{A}_\parallel \, f$$

- use symmetry to freely add $\widehat{\phi}\widehat{A}_\parallel v_\parallel F^M$ piece

$$\mathcal{U} = \mathcal{U}_E + \mathcal{U}_M + \mathcal{U}_F = \int d\Lambda \, \frac{1}{2} \, \frac{T}{F^M} \, hg$$

recall

$$g = f + e\frac{v_\parallel}{c} \widehat{A}_\parallel \frac{F^M}{T} \qquad h = f + e\widehat{\phi} \frac{F^M}{T}$$

Delta-F Energy Conservation

- similar tricks used to get field energy time derivatives, e.g.,

$$\frac{\partial \mathcal{U}_E}{\partial t} = \int d\mathcal{V}\,\phi \sum_{\text{species}} \left[\frac{e^2}{T}\left(1-\Gamma_0\right)\phi\right] = \int d\Lambda\, e\widehat{\phi}\,\frac{\partial f}{\partial t} = \int d\Lambda\, e\widehat{\phi}\,\frac{\partial g}{\partial t}$$

- ExB, magnetic, and thermal free energy pieces

$$\frac{\partial \mathcal{U}_E}{\partial t} = \int d\Lambda\, e\widehat{\phi}\,\frac{\partial g}{\partial t} \qquad \frac{\partial \mathcal{U}_M}{\partial t} = \int d\Lambda\, f\, e\frac{v_\parallel}{c}\frac{\partial \widehat{A}_\parallel}{\partial t} \qquad \frac{\partial \mathcal{U}_F}{\partial t} = \int d\Lambda\,\frac{T}{F^M}\, f\,\frac{\partial f}{\partial t}$$

- use these individually to get transfer effects (and contributions per species to them)
- add them up to find total free energy conservation

Energy Theorem: $$\frac{\partial \mathcal{U}}{\partial t} = \int d\Lambda\,\frac{T}{F^M}\, h\frac{\partial g}{\partial t} = 0$$

since each term in the gyrokinetic equation has the form of a bracket, e.g.,

$$\int d\Lambda\,\frac{T}{F^M}\, h[\psi, h] = \int d\Lambda\,\frac{T}{F^M}\,\left[\psi, \frac{h^2}{2}\right] = 0$$

Procedure in FEFC code

- model is same as F Jenko's GENE code but scheme is different
- known quantities (last 3 time steps): f and ϕ and $A_\|$ hence also g and $\widehat{\phi}$ and $\widehat{A}_\|$
- find h and use gyrokinetic equation to update g

$$\frac{\partial g}{\partial t} + [\psi, h] + v_\| \frac{\partial h}{\partial s} = \mathcal{K}(h)$$

- sum up charge density and current moments, find field potentials in $\mathbf{k}_\perp$-space

$$\sigma_E^2 \phi = \sum_{\text{species}} \int d\mathcal{W}\, e\, \widehat{g} \qquad \sigma_A^2 A_\| = \frac{4\pi}{c} \sum_{\text{species}} \int d\mathcal{W}\, e v_\| \,\widehat{g}$$

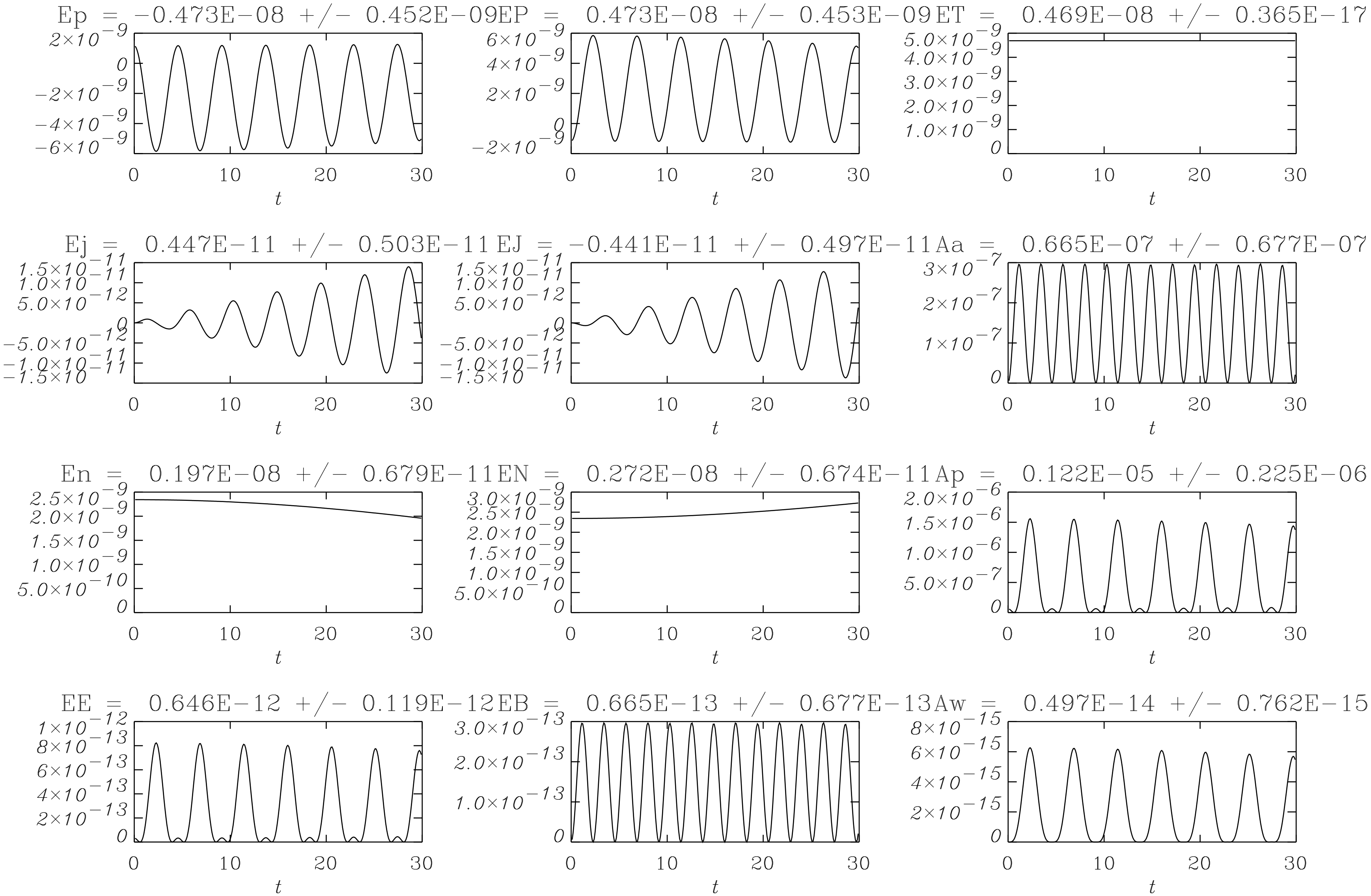
with screening functions (used in $\mathbf{k}_\perp$-space)

$$\sigma_E^2 = \sum_{\text{species}} \int d\mathcal{W}\, e^2 \left(1 - J_0^2\right) \frac{F^M}{T} \qquad \sigma_A^2 = k_\perp^2 + \sum_{\text{species}} \int d\mathcal{W}\, \frac{4\pi e^2}{c^2} \left(v_\|^2\, J_0^2\right) \frac{F^M}{T}$$

- all terms involving h are bracket forms $\rightarrow$ Arakawa spatial scheme
- central dynamics are waves and advection $\rightarrow$ Karniadakis time step scheme

time traces

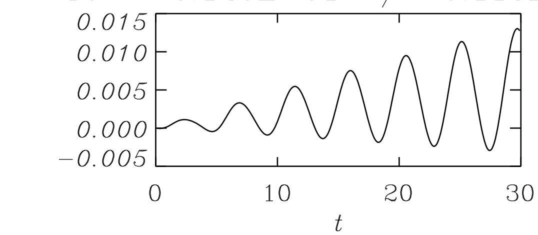
FE 3 Alfvén transient, K = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100



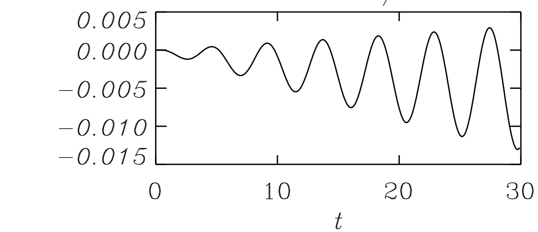
energetics

FE 3 Alfvén transient, K = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100

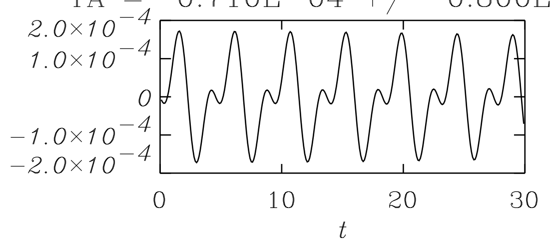
Te = 0.120E−01 +/− 0.118E−02



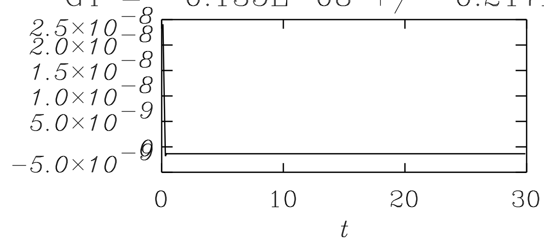
Ti = −0.119E−01 +/− 0.124E−02



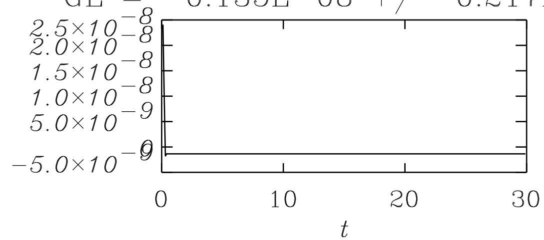
TA = 0.710E−04 +/− 0.800E−04



GT = −0.135E−08 +/− 0.217E−13

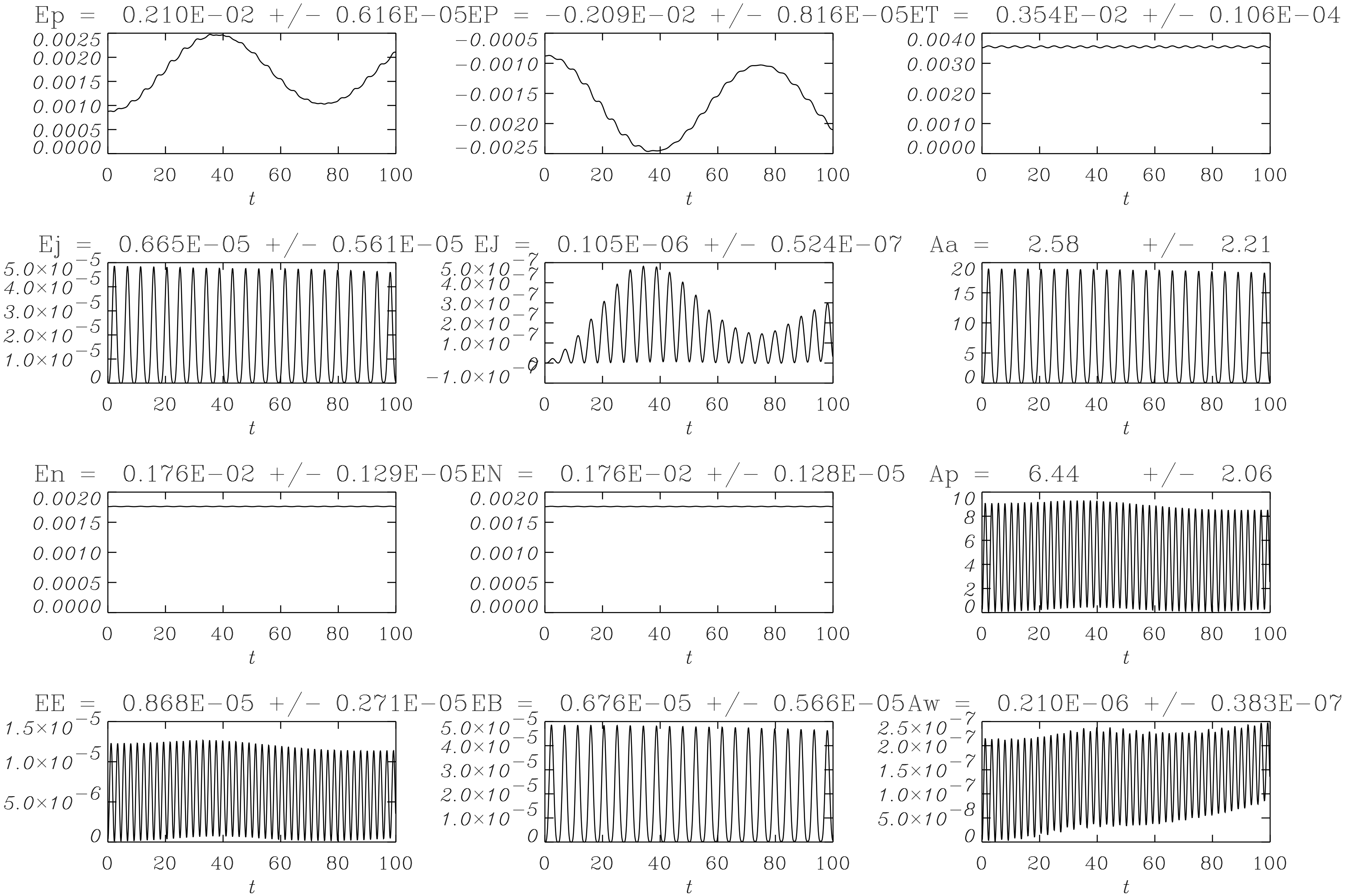


GE = −0.135E−08 +/− 0.217E−13



time traces

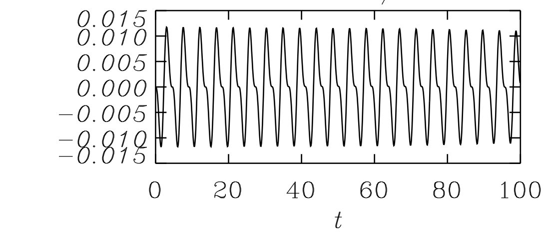
FE 4 Pfirsch-Schlüter transient, d = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100



energetics

FE 4 Pfirsch-Schlüter transient, d = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100

Te = 0.521E−02 +/− 0.317E−02



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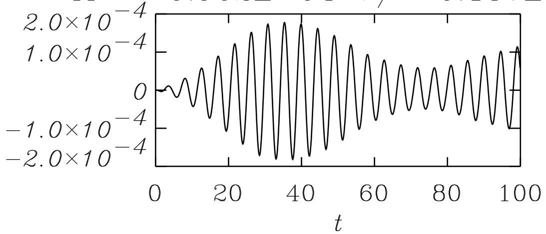
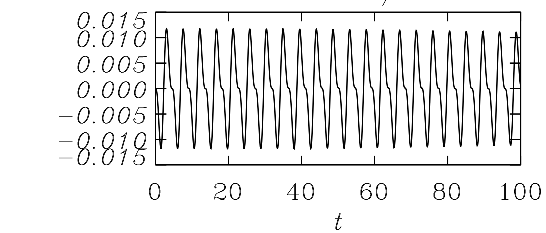
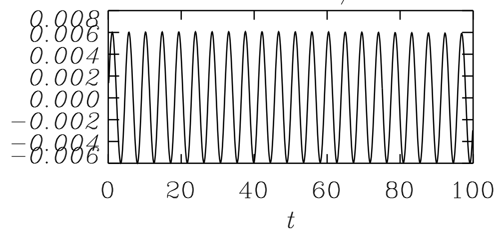


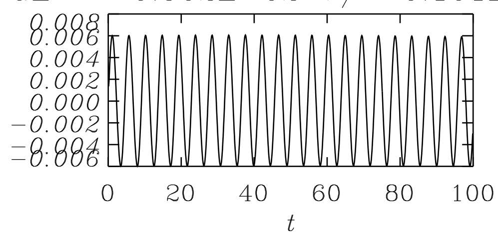
TA = 0.531E−02 +/− 0.319E−02



GT = −0.502E−02 +/− 0.101E−02

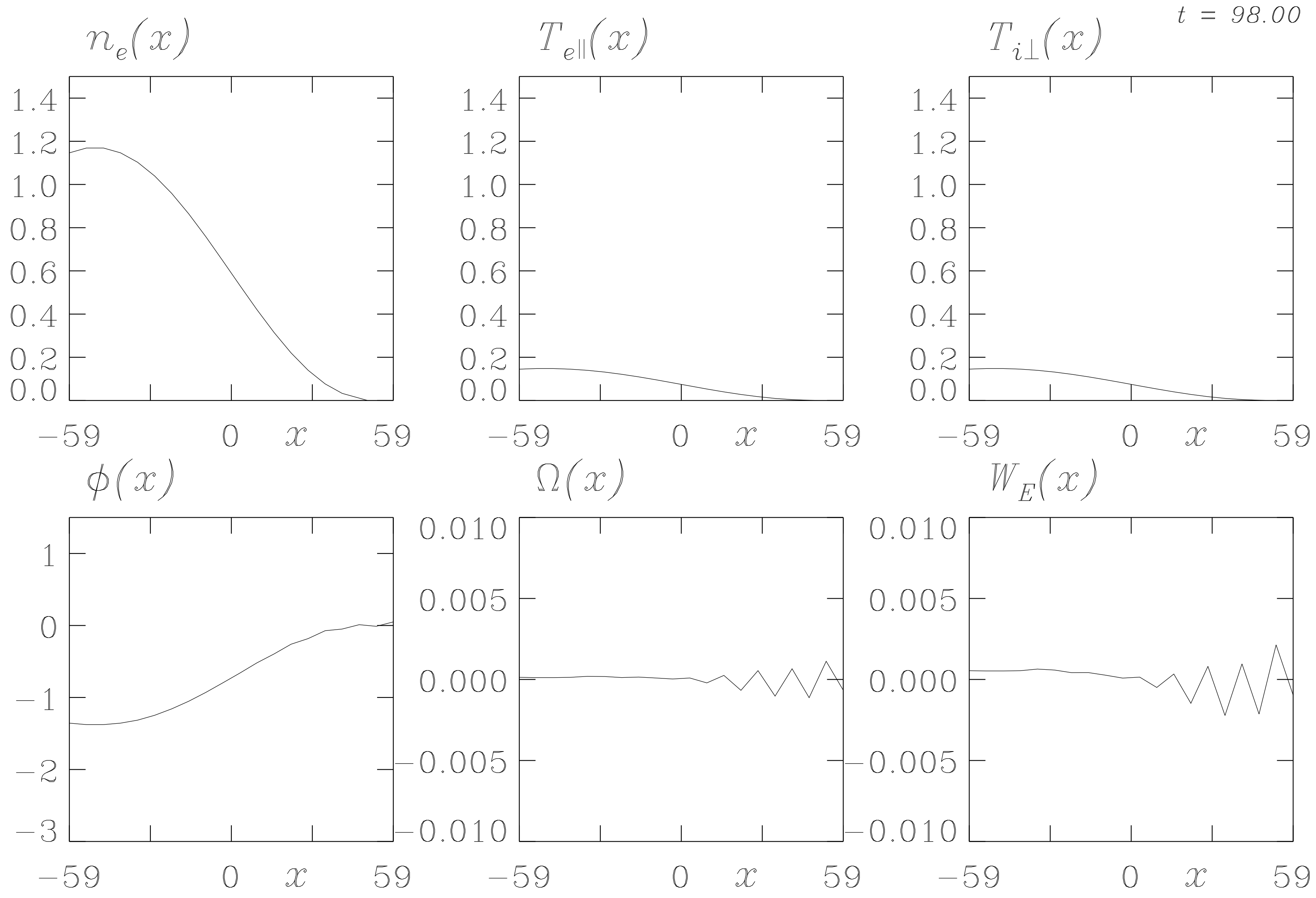


GE = −0.502E−02 +/− 0.101E−02



zonal profiles

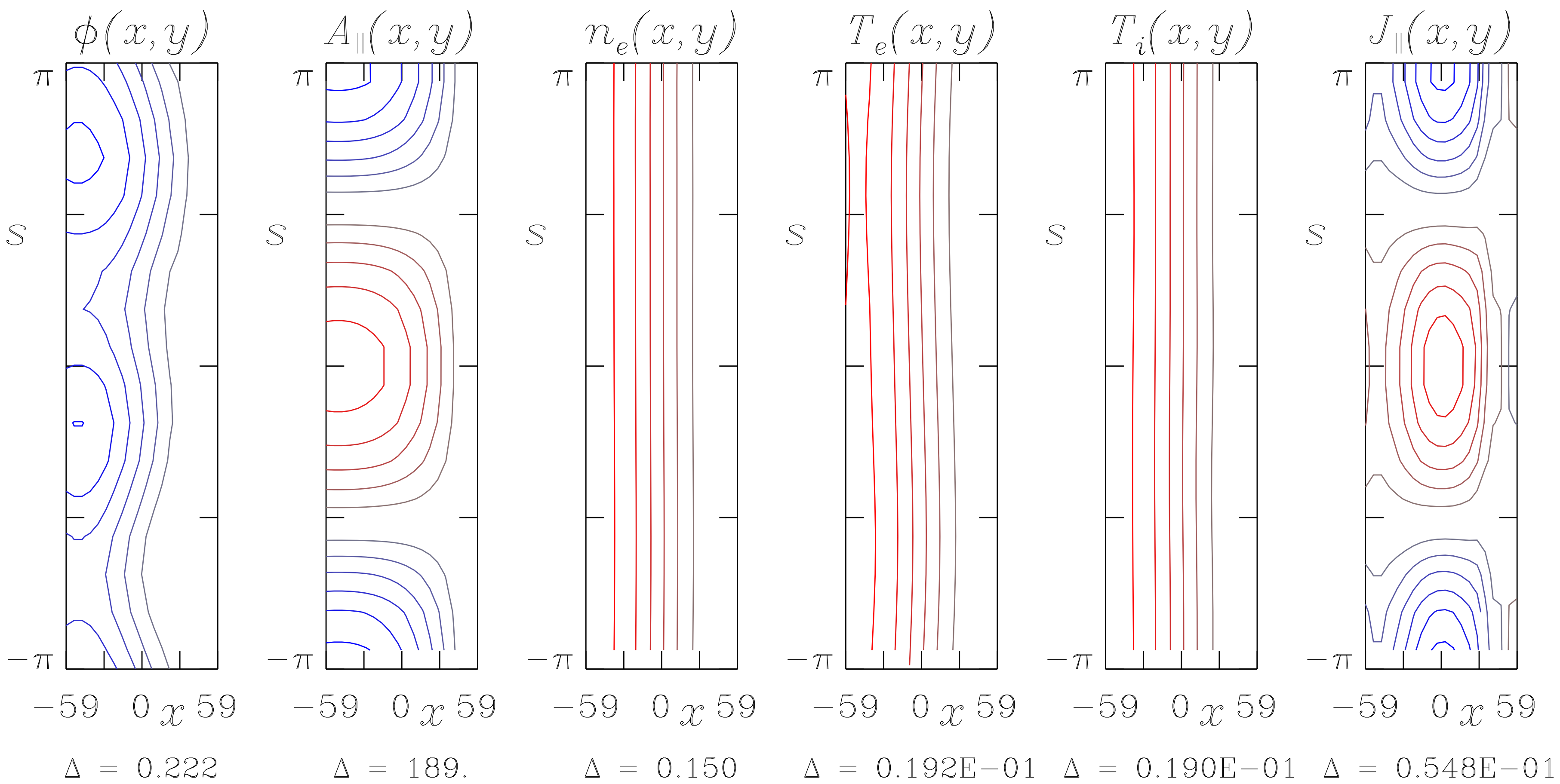
FE 4 Pfirsch-Schlüter transient, d = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100



poloidal plane morphology

FE 4 Pfirsch-Schlüter transient, d = 0.01, beta = 0.0001, mu = me/MD, qR/L = 100

t = 98.00



Main Points

- **Gyrokinetic Equation**

  — Hamiltonian structure follows Lagrangian formulation

  — polarisation equation follows from field variation

- **Energy conservation**

  — split between particle and field Hamiltonian

  — polarisation equation $\rightarrow$ ExB energy

  — flows (zonal): field energy (zonal)

- **Delta-F Model**

  — delta-f approximations

  — free energy (not energy) conservation

  — inhomogeneity through v-dependence, but F^M is spatially constant

- **Delta-F Code FEFC**

  — Alfvén energy conservation performs well

  — geodesic (2D) equilibrium recovered up to curvature + boundaries

  — trapping can be included up to minor modifications

  — expect to recover self consistent pedestal dynamics