Edge Turbulence Transport Scaling

Comparison with Transport Models

B. Scott

Max Planck Institut für Plasmaphysik
Euratom Association
D-85748 Garching, Germany

Dec 2007

Main Points

- parameters, time scales, what defines the edge
- meaning of nonlinearity
  - the "flow shear" is the native vorticity of the turbulence
- gyrofluid turbulence scaling, and H-Mode models
  - no threshold, all gradients can drive, generally
  - sensitivity to ExB shear (externally imposed values)
  - no "S-curve" even for grad-T scaling assisted by ExB shear

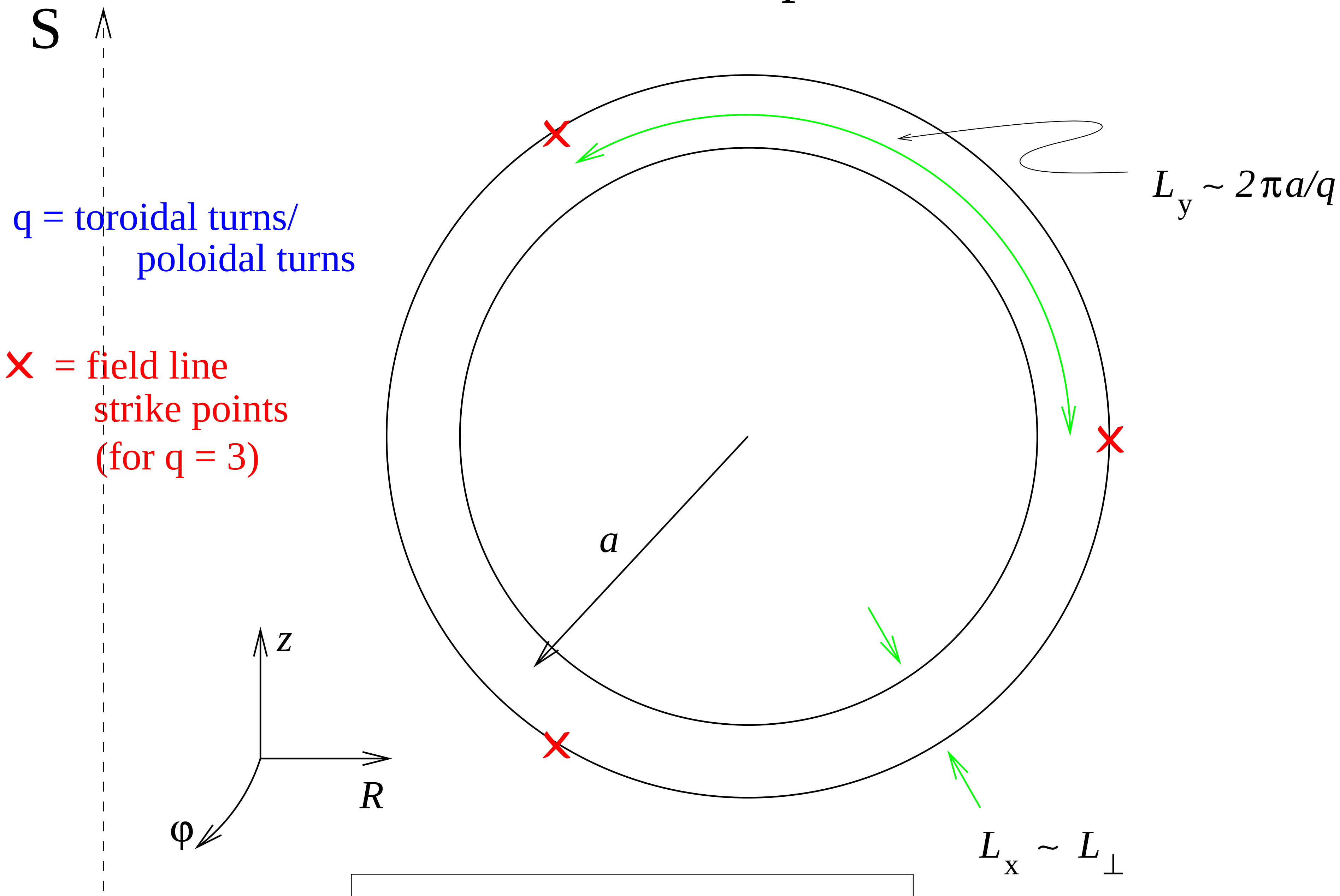
RITM model result thanks to Dennis Kalupin
(auspices of JET Task Force T work group on L-H Transition)

The Tokamak Edge

- coordinates: radial (x), electron drift (y), parallel (s)
- "thin atmosphere" property: $L_\perp/a \ll 1$ hence $L_y \gg L_x$ (in a code, $L_y/L_x \sim 4$)
- field line connection: $L_s = 2\pi qR$, with property that $k_\parallel \neq 0$ for $k_y \neq 0$
- small but moderate drift scale $\delta = \rho_s/L_\perp \ll 1$ but $\delta > 10^{-2}$
- turbulence, small scale isotropy: both $\mathrm{Max}(k_x\rho_s) > 1$ and $\mathrm{Max}(k_y\rho_s) > 1$
- two-fluid adiabatic response: $n_e e E_\parallel \sim \nabla_\parallel p_e$
- NB: if $T_i \sim T_e$ then $\rho_i \sim \rho_s$ hence $k_\perp\rho_i > 1$ hence full FLR

well constructed computations respect all of these
in every run

Computational Domain



large aspect ratio: $L_y \gg L_x$

Edge Parameters

- typical situation: Alfvén/electron transit, collision, and drift frequencies comparable
- drift frequency is $c_s/L_\perp$, spectral range of main interest is $0.1 < k_y\rho_s < 1$
- steep gradient

$$\hat{\mu} \equiv \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{V_e/qR}\right)^2 > 1$$

- collisional

$$C \equiv \frac{0.51\nu_e}{c_s/L_\perp}\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = 0.51\frac{\nu_e\, c_s/L_\perp}{(V_e/qR)^2} > 1$$

- electromagnetic

$$\hat{\beta} \equiv \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{v_A/qR}\right)^2 \gtrsim 1$$

What Determines the Edge?

- mainly, the first of the conditions: $\hat{\mu} > 1$
- consider the boundary, $\hat{\mu} = 1$

$$\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = 1$$

- solve this for the profile scale length

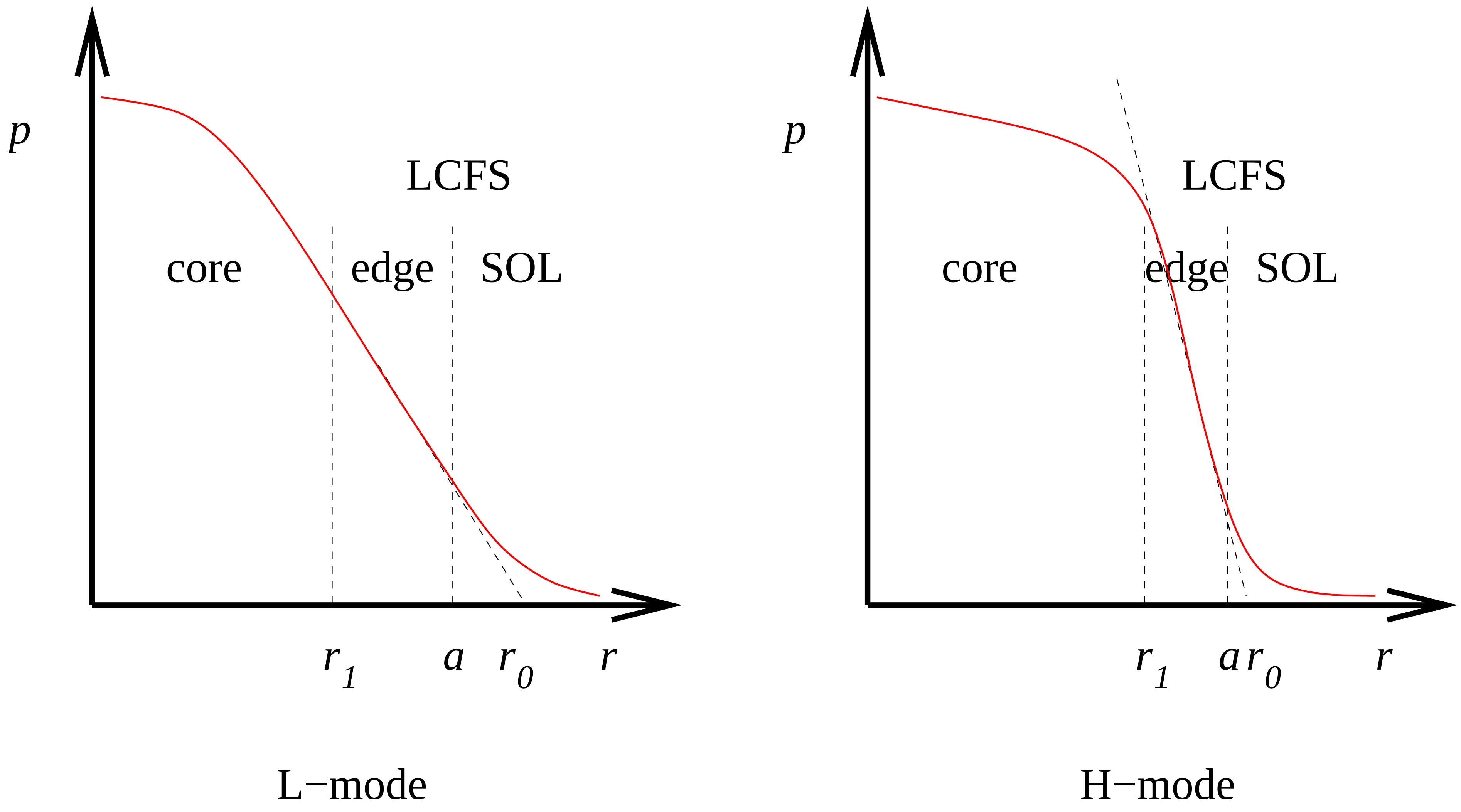
$$L_\perp = \sqrt{m_e/M_D}\, qR$$

- for linear profile gradients this is typically about 8 cm
  - and it holds over about the last 4 cm within the LCFS

if a pedestal exists, the top is the edge/core boundary

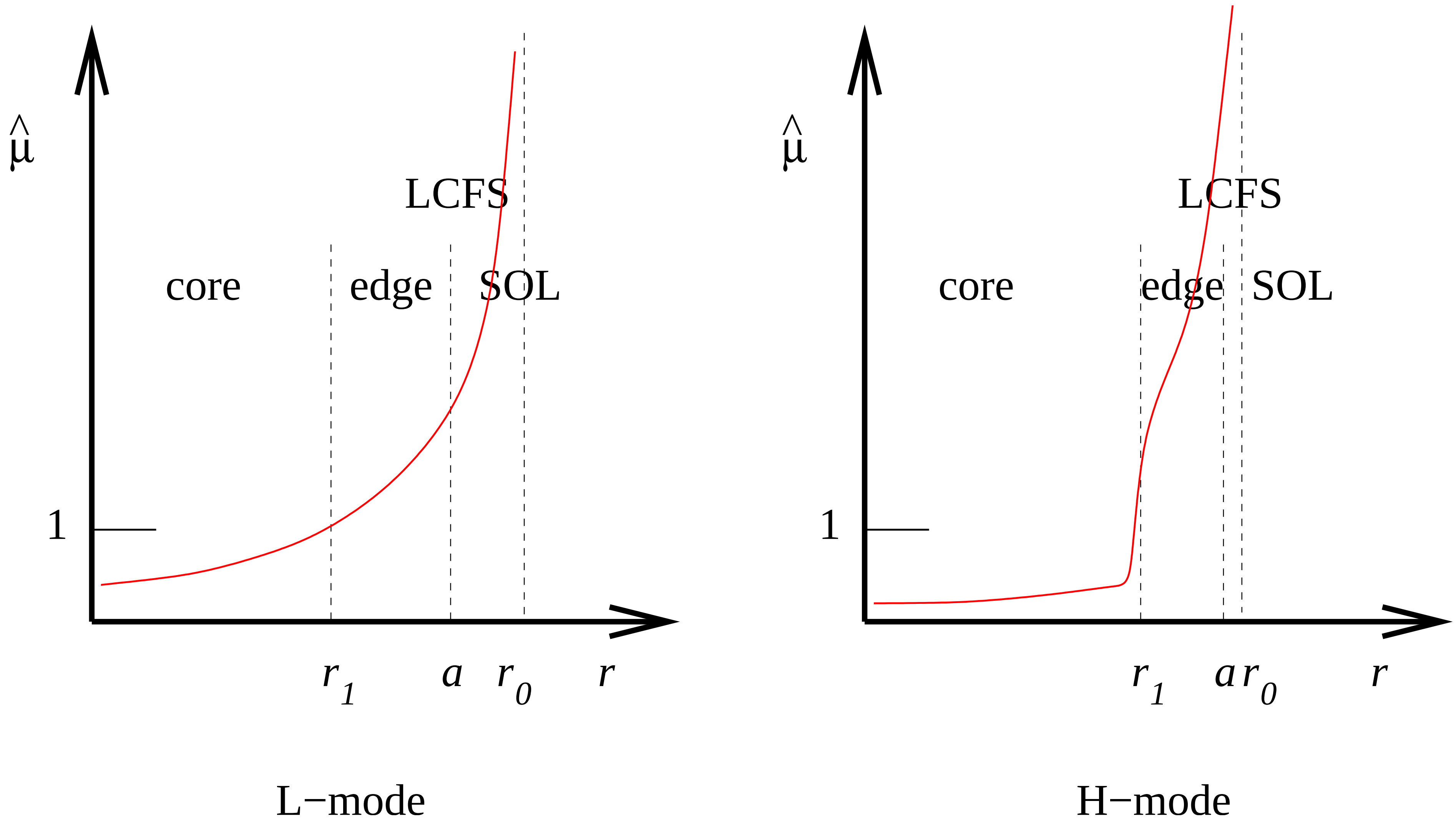
Edge Layer Extent

$$\hat{\mu} = 1 \text{ at } r = r_1$$

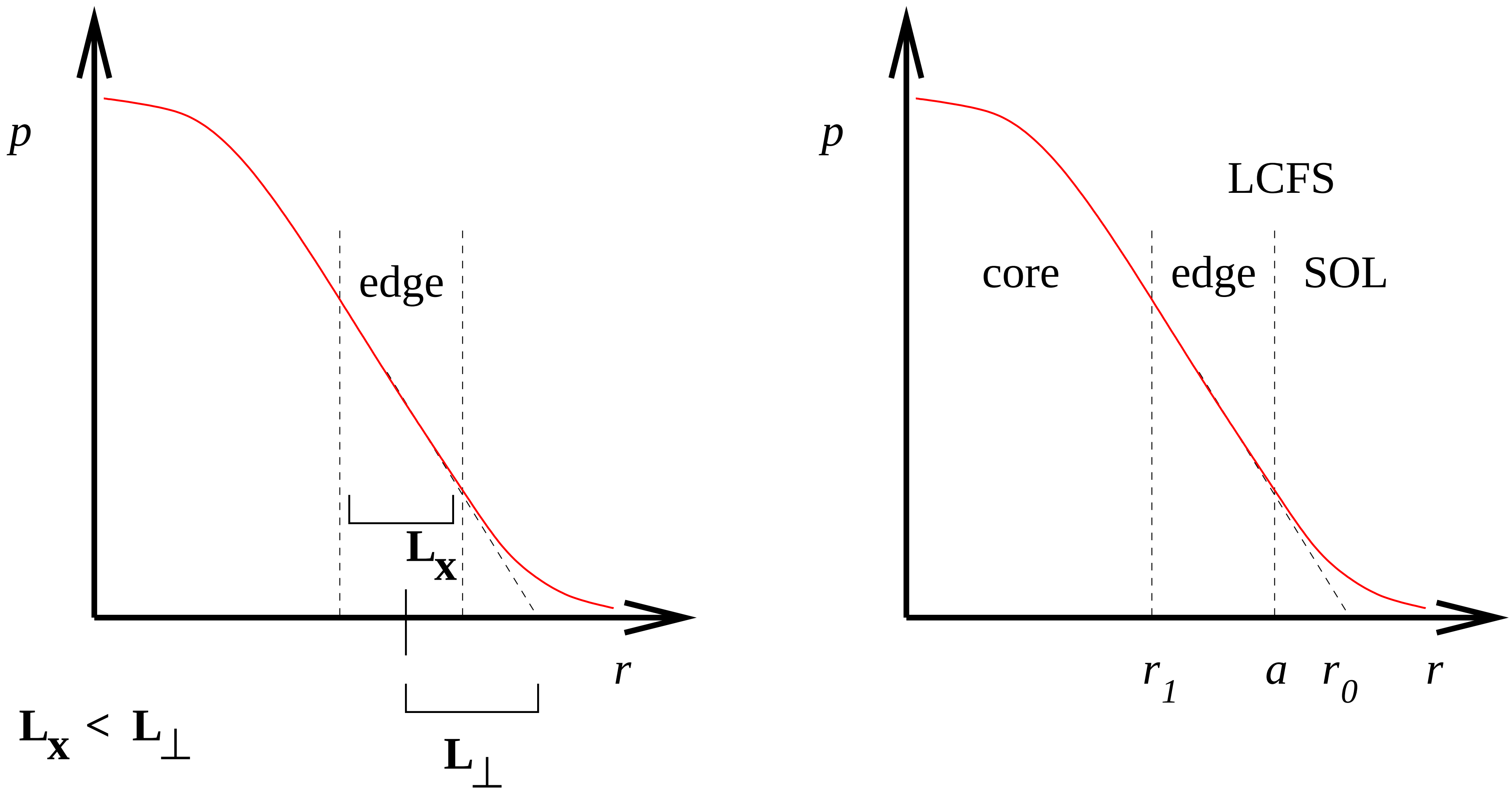


Edge Layer Extent

$\hat{\mu} = 1$ at $r = r_1$



Edge Turbulence Computation Arrangement



Meaning of Nonlinearity, Self Sustained Turbulence

- mode coupling is dominant
  - ∘ linear results depend on details of eigenmode structure
  - ∘ nonlinearity (statistical energy transfer) breaks this up
  - ∘ stable modes can collectively destabilise each other

- turbulence excites all eigenmodes, not just unstable ones
  - ∘ significant energy resident in damped Alfvén transients

- observe clear changes in mode structure linear → saturation → turbulence

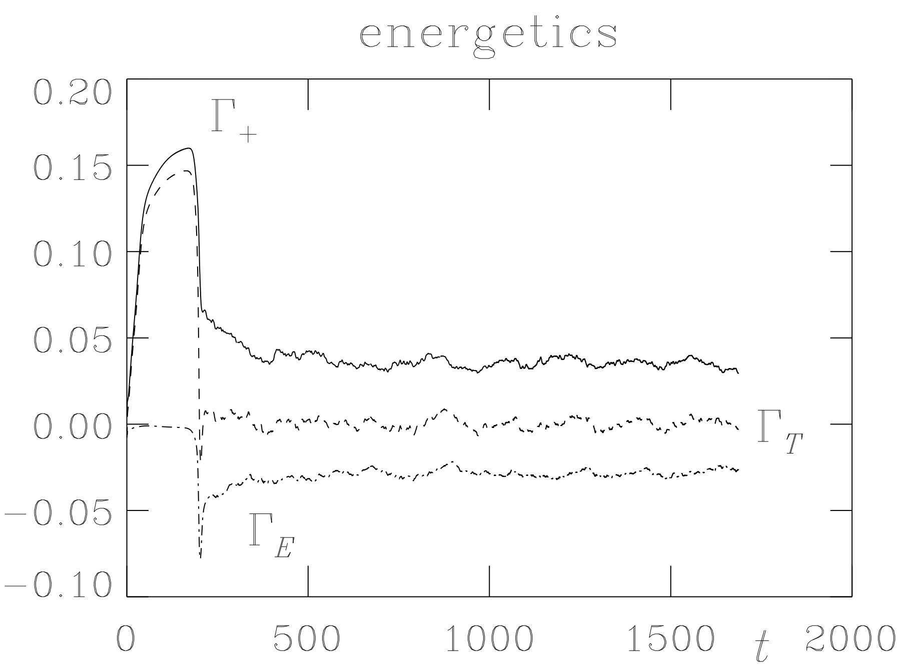
- why does it work? vorticity! $\omega_{\mathrm{turb}} > \gamma_L$

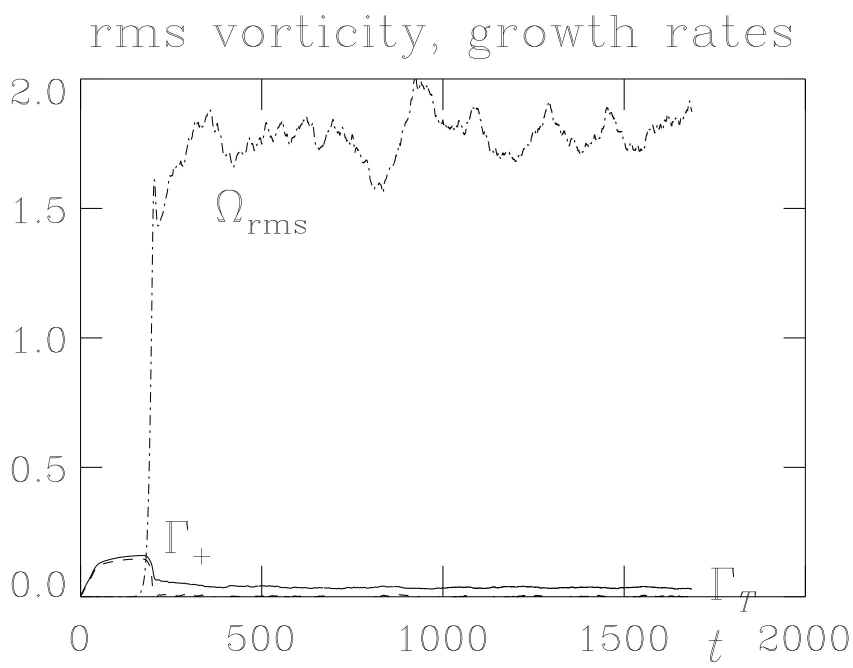
> native drift wave turbulence vorticity
> shears eigenmodes apart faster than they can form

(B Scott PPCF 12/2003, Phys Plasmas 6/2005)

Growth Rate and Vorticity in the Transition to Turbulence

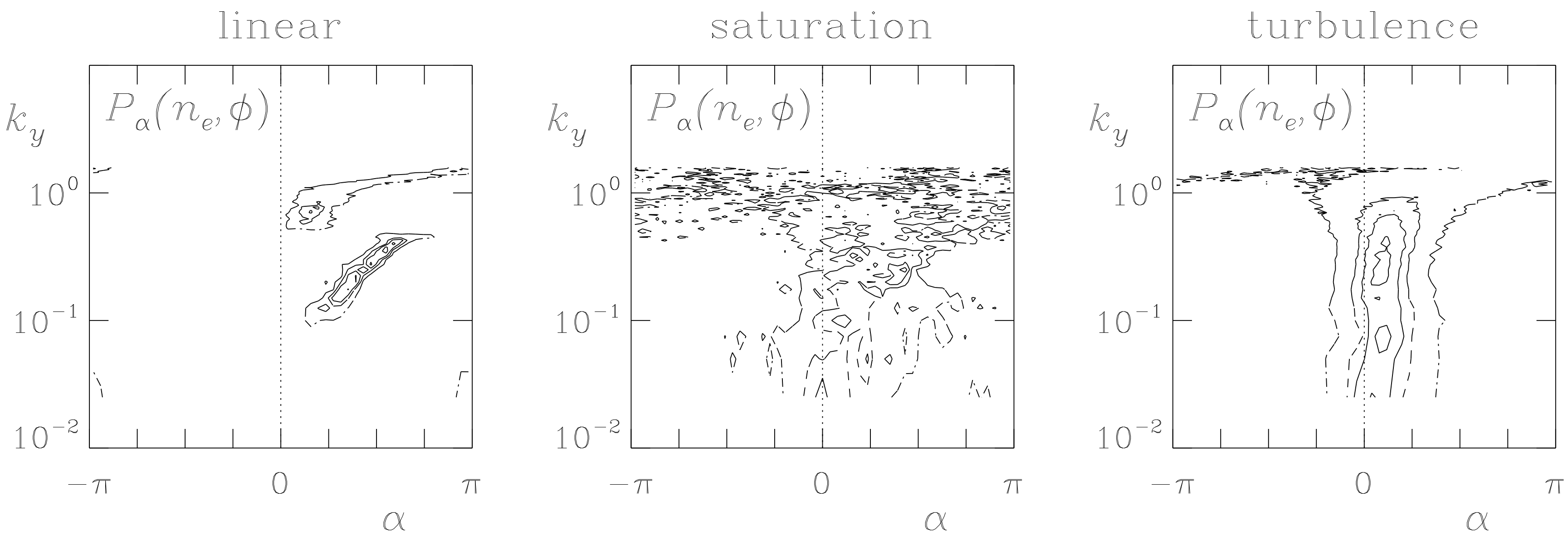
- linear start, eigenmode forms, growth rate maximizes
- saturation: coupled modes arise, growth rate crashes, turbulence vorticity takes over
- at late times, subgrid dissipation (transfer out of spectrum) balances gradient drive





Self Sustained Turbulence Erases Linear Instabilities

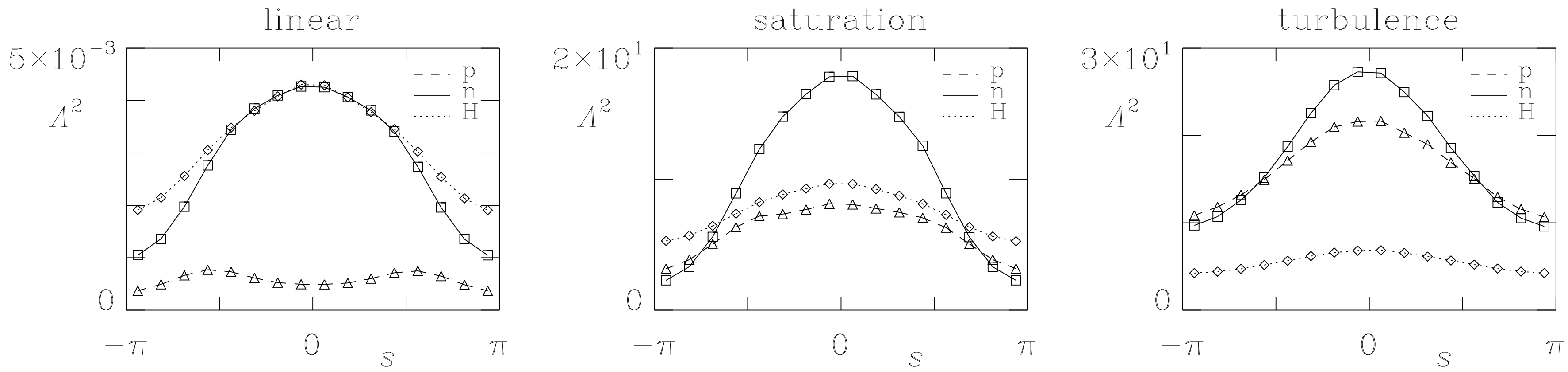
e.g., phase shift distributions for each wavelength



- for linear modes: part of the dispersion relation

- for turbulence: part of the statistical character
  - involves damped or stable, as well as driven, transients
  - this pattern is a clear signature of drift wave mode structure

Parallel Mode Structure Changes



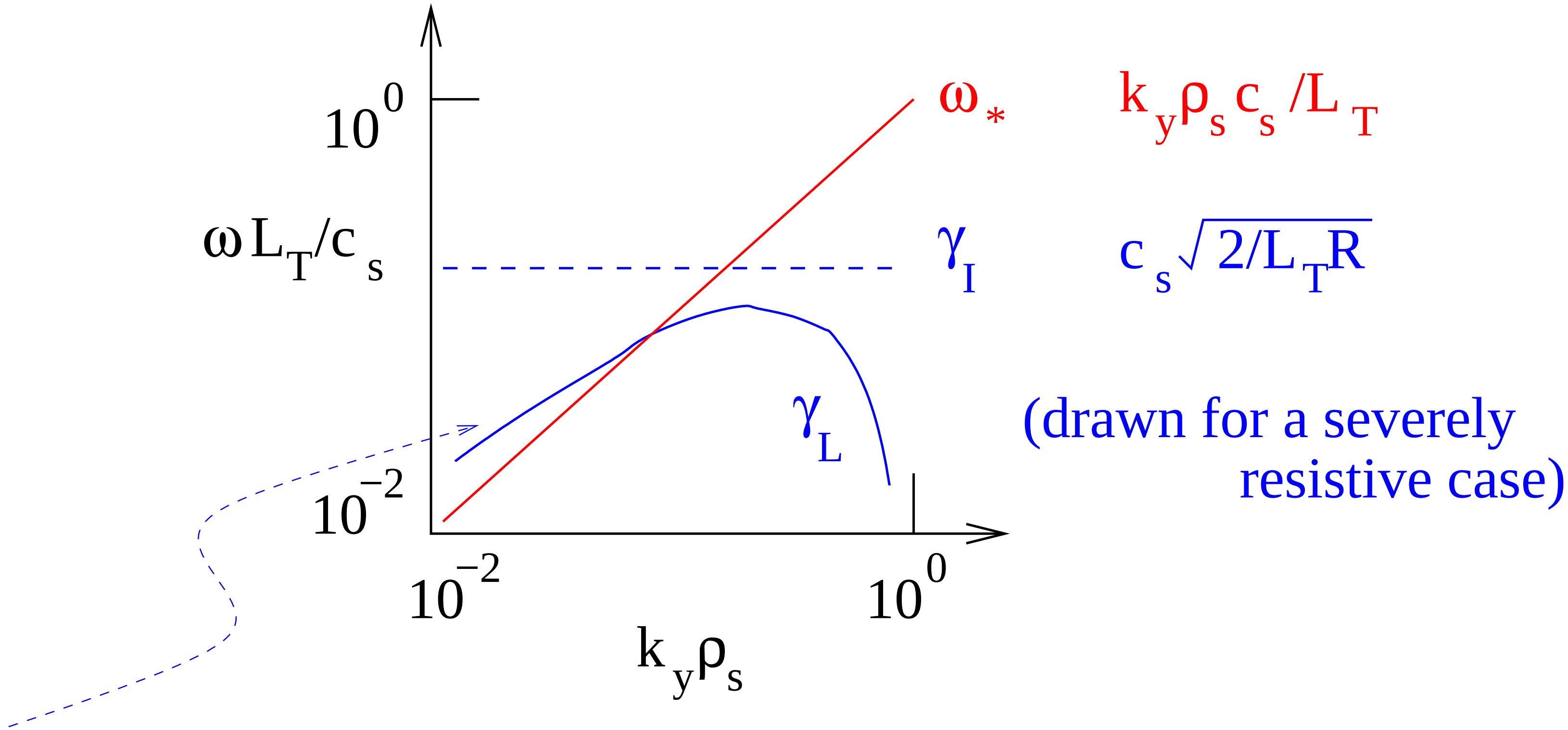
- for linear modes: eigenmode structure

- for turbulence: strong effect of Alfvén dynamics
  (electron pressure/electric field/currents)

$$\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \frac{m_e}{n_e e^2}\frac{dJ_\parallel}{dt} + \eta_\parallel J_\parallel = \frac{1}{n_e e}\nabla_\parallel p_e - \nabla_\parallel \phi$$

Relevance Range for Linear Instabilities

dispersion space bounded by ideal interchange and diamagnetic rates



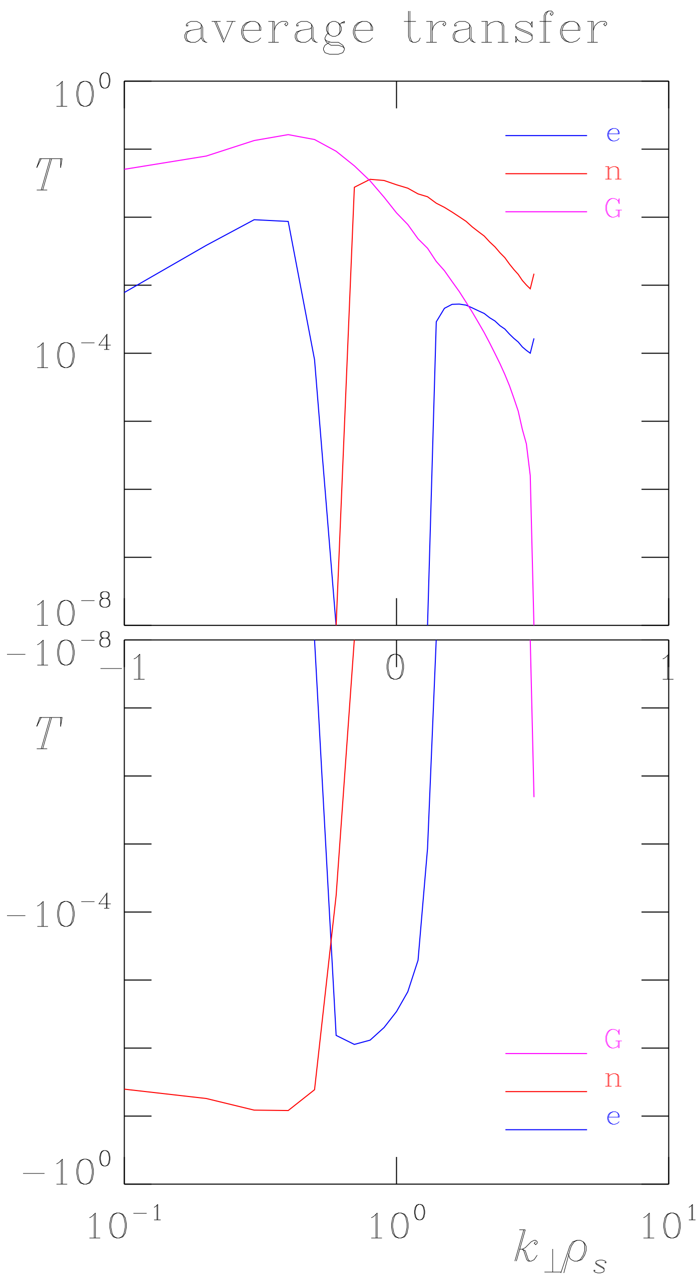
if the linear growth rate is above the red line then the instability is relevant

usually, this is not the case anywhere in the spectrum (unless: MHD threshold)

this situation is a direct consequence of very large $R/L_T >> 1$ in the edge

(B Scott New J Phys 2002, Phys Plasmas 2005)

Nonlinear Free Energy Cascade



direct cascade

--> nonlinear drive at small scales

==> passive scalar regime

frequency/scale correlation

matches with frequency break

evidence for onset of passive scalar regime

Outline of the Local GEM Model

- electromagnetic full-FLR gyrofluid equations for electrons, ions
  ◦ reference: B Scott, Phys Plasmas 12 (2005) 102307

- local fluxtube geometry, with global consistency, shifted metric form
  ◦ reference: B Scott, Phys Plasmas 8 (2001) 447-458

- run setup criteria in B Scott, Plasma Phys Contr Fusion 45 (2003) A385-A398
  ◦ initial state: random bath for $\widetilde{n}_e = \widetilde{n}_i$ others zero
  ◦ run to $c_s t/L_{Te} = 4000$
  ◦ statistics taken over saturated state $1000 < c_s t/L_{Te} < 4000$

- "PET05 Base Case" (B Scott, Contrib Plasma Phys 46:714 2006)

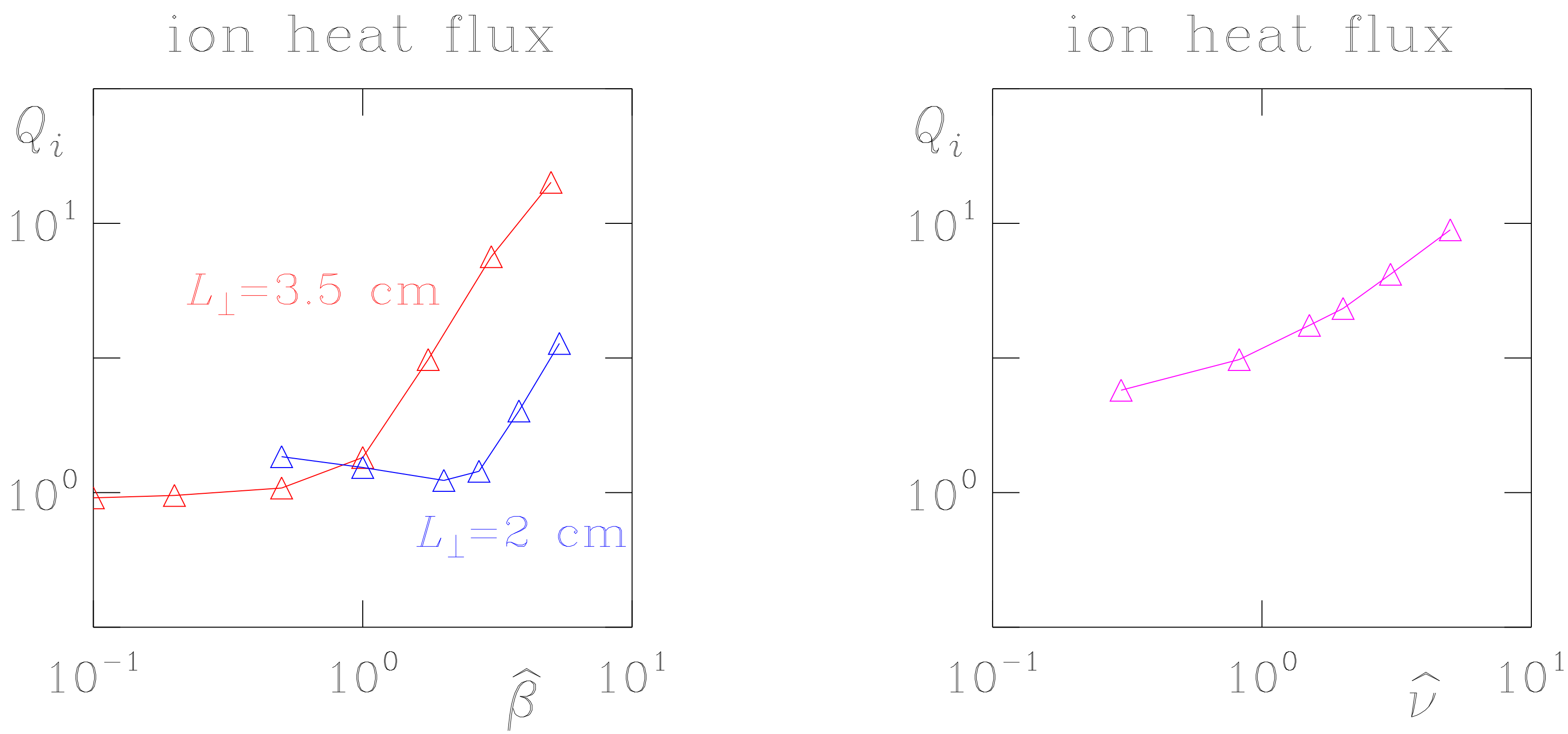
$$T_e = T_i = 100\,\mathrm{eV} \qquad n_e = n_i = 2.0 \times 10^{13}\,\mathrm{cm}^{-3} \qquad B = 2.5\,\mathrm{T}$$

$$R = 165\,\mathrm{cm} \qquad L_T = L_\perp = 3.5\,\mathrm{cm} \qquad L_n = 7.0\,\mathrm{cm} \qquad q = 3.5 \qquad \hat{s} = 1.14$$

- normalised parameters $\hat{\beta} = 1.75, \quad \hat{\mu} = 7.41, \quad C = 3.11, \quad \nu_B = 0.13$
  (B Scott Phys Plasmas 6/2005, PPCF 12/2003 and 12/2006)

Basic Transport Scaling (IAEA 06)

- total ion ExB thermal transport (electron is very similar)



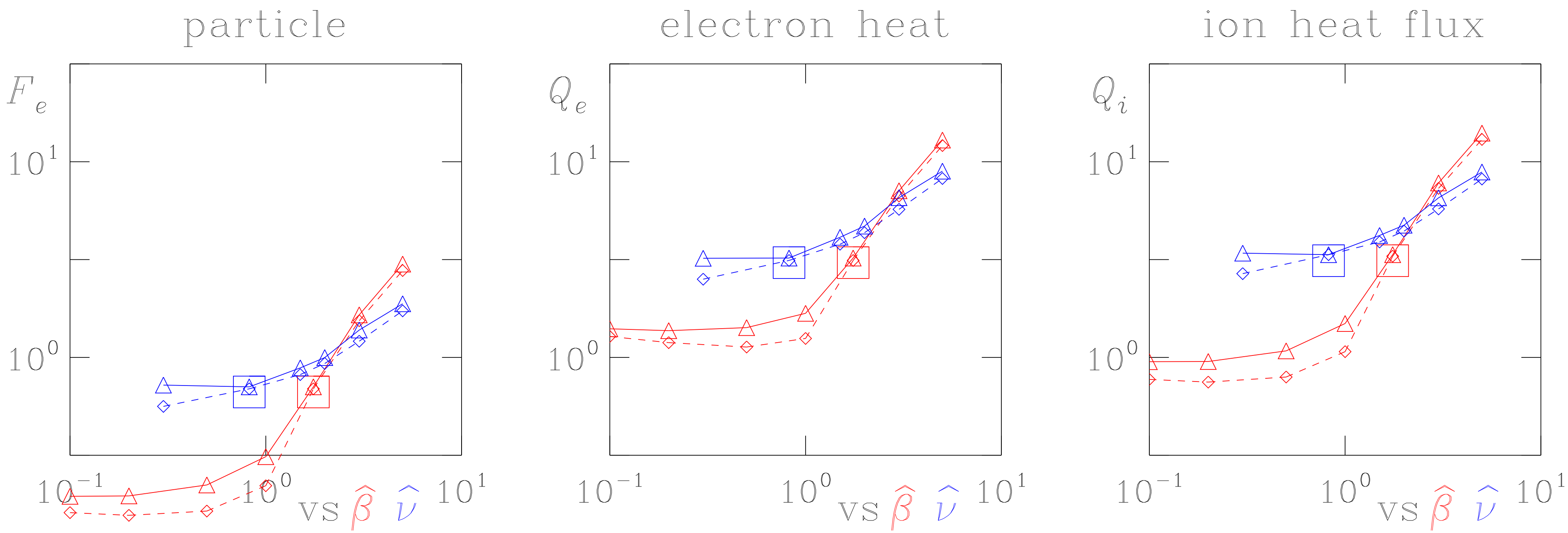
- mangetic flutter: small and negative — dynamic EM, transport ES
  ◦ rise at different $\hat\beta$ for different $L_\perp$ is the MHD α_M-effect

- delta-f gyrokinetic result is similar with differences (IAEA TM Instabilities, 2005)

- on top of this: rho-star scaling $D, \chi \sim \rho_s^2 c_s / L_\perp$ (nominal: $0.66\,\mathrm{m}^2/\mathrm{sec}$)

All Transport Channels

- particle and total e/i thermal transport (ambipolar to machine accuracy)
  - ◦ large squares give "PET05 Base Case"

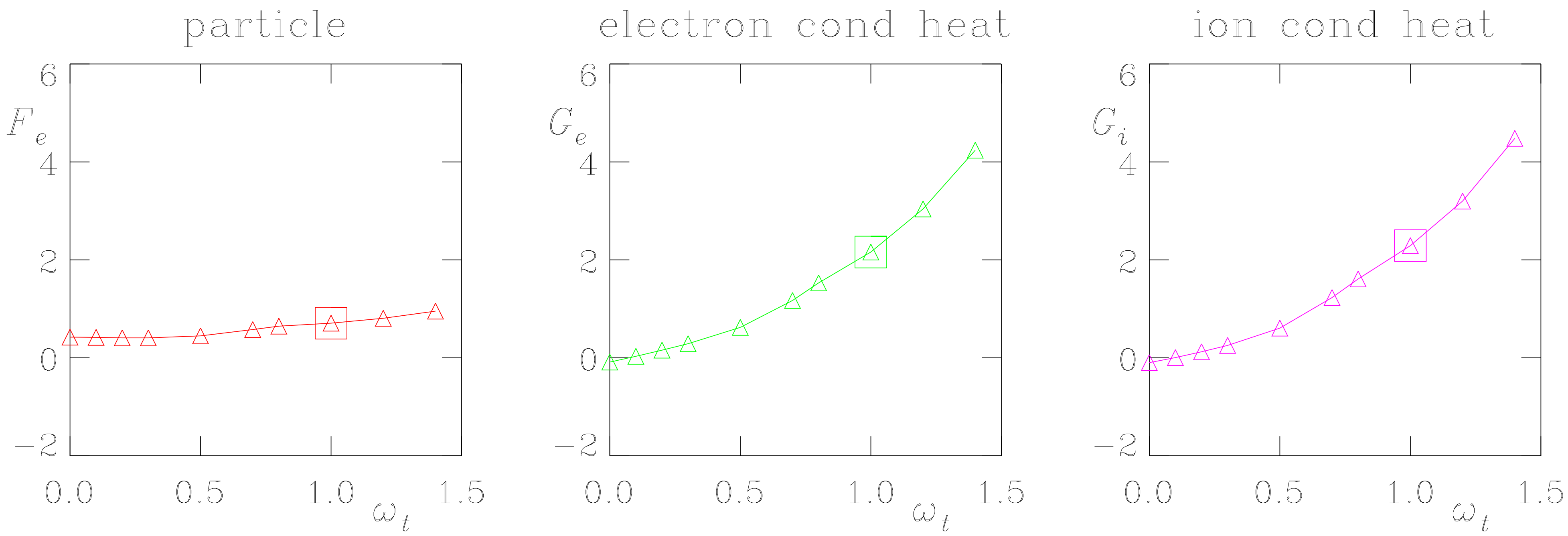


- dashed lines give doubled resolution (strongest effect at low-C low-$\hat{\beta}$)

- all transport channels give similar scaling (gradient synergy)
  - ◦ ratios for these ITG conditions are about $\chi_i \sim \chi_e \sim 1.5D$

Fix grad-n and Vary grad-T Scale Lengths

- particle and conductive e/i transport, with $\omega_t = L_\perp / L_T$ at $L_n = 2L_\perp$
  - large squares give "PET05 Base Case"

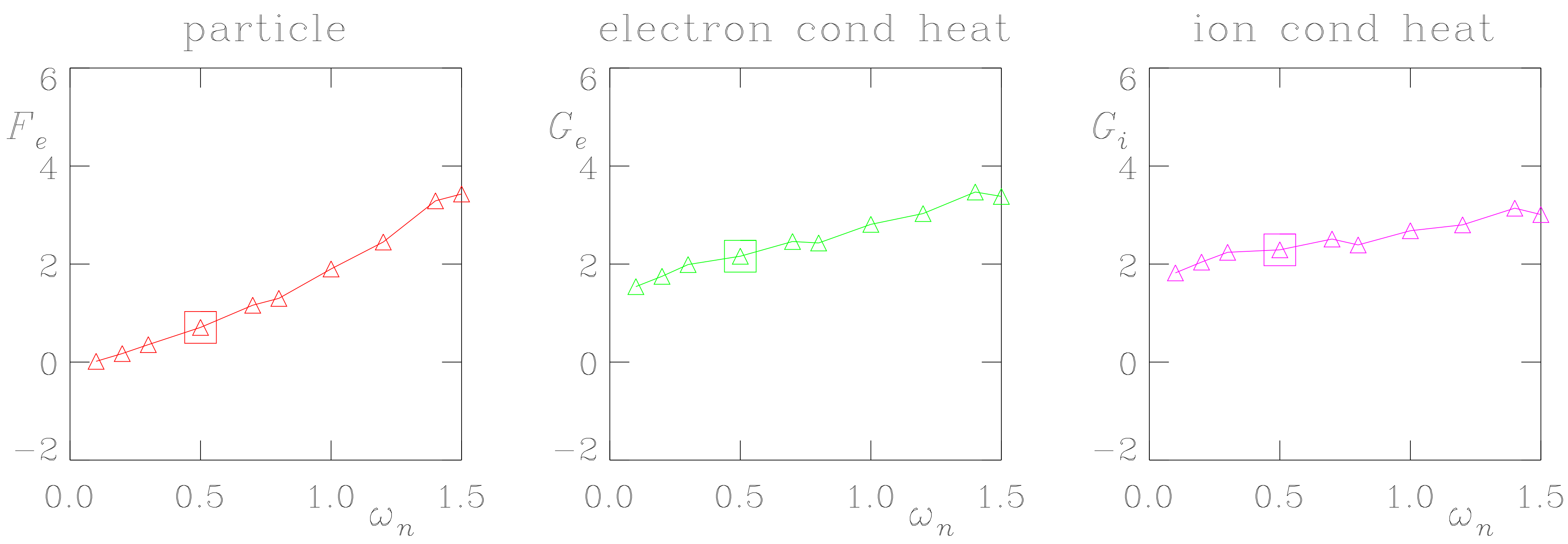


- obvious conduction effect, while particle flux follows total grad-p

$$\text{here,} \qquad L_\perp \nabla \log p = 0.5 + \omega_t$$

Fix grad-T and Vary grad-n Scale Lengths

- particle and conductive e/i transport, with $\omega_n = L_\perp/L_n$ at $L_T = L_\perp$
  - large squares give "PET05 Base Case"

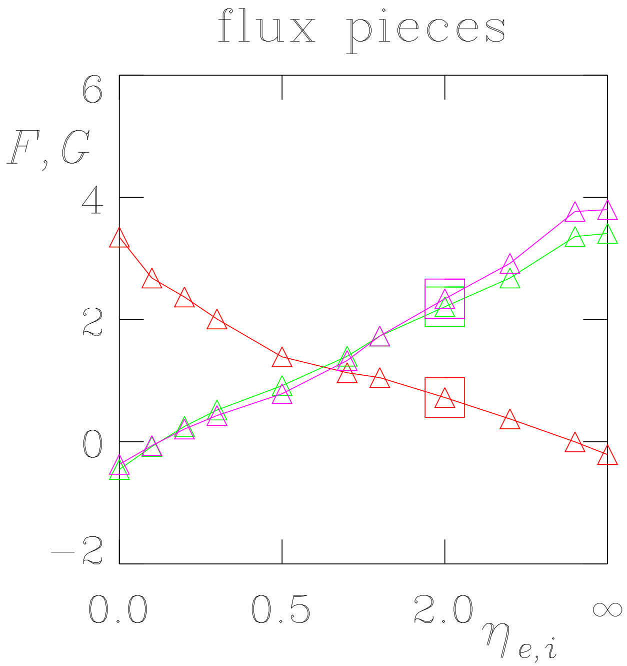


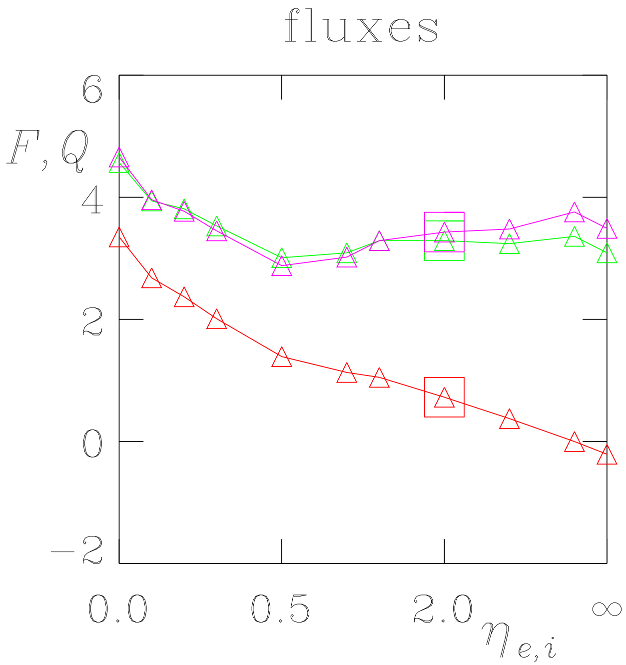
- conductive fluxes follow grad-p while particle convection follows grad-n

$$\text{here,} \qquad L_\perp \nabla \log p = 1 + \omega_n$$

Now Fix grad-p and Vary Both Scale Lengths

- (left) particle and conductive e/i transport (right) particle and total e/i transport, with $\eta_{\{i,e\}} = L_T/L_n$ and $\omega_n + \omega_t = 1.5$
  - large squares give “PET05 Base Case”





- very weak pinch effects

- flux pieces follow resp. gradients, total follows mostly grad-p

Diamagnetic Stabilisation

- a result from simple ballooning analysis (Hahm and Diamond 1986)
  - talked up in a big way by Drake and Rogers (late 1990s)
  - still is a central part of Guzdar's H-mode model
- is an effect of ∇p_i in the vorticity (ω_{*i})
  - adds to ballooning dispersion relation
  - works on high-$k_\perp$ side where resistive instability is effective
- gyrofluid correspondence: FLR nonlinearity $\leftrightarrow$ polarisation (B Scott Phys Plasmas 10/2007)
- next pages: Braginskii correspondence example, control test

FLR and Polarisation — Gyrofluid versus Braginskii

- gyrofluid FLR nonlinearities reduce to Braginskii "gyroviscosity" tensor/polarisation
  - ◦ isothermal version shown

$$\frac{\partial n_i}{\partial t} + [\phi_G, n_i] + \nabla_\parallel u_\parallel = \text{curvature terms}$$

with low-$k_\perp$ limits

$$\phi_G = \left(1 + \frac{\rho_i^2}{2}\nabla_\perp^2\right)\phi \qquad\qquad n_i = \left(1 - \frac{\rho_i^2}{2}\nabla_\perp^2\right) n - n\frac{e}{T_e}\rho_s^2\nabla_\perp^2\phi$$

becomes the density equation with polarisation divergence

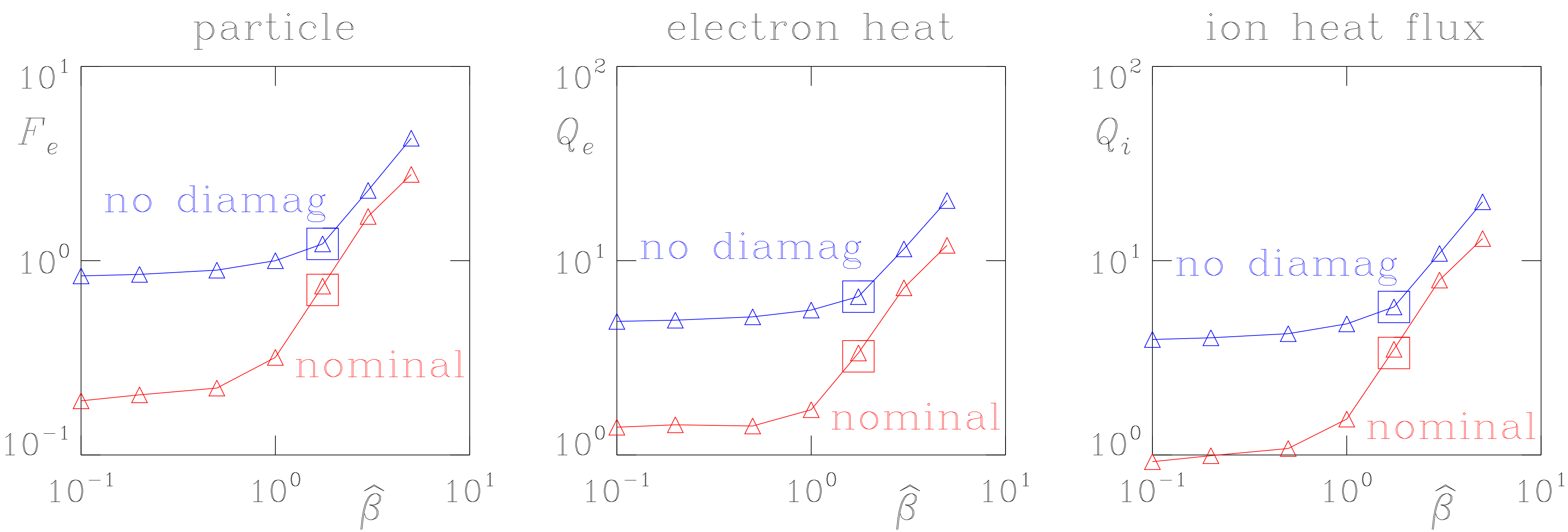
$$\frac{\partial n}{\partial t} + [\phi, n] - \nabla\cdot\left(\frac{\partial}{\partial t} + [\phi, \ ]\right) n\frac{e}{T_e}\rho_s^2\left(\nabla_\perp\phi + \frac{\nabla_\perp p_i}{ne}\right) + n\nabla_\parallel u_\parallel = \text{curvature terms}$$

and hence the vorticity equation

$$\nabla\cdot\left(\frac{\partial}{\partial t} + [\phi, \ ]\right) n\frac{e^2}{T_e}\rho_s^2\left(\nabla_\perp\phi + \frac{\nabla_\perp p_i}{ne}\right) = \nabla_\parallel J_\parallel + \text{curvature terms}$$

Diamagnetic Polarisation Effect — Transport Scaling versus Beta

standard L-mode cases with ITG gradients: $L_T = 0.5\, L_n$



- new GEM results with Arakawa/Karniadakis scheme, full resolution, long runs
- rise of all ExB transport channels with experimentally relevant $\hat{\beta}$ is found
- diamagnetic effects stabilise, more so at low-β
  - but do not affect transport trend

Externally Imposed ExB Shear

- simplest model: velocity shear V' is constant within the domain
- gyro-Bohm normalisation

$$V' = \frac{c}{B}\nabla_\perp^2\phi = \frac{cT_e}{eB}\frac{e}{T_e}\nabla_\perp^2\phi = \frac{c_s}{L_T}\rho_s^2\nabla_\perp^2\left(\frac{e}{T_e}\frac{L_T}{\rho_s}\phi\right) \rightarrow \nabla_\perp^2\phi$$

- adds an extra profile piece to be added in nonlinear advection

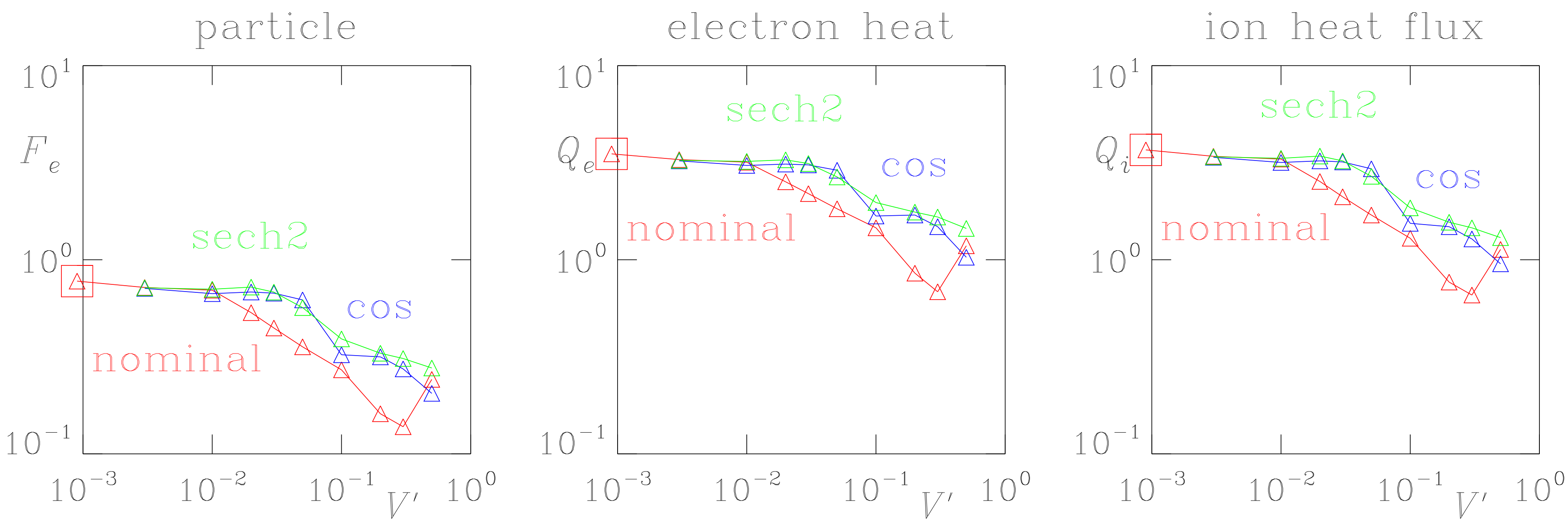
$$[\phi_G, \,] \rightarrow [\phi_G + \phi_0, \,] \qquad \text{with} \qquad \phi_0 = \frac{V'}{2}x^2$$

- also tried other models (motivated by CS Chang)

$$V' \propto \cos(2\pi x/L_x) \qquad\qquad V' \propto \mathrm{sech}^2(8x/L_x)$$

Sensitivity to Externally Imposed ExB Shear

standard L-mode cases with ITG gradients: $L_T = 0.5\, L_n$



- squares give value at zero shear
  - red/blue/green lines give constant/cos/sech2 profiles for applied vorticity
- rolloff is slow, shallower than $Q \sim (V')^{-1}$
- max suppression is only about a factor of 4

ExB Shear and the L-to-H Transition

- Chang's original model:

$$\chi = \frac{\chi_0}{1 + c_0 V'^2}$$

- GEM result is much shallower:

$$\chi = \frac{\chi_0}{1 + c_1 V'^{0.7}}$$

in XGC0 Chang found that this change eliminates
this simple ExB shear induced L-to-H transition scenario

- furthermore, the suppression itself is weak — return to this after the grad-T scalings (cf. also B Scott PPCF 12/92 and Phys Plasmas 5/00)

Scalings with Physical Parameters

- gyro-Bohm results are useful for physics

> but any experimental comparisons should be done in physical units

- minimal reason: normalisation contains factor $T^{5/2}$ for heat flux

$$Q_{GB} = n_e T_e c_s \times \frac{\rho_s^2}{L_\perp^2}$$

- one other interesting experimental observation:
  - gradient steepening is often self similar
  - so a pure grad-T scaling is best done through T, not L_T
  - and this involves all the normalisation considerations ($\hat{\beta}, C$ parameters)

grad-T Scalings

- assume profile "pivots" around its "zero point" ($r = r_0$)

$$T_e = T_0 \frac{r_0 - r}{L_\perp} \qquad \text{with} \quad T_0 \quad \text{varying, and} \quad L_\perp \quad \text{constant}$$

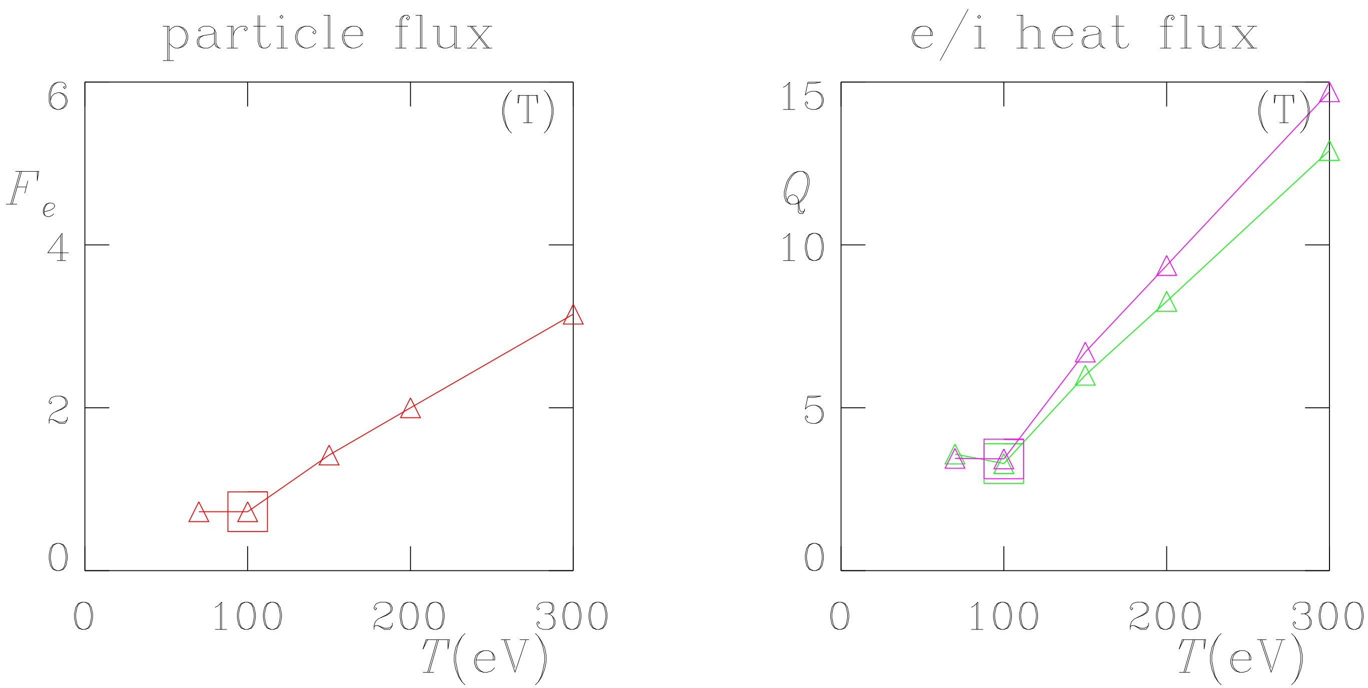
- hence all parameters except T_0 are fixed
- scale lengths remain fixed, while main parameters vary as

$$\hat{\beta} = 1.75 \times (T_0/100\,\mathrm{eV}) \qquad C = 3.11 \times (T_0/100\,\mathrm{eV})^{-2}$$

- results shown in both normalised and physical units
  - ○ for latter assume PET05 Base Case with $a = b = 50\,\mathrm{cm}$

Basic grad-T Scaling

- particle and total e/i heat transport
  - large squares give “PET05 Base Case”

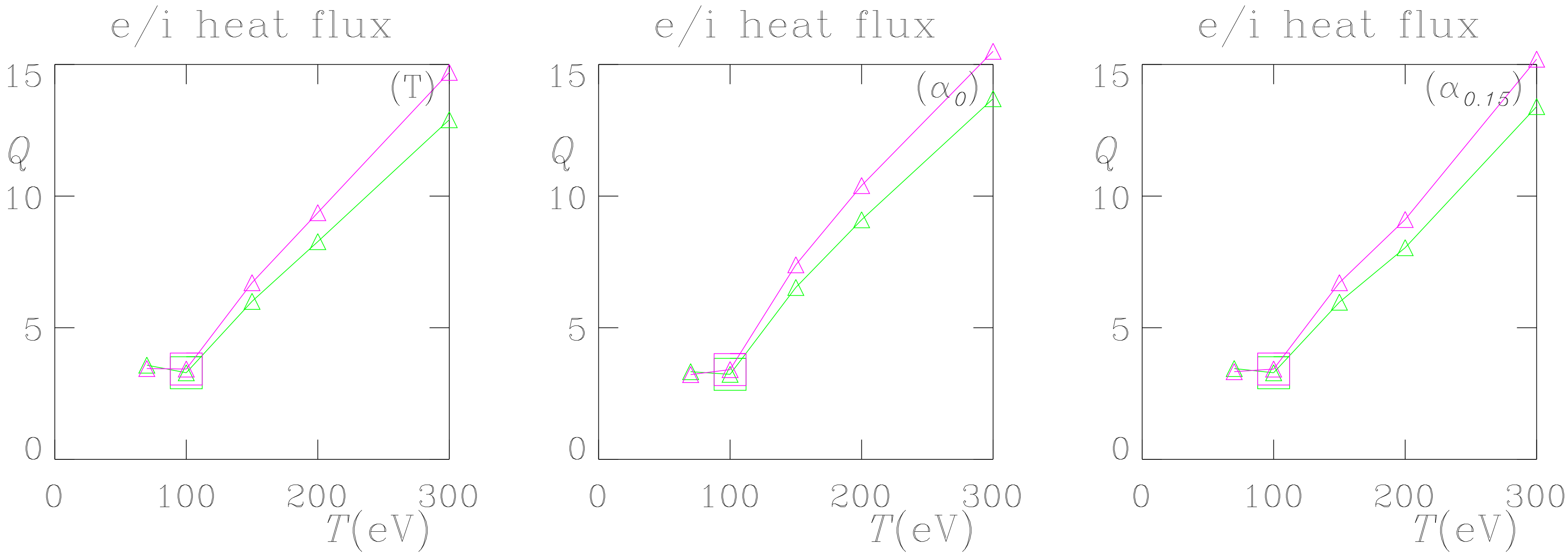


- trend is up, beta effect overcomes collisionality

- this is in gyro-Bohm units, physical result is steeper still

Shafranov Shift Effect

- total e/i heat transport, large squares give "PET05 Base Case"
  (left) nominal (center) with no shift (right) with constant $\alpha = 0.15$



- within S-α model, very little effect
  - shaping results shown later

Externally Imposed ExB Shear in grad-T Scalings

- simple equilibrium models yield $V' \propto |\nabla T_e|$
  - also the result of study by Heikkinen et al (Phys Rev Lett 2000)

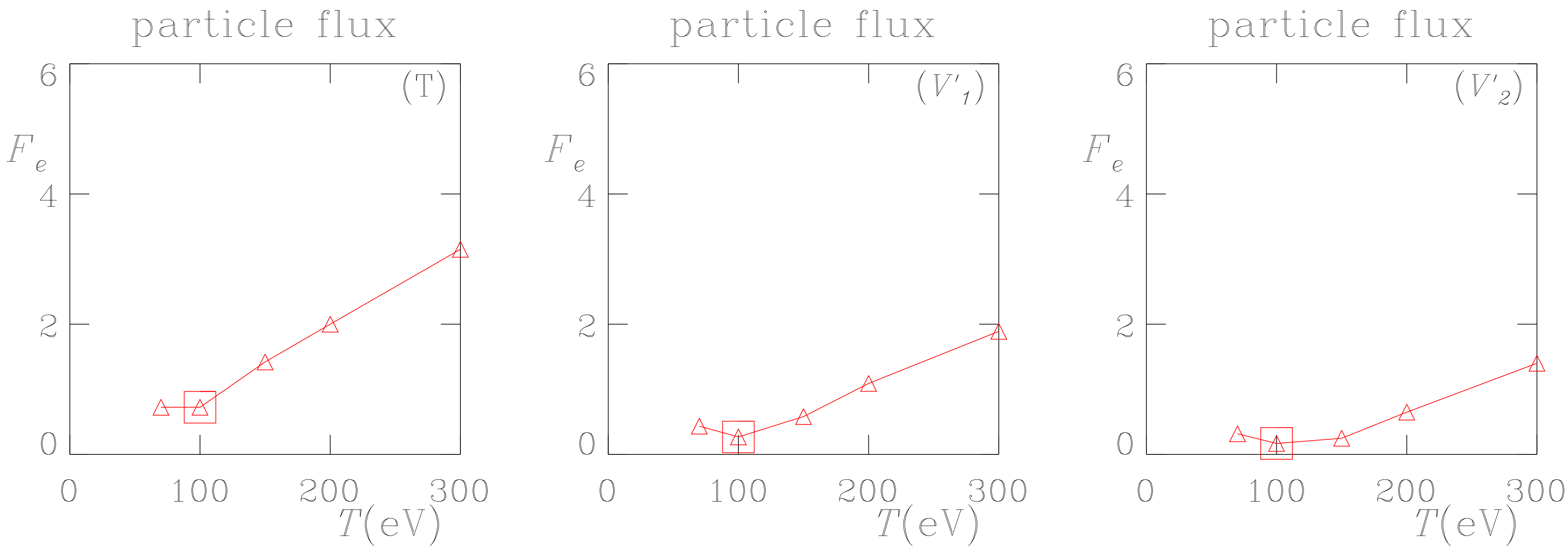
- gyro-Bohm normalisation gives $V' \propto \rho_s/L_T$
  - from "diamagnetic level flow shear" or flow at level v_D on scale L_T

$$\frac{v_D}{L_T} = \frac{\rho_s}{L_T} \times \frac{c_s}{L_T}$$

- try two models: $V' = 0.1$ or $0.2 \times (T/100\,\mathrm{eV})^{1/2}$
  - this is about 6 or 12 times ρ_s/L_T

ExB Shear Effects

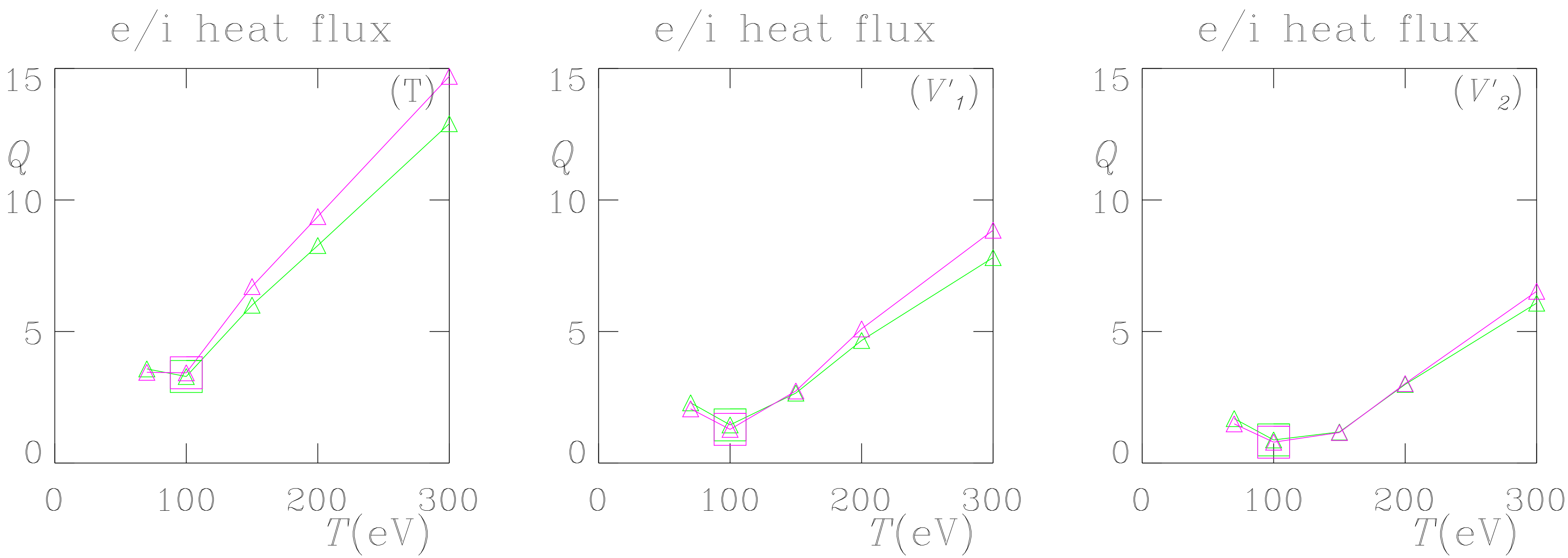
- particle transport, assuming $V' = 0$ or 0.1 or 0.2 times $(T_0/100\,\mathrm{eV})^{1/2}$
  - large squares give "PET05 Base Case"



- only moderately stabilising, does not affect trend
  - more on the $70 < T_{\mathrm{eV}} < 100$ range later (this is still gyro-Bohm units)

ExB Shear Effects

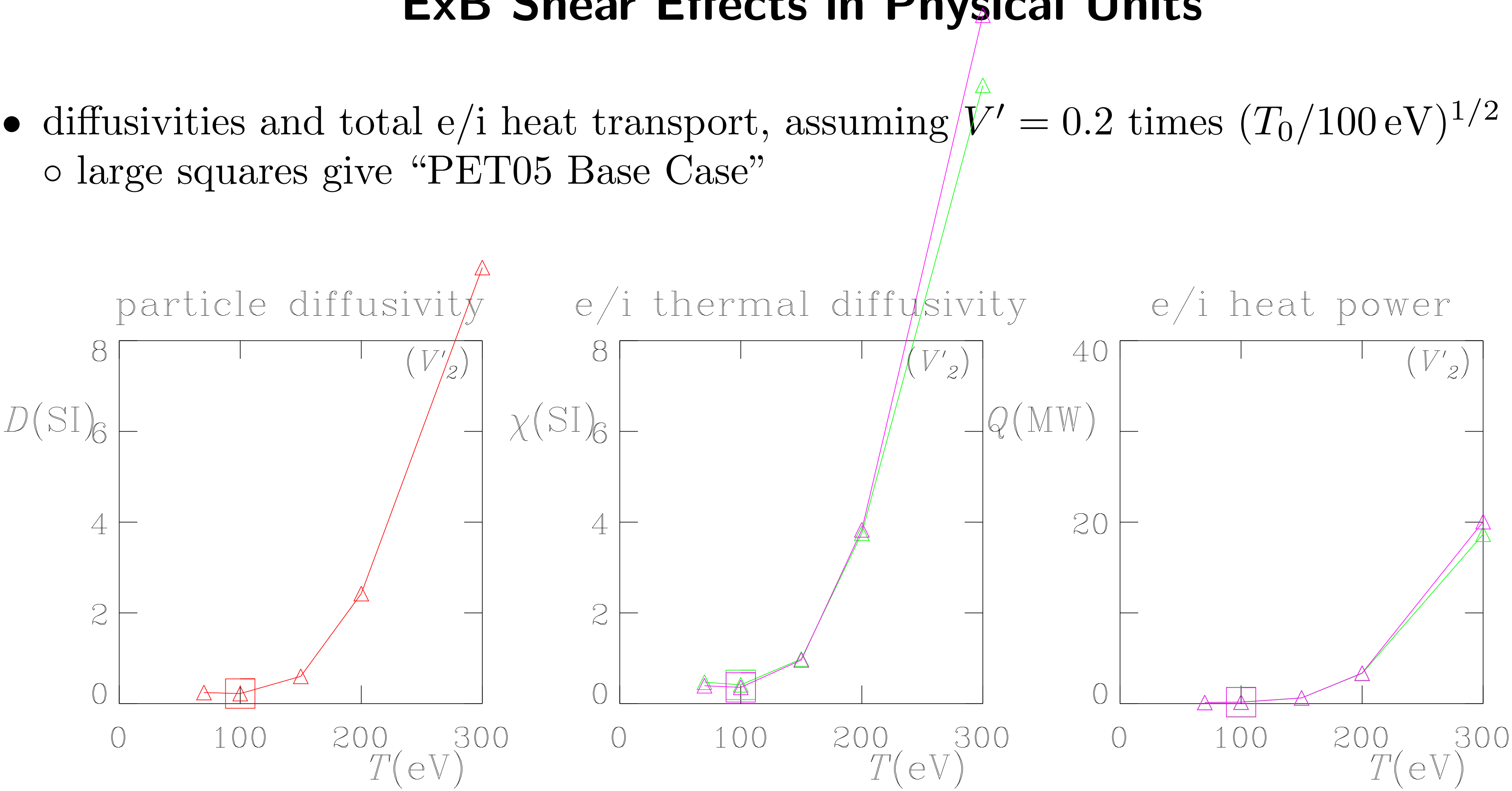
- total e/i heat transport, assuming $V' = 0$ or 0.1 or 0.2 times $(T_0/100\,\mathrm{eV})^{1/2}$
  - large squares give "PET05 Base Case"



- only moderately stabilising, does not affect trend
  - more on the $70 < T_{\mathrm{eV}} < 100$ range later (this is still gyro-Bohm units)

ExB Shear Effects in Physical Units

- diffusivities and total e/i heat transport, assuming $V' = 0.2$ times $(T_0/100\,\mathrm{eV})^{1/2}$
  - large squares give “PET05 Base Case”



- note in even the $70 < T_{\mathrm{eV}} < 100$ range
  - the factors of T_0 in the physical units overcome the normalisation
  - the trend is monotonically rising

The Results with Shaping Effects

- use HELENA code in standard kappa/delta model
  - details in B Scott, Phys Plasmas 5/00

- same model for ExB flow shear, with $V' = 4v_D/L_T$

- shaping gives more realistic trend
  - stronger at low beta, weaker MHD effects hence weaker at high beta

- parameters (beta rises by 4)

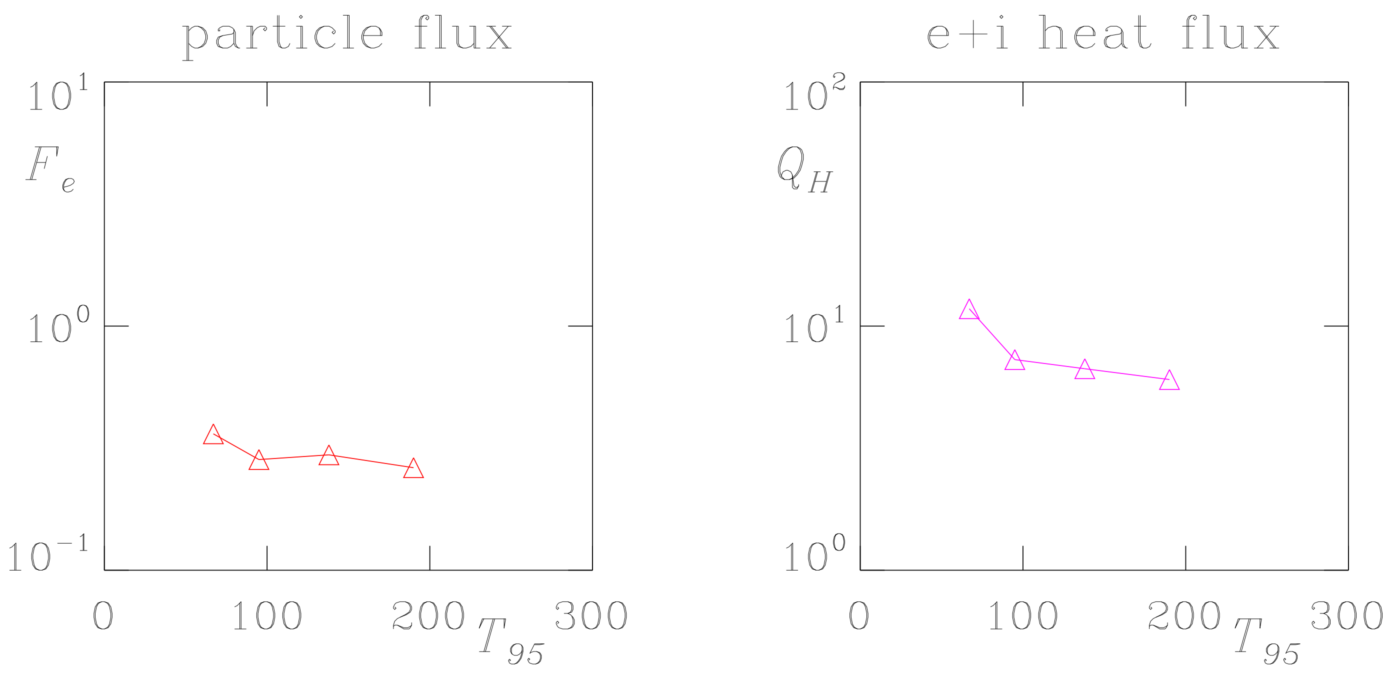
$$n_{95} = 3.9, \quad 4.3, \quad 4.9, \quad 5.4 \times 10^{13}\,\mathrm{cm}^{-3}$$

$$T_{95} = 74, \quad 105, \quad 152, \quad 210\,\mathrm{eV}$$

- the result was essentially the same (i.e., the original):
  - falling trend in normalised units, rising trend in physical units

HELENA/AUG grad-T Scaling with ExB Shear

- particle and total e+i heat transport (log scale)
  - HELENA model with $\kappa/\delta = 1.6/0.2$ with AUG dimensions

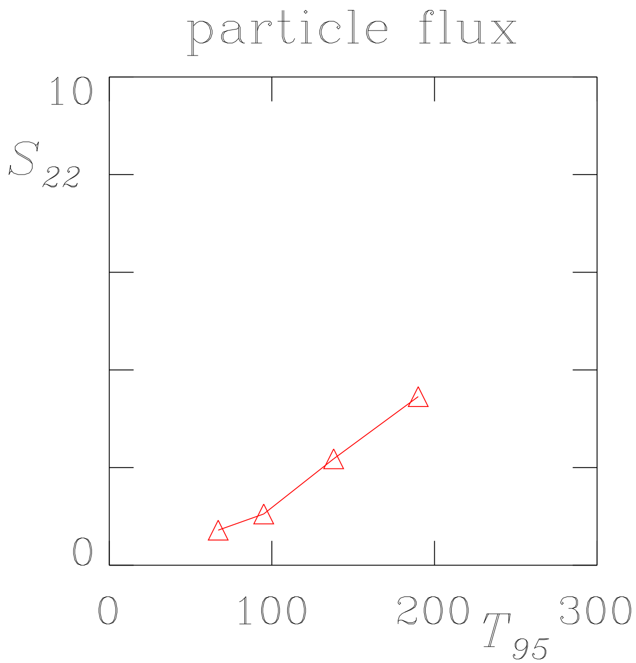


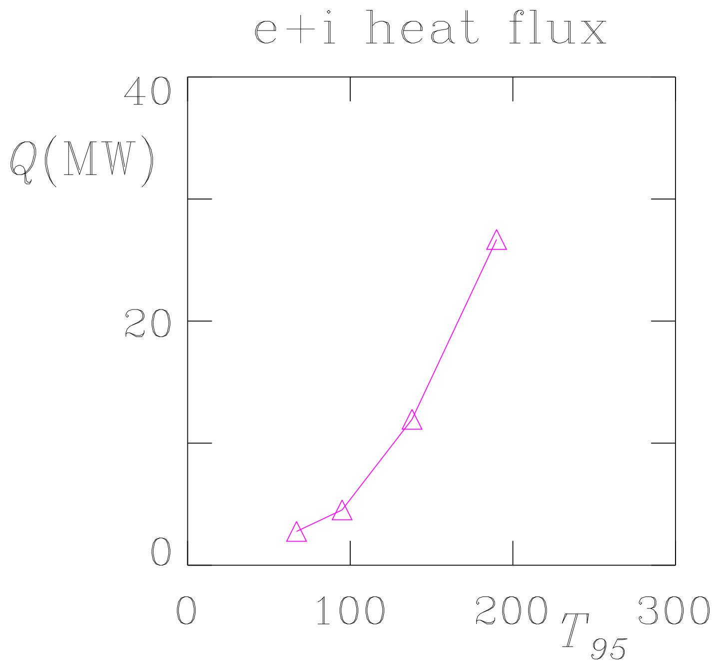
- trend in gyro-Bohm units is down, raising false hopes

- note the concavity is wrong anyway

HELENA/AUG grad-T Scaling with ExB Shear, Physical Units

- particle and total e+i heat transport (linear scale)
  - HELENA model with $\kappa/\delta = 1.6/0.2$ with AUG dimensions





- normalised fluxes multiply by $nT^{3/2}$ and $nT^{5/2}$, respectively

- note this effect of units when assessing theory papers!

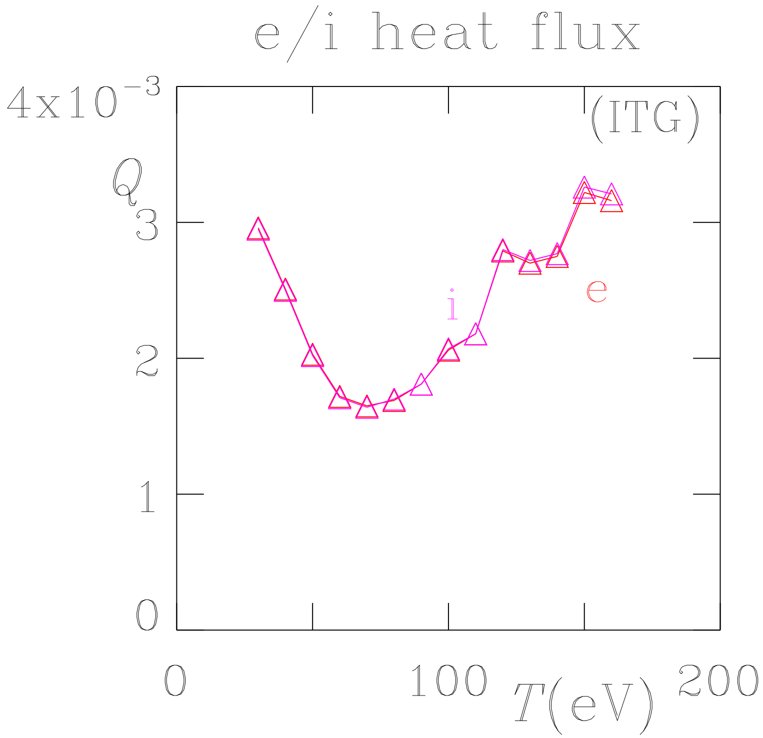
Comparison to the RITM Transport Model

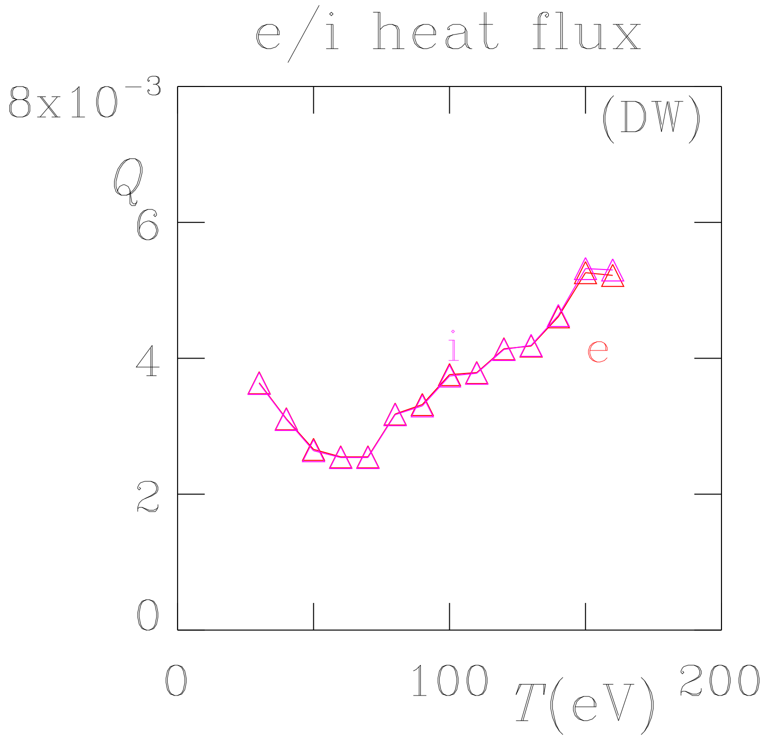
- from Dennis Kalupin: curve for heat flux versus ∇T (else fixed)
- three run series: DW and ITG and FTI
  ◦ drift wave (DW): $L_{Ti} = L_\perp$ and $L_n = L_\perp$ and $T_i = T_e$
  ◦ ITG mode (ITG): $L_{Ti} = L_\perp$ and $L_n = 2L_\perp$ and $T_i = T_e$
  ◦ edge ion temp gradient (FTI): $L_{Ti} = L_n = 2L_\perp$ and $T_i = 1.2\,T_e$
- normalisation inside code: Bohm with $\delta = \rho_s/L_\perp$ as a parameter
  ◦ perp lengths normed to $L_\perp$, time to $L_\perp/c_s$
- normalised flux results: $Q_{e,i}$ in terms of $p_e c_s$
- physical units: flux in MW through reference surface with $a = 1\,\mathrm{m}$ and $R = 3\,\mathrm{m}$
- most important result:

no L-to-H transition in fully developed turbulence in local models

Transport T-Scaling: GEM on JET Cases

ITG and DW gradients: $L_n = L_T \times 2$ or 1

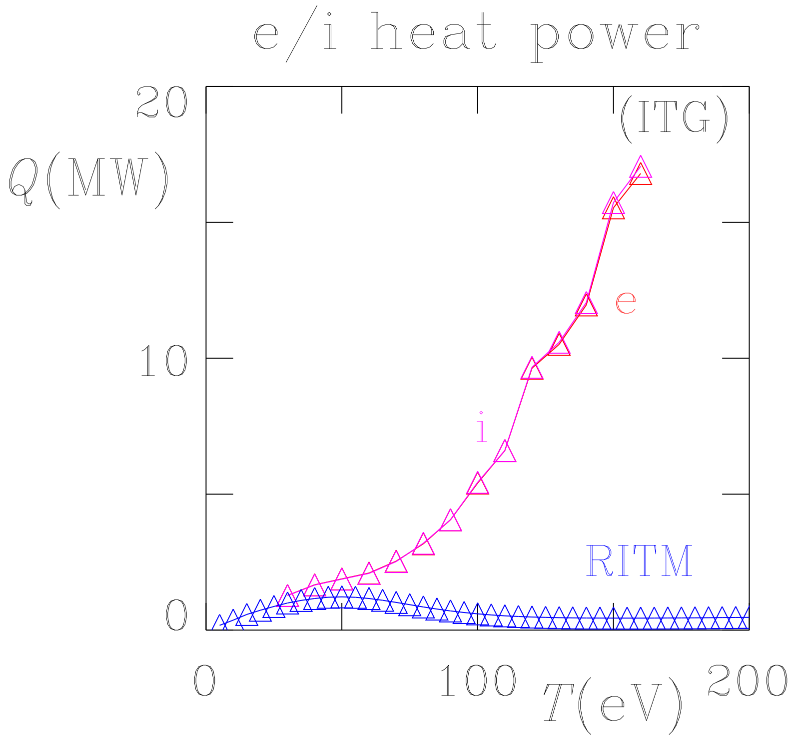


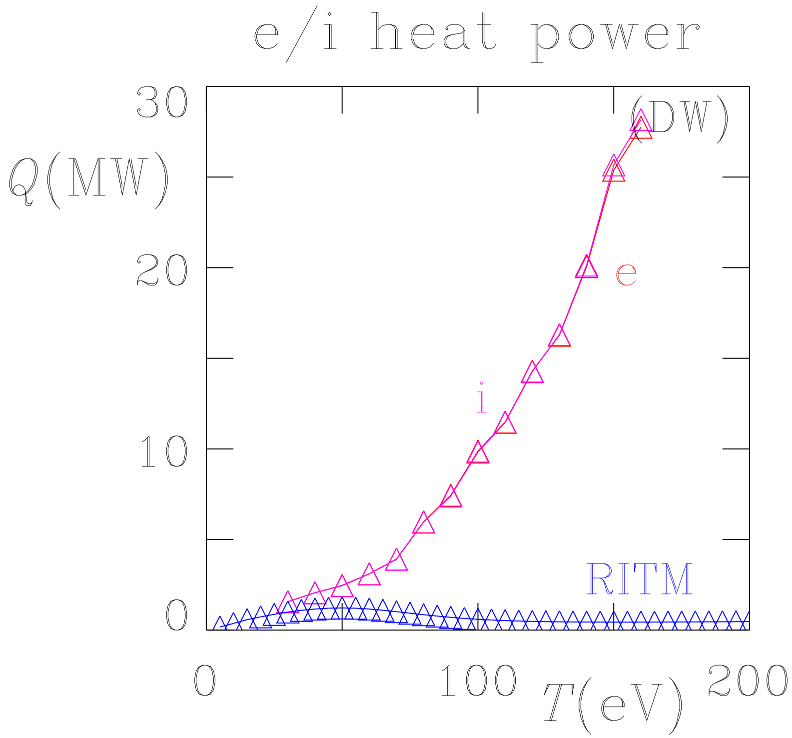


- fluxes in normalised units — n_e or $p_e \times c_s$
- species channels very similar, synergistic drive by gradients
- beta, collisionality, rho_star dependences all enter, roughly cancel

Transport T-Scaling: GEM vs RITM on JET Cases

ITG and DW gradients: $L_n = L_T \times 2$ or 1

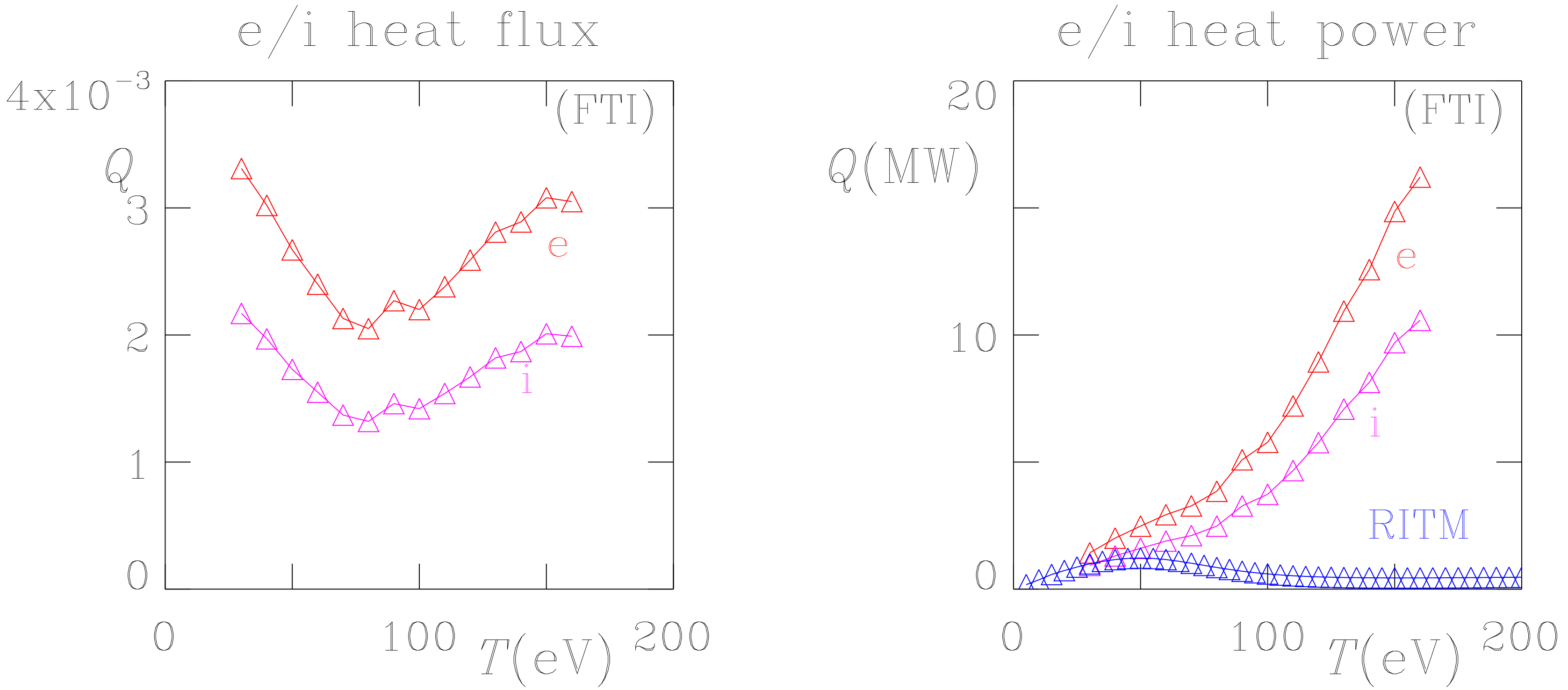




- fluxes as power across the surface in MW for $R = 3a = 300\,\mathrm{cm}$
- peak near $50\,\mathrm{eV}$ not found in nonlinear result
- enter nonlinearly driven large scales which are subdominant as linear instabilities

Transport T-Scaling: GEM vs RITM for the FTI Cases

FTI conditions: $L_n = L_{Ti} = 2 \times L_{Te}$ and $T_i/T_e = 1.2$



- normalised in terms of $p_e\, c_s$ (left), as power through reference surface (right)
- results from DW and ITG and FTI qualitatively similar
- still no L-to-H transition in any of the scenarios

Main Points — Transport Scaling

- trends follow nonlinear, not linear, physics

- for any useful modelling, effects of ion grad-T must be kept...
  - accounts for nonlinearly driven longer wavelengths
  - prevents cutoff of transport towards higher T and grad-T
  - since $\rho_s \sim \rho_i$ this necessitates the gyrofluid model

- no threshold: $R/L_\perp \gg 1$ and each of the gradients drives effectively

- shear flow effects are as expected but not strong enough

- trend with either grad-T or beta always monotonically upward
  - in physical units(!)

no L-to-H transition in fully developed turbulence in local models